John Hermitt
Hw 22

1. i. I did not realize that you do not need $\lambda$ to determine if a matrix is an eigenvector

$$\begin{pmatrix} 6 & 1 \\ -2 & 3 \end{pmatrix}$$

$v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $v_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $v_3 = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$ $v_4 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $v_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 6 & 1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$ Not eigenvector

$\begin{pmatrix} 6 & 1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$ Not an eigenvector

$\begin{pmatrix} 6 & 1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 24 \\ 12 \end{pmatrix}$ Not an eigenvector

$\begin{pmatrix} 6 & 1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 13 \\ -1 \end{pmatrix}$ Not an eigenvector

$\begin{pmatrix} 6 & 1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ Not an eigenvector

$$\begin{pmatrix} 5 & -1 \\ -3 & 1 \end{pmatrix}$$

$\begin{pmatrix} 6 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$

$\begin{pmatrix} 5 & -1 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 29 \\ -17 \end{pmatrix}$ Not an eigenvector

$\begin{pmatrix} 5 & -1 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$ Not an eigenvector

$\begin{pmatrix} 5 & -1 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -17 \\ 11 \end{pmatrix}$ $\lambda = \frac{33}{34}$ is an eigenvector

$\begin{pmatrix} 5 & -1 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 23 \\ -13 \end{pmatrix}$ Not an eigenvector

$\begin{pmatrix} 5 & -1 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ Not an eigenvector

11. I thought it was discrete not continuous

a bacterial population decreases at a rate of one fourth the reciprocal of the square of the current value. $t=1$ the value is 1, what's the value at $t=8$

$$x'(t) = -\frac{1}{4x(t)^2} \qquad x(1)=1$$

$$\frac{dx}{dt} = -\frac{1}{4x^2}$$

$$-4x^2\,dx = dt$$

$$-\frac{4}{3}x^3 + c = t$$

$$-\frac{4}{3}(1)^3 + c = 1$$

$$c = 1 + \frac{4}{3}$$

$$c = \frac{7}{3}$$

$$-\frac{4}{3}x^3 + \frac{7}{3} = t$$

$$x(t) = \sqrt[3]{-\tfrac{3}{4}\left(t - \tfrac{7}{3}\right)}$$

$$x(8) \approx -1.670$$

population is dead at $t=8$

bacterial population increases at a rate of one sixth the reciprocal of the square of the current value. $t=1$ the value is 1, what's the value at $t=8$?

$$x'(t) = \frac{1}{6x(t)^2}$$

$$\frac{dx}{dt} = \frac{1}{6x^2}$$

$$6x^2\,dx = dt$$

$$2x^3 = t + C$$

$$2(1)^3 = 1 + C$$

$$2 = 1 + C$$

$$1 = C$$

$$2x^3 = t + 1$$

$$x(t) = \sqrt[3]{\tfrac{1}{2}(t+1)}$$

$$x(8) \approx 1.651$$

5. $x(n) = \frac{x(n-1)}{10 + x(n-1)}$

$x = \frac{x}{10+x}$

$x(10+x) = x$

$x^2 + 10x = x$

$x^2 + 9x = 0$

$x(x+9) = 0$

$x = 0, -9$

$x' = \frac{(x+10) - x}{(10+x)^2}$

$|x'(0)| = \left|\frac{10}{100}\right| = \frac{1}{10}$ stable

$|x'(-9)| = \left|\frac{-8}{1}\right| = 8$ unstable

```
> #John Hermitt hw22
  read "/John/Rutgers/Senior Fall/Dynamic Models/DMB.txt" :
```

*First Written: Nov. 2021*

*This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)*
*accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)*

*The most current version is available on WWW at:*
*http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .*
*Please report all bugs to: DoronZeil at gmail dot com .*

*For general help, and a list of the MAIN functions,*
*type "Help();". For specific help type "Help(procedure_name);"*

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*For a list of the supporting functions type: Help1();*
*For help with any of them type: Help(ProcedureName);*

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*For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();*
*For help with any of them type: Help(ProcedureName);*

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*For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();*
*For help with any of them type: Help(ProcedureName);*

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```
proc(ODEs::{anything} := NULL, {atomizenames::truefalse := true, build::truefalse := false,      (1)
    type::name := 'none'})
    ...
end proc
```

```
> #2
  #a
  #x(n) = 2x(n-1)+3y(n-1)   x(1) = 20
```

```
#y(n) = 3x(n-1) + y(n-1)   y(1) = 10
#in the 10th year there are 13619620 lynxes and 11537890 hares
 Orb([2·x + 3·y, 3·x + y], [x, y], [20, 10], 8, 9);
```

$$[[2999150, 2540440], [13619620, 11537890], [61852910, 52396750]] \quad \textbf{(2)}$$

```
> #b
  # x'(t) = 2x(t) +3y(t)   x(0)=20
  # y'(t) = 3x(t) + y(t)   y(0) = 10
  #
  sys := diff(x(t), t) = 2·x(t) + 3·y(t), diff(y(t), t) = 3·x(t) + y(t) :
  F := {x(t), y(t)} :
  P := dsolve({sys, x(0) = 20, y(0) = 10}, F, );
  expand(subs(t = 10., P));
```

$$P := \left\{ x(t) = \left(10 + \frac{40\sqrt{37}}{37}\right) e^{\frac{(3+\sqrt{37})t}{2}} + \left(10 - \frac{40\sqrt{37}}{37}\right) e^{-\frac{(-3+\sqrt{37})t}{2}}, y(t) \right.$$

$$= \frac{\left(10 + \frac{40\sqrt{37}}{37}\right) e^{\frac{(3+\sqrt{37})t}{2}} \sqrt{37}}{6} - \frac{\left(10 - \frac{40\sqrt{37}}{37}\right) e^{-\frac{(-3+\sqrt{37})t}{2}} \sqrt{37}}{6}$$

$$\left. - \frac{\left(10 + \frac{40\sqrt{37}}{37}\right) e^{\frac{(3+\sqrt{37})t}{2}}}{6} - \frac{\left(10 - \frac{40\sqrt{37}}{37}\right) e^{-\frac{(-3+\sqrt{37})t}{2}}}{6} \right\}$$

$$\{x(10.) = 3.269017372 \times 10^7 e^{5.000000000\sqrt{37}} + 3.534072835 \times 10^6 \sqrt{37}\, e^{5.000000000\sqrt{37}} \quad \textbf{(3)}$$

$$+ 3.269017372 \times 10^7 e^{-5.000000000\sqrt{37}} - 3.534072835 \times 10^6 \sqrt{37}\, e^{-5.000000000\sqrt{37}}, y(10.)$$

$$= 4.859350148 \times 10^6 \sqrt{37}\, e^{5.000000000\sqrt{37}} + 1.634508686 \times 10^7 e^{5.000000000\sqrt{37}}$$

$$- 4.859350148 \times 10^6 \sqrt{37}\, e^{-5.000000000\sqrt{37}} + 1.634508686 \times 10^7 e^{-5.000000000\sqrt{37}} \}$$

```
> #3
  #a = 6, b = .1, c = .2
  #R0 = 20
   a := 6;
  b := 0.1;
  c := 0.2;
  Eq1 := x·(1 − b − c) + y·(1 exp(-a·x));
  Eq2 := (1 − y)·b + y·exp(-a·x);
  evalf(OrbF([evalf(Eq1), evalf(Eq2)], [x, y], [69, 420], 1000, 1001));
```

$$a := 6$$

$$b := 0.1$$

$$c := 0.2$$

$$Eq1 := 0.7\,x + y\,e^{-6x}$$

$$Eq2 := 0.1 - 0.1\,y + y\,e^{-6x}$$

[[0.1669970519, 0.1364537415], [0.1669970519, 0.1364537415], [0.1669970519, 0.1364537415]] **(4)**

```
> #4
  evalf(OrbF([x·(1 − b) + (1 − x)·(1 − exp(−a·x))], [x], [5], 1000, 1001));
  xasterisk := b·x − (1 − x)·(1 − exp(−a·x)) = 0;
  eval(xasterisk, x = 0);
```

[[0.9087353125], [0.9087353124], [0.9087353125]]

$$xasterisk := 0.1\,x - (1 - x)\left(1 - e^{-6x}\right) = 0$$

$$0. = 0$$ **(5)**

```
>
```