Homework 22

Question 1

For first question on the attendance quiz, I didn't get it wrong but since I didn't do it the way it was intended I will redo a similar question. I can only say that I didn't use the $A\vec{v} = \lambda\vec{v}$ method due to it slipping my mind, but as soon as Dr. Z was solving it in class I realized my mistake.

For each of the following vectors in R^2

$$v_1 = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \quad v_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad v_3 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad v_4 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad v_5 = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

decide whether or not they are an eigenvector of the matrix

$$A = \begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$$

If it is, find corresponding eigenvalues

Using $A\vec{v} = \lambda\vec{v}$

1. v_1 $\begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 20 \\ 10 \end{pmatrix} = 5\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ $\lambda = 5$ ✓ eigenvector of A

2. v_2 $\begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 18 \\ 9 \end{pmatrix}$ not a scalar multiple of $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ so not an eigenvector of A

3. v_3 $\begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 14 \\ 7 \end{pmatrix}$ not a scalar multiple of $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ so not an eigenvector of A

4. v_4 (Note: $\overset{v_1}{\begin{pmatrix}4\\2\end{pmatrix}}$ is a $\overset{\text{non-zero}}{\vee}$ multiple of $\overset{v_4}{\begin{pmatrix}2\\1\end{pmatrix}}$ so since v_1 was an eigenvector then so should v_4, we'll calculate anyway)

$$\begin{pmatrix}2 & 6\\1 & 3\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}10\\5\end{pmatrix} = 5\begin{pmatrix}2\\1\end{pmatrix} \quad \lambda=5 \checkmark \text{ eigenvector of } A$$

5. v_5 $\begin{pmatrix}2 & 6\\1 & 3\end{pmatrix}\begin{pmatrix}-3\\1\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} = 0\begin{pmatrix}-3\\1\end{pmatrix}$ $\lambda=0 \checkmark$ eigenvector of A

Same Question Different Eigenvectors & Matrix

$v_1 = \begin{pmatrix}2\\1\end{pmatrix}$ $v_2 = \begin{pmatrix}3\\-6\end{pmatrix}$ $v_3 = \begin{pmatrix}1\\1\end{pmatrix}$ $v_4 = \begin{pmatrix}-1\\1\end{pmatrix}$ $v_5 = \begin{pmatrix}5\\2\end{pmatrix}$

$$A = \begin{pmatrix}-5 & -2\\-1 & -4\end{pmatrix}$$

1. v_1 $\begin{pmatrix}-5 & -2\\-1 & -4\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}-12\\-6\end{pmatrix} = -6\begin{pmatrix}2\\1\end{pmatrix}$ $\lambda=-6$ ✓

Yes, eigenvector of A

2. v_2 $\begin{pmatrix}-5 & -2\\-1 & -4\end{pmatrix}\begin{pmatrix}3\\-6\end{pmatrix} = \begin{pmatrix}-3\\21\end{pmatrix}$ not a scalar multiple of $\begin{pmatrix}3\\-6\end{pmatrix}$ so no not an eigenvector of A

3. v_3 $\begin{pmatrix}-5 & -2\\-1 & -4\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}-7\\-5\end{pmatrix}$ not a scalar multiple of $\begin{pmatrix}1\\1\end{pmatrix}$ so no not an eigenvector of A

4. v_4 $\begin{pmatrix}-5 & -2\\-1 & -4\end{pmatrix}\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}3\\-3\end{pmatrix} = -3\begin{pmatrix}-1\\1\end{pmatrix}$ $\lambda=-3$ ✓

Yes, an eigenvector of A

5. v_5 $\begin{pmatrix}-5 & -2\\-1 & -4\end{pmatrix}\begin{pmatrix}5\\2\end{pmatrix} = \begin{pmatrix}-29\\-13\end{pmatrix}$ not a scalar multiple of $\begin{pmatrix}5\\2\end{pmatrix}$ so no not an eigenvector of A

My mistake on Number 2 of the attendance was a careless integration error, which due to lack of time I didn't catch and fix.

The population of a certain bacteria increases at a rate that equal to a half of the reciprocal of its current value. If time $t=1$ value is 1 what is value at $t=16$

$$x'(t) = \frac{1}{2x(t)} \qquad x(1)=1$$

$$\frac{dx}{dt} = \frac{1}{2x}$$

$$\int 2x\,dx = \int dt$$

$$x^2 = t + C \rightarrow 1^2 = 1 + C \quad C = 0$$

$$x^2 = t$$

$$x = \sqrt{t}$$

$$x(t) = \sqrt{t} \quad \text{at } t=16 \quad \boxed{x(16) = \sqrt{16} = 4}$$

Same premise but now the population decreases at a rate equal to a quarter of its current value. At time $t=1$ the value is 4 what's the value at $t=3$.

$$x'(t) = -\frac{x(t)}{4}$$

$$\frac{dx}{dt} = -\frac{x}{4} \Rightarrow -4\int \frac{1}{x}\,dx = \int dt$$

$$-4\ln x = t + C$$

$$-4\ln 4 = 1 + C$$

$$c = -4\ln 4 - 1 \approx -6.545$$

$$-4\ln x = t - 6.545$$

$$\ln x = -\frac{t}{4} + 1.63625$$

$$x = e^{-t/4} \cdot e^{1.63625} \quad \text{at } t=3 \quad x = e^{-3/4} \cdot e^{1.63625}$$

$$\boxed{x(3) = 2.426}$$

```
> #Ok to post Homework
> #Jeton Hida, Assignment 22, November 22, 2021

> #Question 2

> #a. Number of lynxes will be denoted by x(n), number of hares will
  be denoted by y(n). We use difference equations, as this is not in
  discrete time, we are not talking about a rate of change, but
  instead we are discussing the number of animals by years.

> #For lynxes -> x(n) = 2*x(n-1) + 3*y(n-1)
  #For hares -> y(n) = 3*x(n-1) + y(n-1)

> F:=[2*x+3*y,3*x+y]
```

$$F := [2x+3y, 3x+y] \tag{1}$$

```
> read "/Users/jeton/Desktop/Math 336/DMB.txt"
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ (2)

```
> Help(Orb)
```

Orb(F,x,x0,K1,K2): Inputs a transformation F in the list of variables x with initial point pt, outputs the trajectory of

of the discrete dynamical system (i.e. solutions of the difference equation): x(n)=F(x(n-1)) with x(0)=x0 from n=K1 to n=K2.

For the full trajectory (from n=0 to n=K2), use K1=0. Try:

*Orb(5/2*x*(1-x),[x], [0.5], 1000,1010);*

*Orb([(1+x+y)/(2+x+y),(6+x+y)/(2+4*x+5*y)],[x,y], [2.,3.], 1000,1010);* (3)

```
> Orb(F,[x,y],[20,10],9,10)[2]
```

$$[61852910, 52396750] \quad (4)$$

```
> #Number of lynxes and hares present at year 10, respectively.
```

```
> #b. Continous differential equation because we are asked about the
  rate of change of a population within a year. Anytime rate is
  mentioned it is CONTINUOUS. Lynx denoted by x(t), hares denoted by
  y(t)
```

```
> #x'(t) = 2*x(t)+3*y(t)
  #y'(t) = 3*x(t)+y(t)
```

```
> dsolve({diff(x(t),t)=2*x(t)+3*y(t),diff(y(t),t)=3*x(t)+y(t),x(0)=
  20,y(0)=10},{x(t),y(t)})
```

$$\left\{x(t) = \left(10+\frac{40\sqrt{37}}{37}\right)e^{\frac{(3+\sqrt{37})t}{2}} + \left(10-\frac{40\sqrt{37}}{37}\right)e^{-\frac{(-3+\sqrt{37})t}{2}}, y(t) = -\frac{\left(10-\frac{40\sqrt{37}}{37}\right)e^{-\frac{(-3+\sqrt{37})t}{2}}\sqrt{37}}{6} + \frac{\left(10+\frac{40\sqrt{37}}{37}\right)e^{\frac{(3+\sqrt{37})t}{2}}\sqrt{37}}{6} - \frac{\left(10-\frac{40\sqrt{37}}{37}\right)e^{-\frac{(-3+\sqrt{37})t}{2}}}{6} - \frac{\left(10+\frac{40\sqrt{37}}{37}\right)e^{\frac{(3+\sqrt{37})t}{2}}}{6}\right\} \quad (5)$$

```
> %[1]
```

$$x(t) = \left(10+\frac{40\sqrt{37}}{37}\right)e^{\frac{(3+\sqrt{37})t}{2}} + \left(10-\frac{40\sqrt{37}}{37}\right)e^{-\frac{(-3+\sqrt{37})t}{2}} \quad (6)$$

```
> evalf(subs(t=10,%))
```

$$x(10) = 8.758846449\ 10^{20} \quad (7)$$

```
> dsolve({diff(x(t),t)=2*x(t)+3*y(t),diff(y(t),t)=3*x(t)+y(t),x(0)=
  20,y(0)=10},{x(t),y(t)})[2]
```

$$y(t) = -\frac{\left(10-\frac{40\sqrt{37}}{37}\right)e^{-\frac{(-3+\sqrt{37})t}{2}}\sqrt{37}}{6} + \frac{\left(10+\frac{40\sqrt{37}}{37}\right)e^{\frac{(3+\sqrt{37})t}{2}}\sqrt{37}}{6} \quad (8)$$

$$-\frac{\left(10-\frac{40\sqrt{37}}{37}\right)e^{-\frac{(-3+\sqrt{37})t}{2}}}{6}-\frac{\left(10+\frac{40\sqrt{37}}{37}\right)e^{\frac{(3+\sqrt{37})t}{2}}}{6}$$

```
> evalf(subs(t=10,%))
```

$$y(10) = 7.419856091\ 10^{20} \tag{9}$$

```
> #These are the numbers of lynxes and hares at t=10, respectively.
```

```
> #Number 3
```

```
> #0 < b+c ≤ 1, 0 < a, 0 < b, 0 < c, 0 < x0 + y0 ≤ 1, 0 < x0, 0 < y0,
  R0 = a/(b+c)
```

```
> F:=AllenSIR(2,.3,.4,x,y)
```

$$F := \left[0.3\,x + y\left(1 - e^{-2\,x}\right), 0.3 - 0.3\,y + y\,e^{-2\,x}\right] \tag{10}$$

```
> OrbF(F,[x,y],[.3,.4],1000,1010)
```

(11) [[0.2397464890, 0.4405915258], [0.2397464890, 0.4405915258], [0.2397464890, 0.4405915258], [0.2397464890, 0.4405915258], [0.2397464890, 0.4405915258], [0.2397464890, 0.4405915258], [0.2397464890, 0.4405915258], [0.2397464890, 0.4405915258], [0.2397464890, 0.4405915258], [0.2397464890, 0.4405915258], [0.2397464890, 0.4405915258]]

```
> F:=AllenSIR(1,.2,.3,x,y)
```

$$F := \left[0.5\,x + y\left(1 - e^{-x}\right), 0.2 - 0.2\,y + y\,e^{-x}\right] \tag{12}$$

```
> OrbF(F,[x,y],[.2,.5],1000,1010)
```

(13) [[0.1813203172, 0.5466992068], [0.1813203172, 0.5466992068], [0.1813203172, 0.5466992068], [0.1813203172, 0.5466992068], [0.1813203172, 0.5466992068], [0.1813203172, 0.5466992068], [0.1813203172, 0.5466992068], [0.1813203172, 0.5466992068], [0.1813203172, 0.5466992068], [0.1813203172, 0.5466992068], [0.1813203172, 0.5466992068]]

```
> 1-.1813203172*(1+.3/.2)=.546692068
```

$$0.5466992070 = 0.546692068 \tag{14}$$

```
> (.2+.3)*.1813203172=.5466992068*(1-exp(-1*.1813203172))
```

$$0.09066015860 = 0.0906601586 \tag{15}$$

```
> 1-.2397464890*(1+.4/.3)=.440591528
```

$$0.4405915257 = 0.440591528 \tag{16}$$

```
> (.3+.4)*.2397464890=.440591528*(1-exp(-2*.2397464890))
```

$$0.1678225423 = 0.1678225431 \tag{17}$$

```
> #Conjecture 1 said that if R0 = a/(b+c) > 1 then as we take the
  limit, or in our case, go through many iterations of the AllenSIR
  transformation with starting points 0 < x0 + y0 ≤ 1 we will reach a
  fixed point called [x*,y*]. This fixed point will be the solutions
  to the equation where (b+c)x*=y*(1-exp(-ax*) and y*=1-x*(1+c/b).
  Which I have showed. I will now show when R0 < 1 then we will get
  close to point[0,1].
```

```
> F:=AllenSIR(.3,.2,.5,x,y)
```

$$F := [0.3\,x + y\,(1 - e^{-0.3\,x}),\, 0.2 - 0.2\,y + y\,e^{-0.3\,x}] \quad (18)$$

```
> OrbF(F,[x,y],[.5,.3],1000,1010)
```

$$[[2.\ 10^{-10}, 0.9999999993], [2.\ 10^{-10}, 0.9999999993], [2.\ 10^{-10}, 0.9999999993], [2.\ 10^{-10}, 0.9999999993], [2.\ 10^{-10}, 0.9999999993], [2.\ 10^{-10}, 0.9999999993], [2.\ 10^{-10}, 0.9999999993], [2.\ 10^{-10}, 0.9999999993], [2.\ 10^{-10}, 0.9999999993], [2.\ 10^{-10}, 0.9999999993], [2.\ 10^{-10}, 0.9999999993], [2.\ 10^{-10}, 0.9999999993]] \quad (19)$$

```
> #With this I confirm the claims of Conjecture 1
```

```
> #Number 4
```

```
> #Conjecture 2 If R0 = a/b > 1, then the solution x(n)=x(n-1)*(1-b)+
  (1-x(n-1))*(1-exp(-a*x(n-2)) satisfies as the limit as 'n' goes to
  infinity x(n) = x* is the positive solution of
  #                                            bx*=(1-x*)(1-exp(-ax*))
  #where 0 < b < 1, 0 < a and 0 < x0, x1 < 1
```

```
> F:=[z[1]*(1-b)+(1-z[1])*(1-exp(-a*z[2]))]
```

$$F := \left[z_1\,(1-b) + (1-z_1)\left(1 - e^{-a\,z_2}\right)\right] \quad (20)$$

```
> G:=z[1]*(1-.4)+(1-z[1])*(1-exp(-2.*z[2]))
```

$$G := 0.6\,z_1 + (1-z_1)\left(1 - e^{-2.\,z_2}\right) \quad (21)$$

```
> HelpDDM()
```

The procedures giving discrete-time dynamical systems (some famous), by giving the the underlying transformations, followed by the list of variables used are:

AllenSIR, Hassell, HW, HWg, May75, NicholsonBailey, RT, Valery **(22)**

```
> HelpCDM()
```

The procedures giving the underlying transformations, followed by the list of variables used are:

ChemoStat, GeneNet, Lotka, RandNice, SIRS , SIRSdemo, Volterra, VolterraM **(23)**

```
> F:=ToSys(2,z,G)[1]
```

$$F := \left[0.6\,z_1 + (1-z_1)\left(1 - e^{-2.\,z_2}\right), z_1\right] \quad (24)$$

```
> OrbF(F,[z[1],z[2]],[.2,.3],1000,1010)
```

$$[[0.6442235141, 0.6442235144], [0.6442235144, 0.6442235141], [0.6442235141, 0.6442235144], [0.6442235144, 0.6442235141], [0.6442235141, 0.6442235144], [0.6442235144, 0.6442235141], [0.6442235141, 0.6442235144], [0.6442235144, 0.6442235141], [0.6442235141, 0.6442235144], [0.6442235144, 0.6442235141], [0.6442235141, 0.6442235144], [0.6442235144, 0.6442235141]] \quad (25)$$

```
> w1:=OrbF(F,[z[1],z[2]],[.2,.3],1000,1010)[1][1]:
  b:=.4:
> a:=2:
> b*w1=(1-w1)*(1-exp(-a*w1))
```

$$0.2576894056 = 0.2576894058 \quad (26)$$

```
> G:=z[1]*(1-.234)+(1-z[1])*(1-exp(-1.421*z[2]))
```

$$G := 0.766\, z_1 + (1 - z_1)\left(1 - e^{-1.421\, z_2}\right) \quad (27)$$

```
> F:=ToSys(2,z,G)[1]
```

$$F := \left[0.766\, z_1 + (1 - z_1)\left(1 - e^{-1.421\, z_2}\right), z_1\right] \quad (28)$$

```
> OrbF(F,[z[1],z[2]],[.333,.721],1000,1010)
```

[[0.7346784076, 0.7346784076], [0.7346784076, 0.7346784076], [0.7346784076, 0.7346784076], [0.7346784076, 0.7346784076], [0.7346784076, 0.7346784076], [0.7346784076, 0.7346784076], [0.7346784076, 0.7346784076], [0.7346784076, 0.7346784076], [0.7346784076, 0.7346784076], [0.7346784076, 0.7346784076], [0.7346784076, 0.7346784076]] **(29)**

```
> w2:=OrbF(F,[z[1],z[2]],[.333,.721],1000,1010)[1][1]:
> b:=.234:
> a:=1.421:
> b*w2=(1-w2)*(1-exp(-a*w2))
```

$$0.1719147474 = 0.1719147474 \quad (30)$$

```
> G:=z[1]*(1-.899)+(1-z[1])*(1-exp(-1.001*z[2]))
```

$$G := 0.101\, z_1 + (1 - z_1)\left(1 - e^{-1.001\, z_2}\right) \quad (31)$$

```
> F:=ToSys(2,z,G)[1]
```

$$F := \left[0.101\, z_1 + (1 - z_1)\left(1 - e^{-1.001\, z_2}\right), z_1\right] \quad (32)$$

```
> OrbF(F,[z[1],z[2]],[.973,.427],1000,1010)
```

[[0.0700449823, 0.0700449823], [0.0700449823, 0.0700449823], [0.0700449823, 0.0700449823], [0.0700449823, 0.0700449823], [0.0700449823, 0.0700449823], [0.0700449823, 0.0700449823], [0.0700449823, 0.0700449823], [0.0700449823, 0.0700449823], [0.0700449823, 0.0700449823], [0.0700449823, 0.0700449823], [0.0700449823, 0.0700449823]] **(33)**

```
> w3:=OrbF(F,[z[1],z[2]],[.973,.427],1000,1010)[1][1]:
> b:=.899:
> a:=1.001:
> b*w3=(1-w3)*(1-exp(-a*w3))
```

$$0.06297043909 = 0.06297043907 \quad (34)$$

```
> #Have shown 3 times the conjecture holds.

> #Number 5

> forget(F,forgetpermanent=true)
> forget(x,forgetpermanent=true)
> F:=[z/(10+z)]
```

$$F := \left[\frac{z}{10 + z}\right] \quad (35)$$

```
> FP(F,[z])
```

$$\{[-9], [0]\} \quad (36)$$

```
> SFP(F,[z])
```

(37)

$$\{[0.]\}\tag{37}$$

```
> OrbF(F,[z],[2],1000,1010)
```

$$[[1.636363637\ 10^{-1000}], [1.636363637\ 10^{-1001}], [1.636363637\ 10^{-1002}], [1.636363637\ 10^{-1003}], [1.636363637\ 10^{-1004}], [1.636363637\ 10^{-1005}], [1.636363637\ 10^{-1006}], [1.636363637\ 10^{-1007}], [1.636363637\ 10^{-1008}], [1.636363637\ 10^{-1009}], [1.636363637\ 10^{-1010}], [1.636363637\ 10^{-1011}]]\tag{38}$$

```
> OrbF(F,[z],[3],1000,1010)
```

$$[[2.250000000\ 10^{-1000}], [2.250000000\ 10^{-1001}], [2.250000000\ 10^{-1002}], [2.250000000\ 10^{-1003}], [2.250000000\ 10^{-1004}], [2.250000000\ 10^{-1005}], [2.250000000\ 10^{-1006}], [2.250000000\ 10^{-1007}], [2.250000000\ 10^{-1008}], [2.250000000\ 10^{-1009}], [2.250000000\ 10^{-1010}], [2.250000000\ 10^{-1011}]]\tag{39}$$

```
> OrbF(F,[z],[-9.1],1000,1010)
```

$$[[8.190000015\ 10^{-998}], [8.190000015\ 10^{-999}], [8.190000015\ 10^{-1000}], [8.190000015\ 10^{-1001}], [8.190000015\ 10^{-1002}], [8.190000015\ 10^{-1003}], [8.190000015\ 10^{-1004}], [8.190000015\ 10^{-1005}], [8.190000015\ 10^{-1006}], [8.190000015\ 10^{-1007}], [8.190000015\ 10^{-1008}], [8.190000015\ 10^{-1009}]]\tag{40}$$

```
> OrbF(F,[z],[-9],1000,1010)
```

$$[[-9.000000000], [-9.000000000], [-9.000000000], [-9.000000000], [-9.000000000], [-9.000000000], [-9.000000000], [-9.000000000], [-9.000000000], [-9.000000000], [-9.000000000], [-9.000000000]]\tag{41}$$

```
> OrbF(F,[z],[-20],1000,1010)
```

$$[[1.636363637\ 10^{-999}], [1.636363637\ 10^{-1000}], [1.636363637\ 10^{-1001}], [1.636363637\ 10^{-1002}], [1.636363637\ 10^{-1003}], [1.636363637\ 10^{-1004}], [1.636363637\ 10^{-1005}], [1.636363637\ 10^{-1006}], [1.636363637\ 10^{-1007}], [1.636363637\ 10^{-1008}], [1.636363637\ 10^{-1009}], [1.636363637\ 10^{-1010}]]\tag{42}$$