Dyn Mod Bio Hrudai Batthini Hw 22

D I got #2 incorrect, I did not realize it was continuous time.

$x'(t) = \frac{1}{2x(t)^3}$ $x(1) = 1$ $x(5) = ?$

$\int x^3\,dx = \int \frac{1}{2}\,dt$ $\frac{x^4}{4} = t + C$ $\frac{x^4}{2} = t + C$ $\frac{1^4}{2} = 1 + C$ $C = -\frac{1}{2}$

$x^4 = 2t - 1$ $x = \sqrt[4]{2t-1}$ $x(5) = \sqrt[4]{9}$ #

$x'(t) = 3x(t)^2$ $x(1) = 2$ $\frac{dx}{dt} = 3x^2$ $\int \frac{dx}{3x^2} = \int dt$ $x(4) = ?$

$-\frac{1}{3x} = t + C$ $-\frac{1}{6} = 2 + C$ $C = \frac{11}{6}$ $-\frac{1}{3x} = t + \frac{11}{6}$

$\frac{1}{x} = -3t + \frac{11}{2}$ $x(t) = -\frac{1}{3t} + \frac{2}{11}$ $x(4) = -\frac{1}{12} + \frac{2}{11} = -\frac{35}{121}$ #

```
> #Hrudai Battini HW 22
  read "/Users/hb334/Documents/DMB.txt";
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

```
> #2a x = lynxes, y = hares
  #x(n) = 2x(n-1)+3y(n-1), x(1)=20
  #y(n) = 3x(n-1)+y(n-1), y(1)=10
  Orb([2*x+3*y,3*x+y],[x,y],[20,10],9,10)[2];
```

[61852910, 52396750] **(2)**

```
> #2b x = lynxes, y = hares
  #x'(t)=2x(t)+3y(t), x(0)=20
  #y'(t)=3x(t)+y(t), y(0)=10
  S := diff(x(t),t)=2*x(t)+3*y(t),diff(y(t),t)=3*x(t)+y(t);
  F := {x(t),y(t)};
  L:=dsolve({S,x(0)=20.,y(0)=10.},F);
  expand(subs(t=10.,L));
```

$$S := \frac{d}{dt}\, x(t) = 2\, x(t) + 3\, y(t),\ \frac{d}{dt}\, y(t) = 3\, x(t) + y(t)$$

$$F := \{x(t), y(t)\}$$

$$L := \left\{ x(t) = \left(10 + \frac{40\sqrt{37}}{37}\right) e^{\frac{(3+\sqrt{37})\,t}{2}} + \left(10 - \frac{40\sqrt{37}}{37}\right) e^{-\frac{(-3+\sqrt{37})\,t}{2}}, y(t) = \frac{\left(10 + \frac{40\sqrt{37}}{37}\right) e^{\frac{(3+\sqrt{37})\,t}{2}} \sqrt{37}}{6} - \frac{\left(10 - \frac{40\sqrt{37}}{37}\right) e^{-\frac{(-3+\sqrt{37})\,t}{2}} \sqrt{37}}{6} - \frac{\left(10 + \frac{40\sqrt{37}}{37}\right) e^{\frac{(3+\sqrt{37})\,t}{2}}}{6} - \frac{\left(10 - \frac{40\sqrt{37}}{37}\right) e^{-\frac{(-3+\sqrt{37})\,t}{2}}}{6} \right\}$$

$$\{x(10.) = 3.269017372 \times 10^{7}\, e^{5.000000000\sqrt{37}} + 3.534072835 \times 10^{6} \sqrt{37}\, e^{5.000000000\sqrt{37}} + 3.269017372 \times 10^{7}\, e^{-5.000000000\sqrt{37}} - 3.534072835 \times 10^{6} \sqrt{37}\, e^{-5.000000000\sqrt{37}}, y(10.) = 4.859350148 \times 10^{6} \sqrt{37}\, e^{5.000000000\sqrt{37}} + 1.634508686 \times 10^{7}\, e^{5.000000000\sqrt{37}} - 4.859350148 \times 10^{6} \sqrt{37}\, e^{-5.000000000\sqrt{37}} + 1.634508686 \times 10^{7}\, e^{-5.000000000\sqrt{37}}\} \quad (3)$$

```
> #3 Proving Conjecture 1
  #a = 2, b = 0.3, c = 0.3; Ro = 2/(0.3+0.3) > 1
  #x(1) =100,, y(1) =10;
  a:=2;
  b:=0.3;
  c:=0.3;
  E1:= x*(1-b-c)+y*(1-exp(-a*x));
  E2:=(1-y)*b+y*exp(-a*x);
  evalf(OrbF([evalf(E1),evalf(E2)],[x,y],[100,10],100,101));
  #Extremely inefficient calculations performed by the code,
  impossible #to calculate past 5 without many hours invested in
  pure computation.
```

$$a := 2$$

$$b := 0.3$$

$$c := 0.3$$

$$E1 := 0.4\, x + y\left(1 - e^{-2x}\right)$$

$$E2 := 0.3 - 0.3\, y + y\, e^{-2x}$$

[[0.3004465422, 0.3991069157], [0.3004465422, 0.3991069157], [0.3004465422, 0.3991069157]] **(4)**

```
> #4 Proving Conjecture 2
  evalf(OrbF([x*(1-b)+(1-x)*(1-exp(-a*x))],[x],[2],100,101))
  ;#Extremely inefficient calculations performed by the code,
  impossible #to calculate past 5 without many hours invested in
  pure computation.
```

```
xstar := b*x-(1-x)*(1-exp(-a*x))=0;
eval(xstar,x=0);
```

$$[[0.7174810314], [0.7174810314], [0.7174810314]]$$

$$xstar := 0.3\,x - (1-x)\left(1-e^{-2\,x}\right) = 0$$

$$0. = 0 \tag{5}$$

```
> #5
F:= x/(10+x);
FP([F],[x]);
SFP([F],[x]);
```

$$F := \frac{x}{10+x}$$

$$\{[-9], [0]\}$$

$$\{[0.]\} \tag{6}$$

```
>
```