#not okay to post

Anusha Nagar, Homework 22, 11.21.2021

MAPLE Code attached to this doc

(1) For Question 1 on the Attendence quiz, it is a concept I'm very familiar with, but in the moment I had just blanked on the relationship ($A\vec{v} = \lambda\vec{v}$). At the end, I had finally remembered it (+ even wrote it down at the top!) but time was up for the attendence quiz. Regardless, I'll make up two similar problems below.

(i) $\vec{A} = \begin{bmatrix} 1 & 7 \\ -1 & -7 \end{bmatrix}$, $\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\vec{v}_3 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, $\vec{v}_4 = \begin{pmatrix} -7 \\ 1 \end{pmatrix}$, $\vec{v}_5 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

$\vec{A}\vec{v}_1 = \begin{pmatrix} 1 & 7 \\ -1 & -7 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 8 \\ -8 \end{pmatrix}$ ×

$A\vec{v}_2 = \begin{bmatrix} 1 & 7 \\ -1 & -7 \end{bmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ -9 \end{pmatrix}$ ×

$A\vec{v}_3 = \begin{pmatrix} 1 & 7 \\ -1 & -7 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 6 \end{pmatrix} \Rightarrow$ yes, $\lambda = 6$

$A\vec{v}_4 = \begin{bmatrix} 1 & 7 \\ -1 & -7 \end{bmatrix}\begin{pmatrix} -7 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \lambda = \emptyset$

$A\vec{v}_5 = \begin{bmatrix} 1 & 7 \\ -1 & -7 \end{bmatrix}\begin{bmatrix} -1 \\ -2 \end{bmatrix} = \begin{pmatrix} -10 \\ 15 \end{pmatrix}$ ×

(ii) $\vec{A} = \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}$, $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $\vec{v}_3 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\vec{v}_4 = \begin{bmatrix} -3 \\ 6 \end{bmatrix}$, $\vec{v}_5 = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$

$Av_1 = \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \Rightarrow \lambda = 3$

$Av_2 = \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -9 \\ 9 \end{bmatrix} \Rightarrow \lambda = 9$

$Av_3 = \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \end{bmatrix} \Rightarrow$ ×

$A\vec{v}_4 = \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}\begin{bmatrix} -3 \\ 6 \end{bmatrix} = \begin{bmatrix} -36 \\ 27 \end{bmatrix}$ ×

$Av_5 = \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}\begin{bmatrix} 7 \\ 5 \end{bmatrix} = \begin{bmatrix} 21 \\ 9 \end{bmatrix}$ ×

For the second question, I forgot about continuous time. We spent so long on discrete time that my mind always defaults there instead of continuous time.

(i) Population decreases @ rate 4 * current value squared

$x'(t) = -4x(t)^2$

(ii) Pop. increases @ rate 5 times reciprocal of current value

$x'(t) = 5x(t)$

(2) a. Lynxes → x

Hares → y

$x(n) = 2x(n-1) + 3y(n-1)$

$y(n) = 3x(n-1) + y(n-1)$

$x(0) = 20, \ y(0) = 10$

Find $x(10), y(10)$

Year 10:

$x(10) = 6.185 \times 10^7,$

$y(10) = 5.2397 \times 10^7)$

(b) x → Lynxes

y → hares

$x'(t) = 2x(t) + 3y(t)$

$y'(t) = 3x(t) + y(t)$

(3) $\left.\begin{aligned}(b+c)x^* &= y^*(1-\exp(-ax^*)) \\ y^* &= 1 - x^*(1+\tfrac{c}{b})\end{aligned}\right\}$

$R_0 \approx \frac{a}{b+c} > 1$

$x(n+1) = x_n(1-b-c) + y_n(1-\exp(-ax_n))$

$y(n+1) = (1-y(n))(b) + y(n)\exp(-ax_n)$

$0 < b+c \leq 1, \ 0 < a$

(4) $0 < b < 1, \ a > 0$

$bx^* = (1-x^*)(1-\exp(-ax^*))$

$x(n) = x(n-1)(1-b) + (1-x(n-1))(1-\exp(-a\,x(n-2)))$

$z[1](1-b) + (1-z[1])(1-\exp(-a\,z[2]))$

(5) $x(n) = \dfrac{x(n-1)}{10 + x(n-1)}$

Fixed points:

$z = \dfrac{z}{10+z}$

$z(10+z) = z$

$10z + z^2 - z = 0$

$z^2 + 9z = 0$

$z(z+9) = 0 \Rightarrow z = 0, -9$

FP: $\{0, -9\}$, SFP: $= \{0\}$

Stable fixed points:

$F(x) = \dfrac{x}{10+x} \Rightarrow F'(x) = \dfrac{1}{10+x} - \dfrac{x}{(10+x)^2}$

$F'(0) = \frac{1}{10} - \frac{0}{(10)^2} = \frac{1}{10} \Rightarrow$ stable

$F'(-9) = \frac{1}{1} - \frac{-9}{1} = 10$

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>

>

> **read** "C://Users/an646/Documents/DMB.txt"

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

> #Problem 1 done on paper (attached)

> #Problem 2

> #Part a where x is lynxes and y is hares

> *Help(Orb)*

Orb(F,x,x0,K1,K2): Inputs a transformation F in the list of variables x with initial point pt, outputs the trajectory of
of the discrete dynamical system (i.e. solutions of the difference equation): x(n)=F(x(n-1)) with x(0)=x0 from n=K1 to n=K2.
For the full trajectory (from n=0 to n=K2), use K1=0. Try:
*Orb(5/2*x*(1-x),[x], [0.5], 1000,1010);*

Orb([(1+x+y)/(2+x+y),(6+x+y)/(2+4*x+5*y),[x,y], [2.,3.], 1000,1010); **(2)**

> $Orb([2\cdot x+3\cdot y, 3\cdot x+y], [x, y], [20., 10.], 0, 10)$

$[[20., 10.], [70., 70.], [350., 280.], [1540., 1330.], [7070., 5950.], [31990., 27160.], [145460., 123130.], [660310., 559510.], [2.999150\ 10^{6}, 2.540440\ 10^{6}], [1.3619620\ 10^{7}, 1.1537890\ 10^{7}], [6.1852910\ 10^{7}, 5.2396750\ 10^{7}], [2.80896070\ 10^{8}, 2.37955480\ 10^{8}]]$ **(3)**

> # $x(10) = 6.1853 \times 10^{7}$ lynxes, $y(10) = 5.2397 \times 10^{7}$ hares

>

> #Part b where x is lynxes, y is hares

> $sys := diff(x(t), t) = 2\cdot x(t) + 3\cdot y(t), diff(y(t), t) = 3\cdot x(t) + y(t)$

$$sys := \frac{d}{dt}\, x(t) = 2\, x(t) + 3\, y(t),\ \frac{d}{dt}\, y(t) = 3\, x(t) + y(t) \quad \textbf{(4)}$$

> $dsolve([sys, x(0) = 20, y(0) = 10])$

$$\left\{x(t) = \left(10 + \frac{40\sqrt{37}}{37}\right) e^{\frac{(3+\sqrt{37})\, t}{2}} + \left(10 - \frac{40\sqrt{37}}{37}\right) e^{-\frac{(-3+\sqrt{37})\, t}{2}}, y(t) = \frac{\left(10 + \frac{40\sqrt{37}}{37}\right) e^{\frac{(3+\sqrt{37})\, t}{2}} \sqrt{37}}{6} - \frac{\left(10 - \frac{40\sqrt{37}}{37}\right) e^{-\frac{(-3+\sqrt{37})\, t}{2}} \sqrt{37}}{6} - \frac{\left(10 + \frac{40\sqrt{37}}{37}\right) e^{\frac{(3+\sqrt{37})\, t}{2}}}{6} - \frac{\left(10 - \frac{40\sqrt{37}}{37}\right) e^{-\frac{(-3+\sqrt{37})\, t}{2}}}{6}\right\} \quad \textbf{(5)}$$

> $subs(t = 10, \%)$

$$\left\{x(10) = \left(10 + \frac{40\sqrt{37}}{37}\right) e^{15+5\sqrt{37}} + \left(10 - \frac{40\sqrt{37}}{37}\right) e^{15-5\sqrt{37}}, y(10) = \frac{\left(10 + \frac{40\sqrt{37}}{37}\right) e^{15+5\sqrt{37}} \sqrt{37}}{6} - \frac{\left(10 - \frac{40\sqrt{37}}{37}\right) e^{15-5\sqrt{37}} \sqrt{37}}{6} - \frac{\left(10 + \frac{40\sqrt{37}}{37}\right) e^{15+5\sqrt{37}}}{6} - \frac{\left(10 - \frac{40\sqrt{37}}{37}\right) e^{15-5\sqrt{37}}}{6}\right\} \quad \textbf{(6)}$$

> $evalf(\%)$

$$\{x(10) = 8.758846449\ 10^{20}, y(10) = 7.419856091\ 10^{20}\} \quad \textbf{(7)}$$

> #$x(10) = 8.7588 \times 10^{20}$ lynxes, $y(10) = 7.42 \times 10^{20}$ hares

> #Problem 3

> $b\_c := rand(0.0 .. 1.0):$

>

> $b\_1 := b\_c()$

$$b\_1 := 0.2342493224 \quad \textbf{(8)}$$

> $c\_1 := b\_c(\,)$

$$c\_1 := 0.1799302829 \quad (9)$$

> $b\_1 + c\_1$

$$0.4141796053 \quad (10)$$

> $a\_rand := rand(1.0\,..100.0):$

> $a\_1 := a\_rand(\,)$

$$a\_1 := 18.98896361 \quad (11)$$

> $R\_0\_1 = \frac{a\_1}{(b\_1 + c\_1)}$

$$R\_0\_1 = 45.84717202 \quad (12)$$

> $sys1 := (b + c)\cdot x\_star = y\_star\cdot(1 - \exp(-a\cdot x\_star)),\, y\_star = 1 - x\_star\cdot\left(1 + \frac{c}{b}\right):$

> $Help(Orb):$

Orb(F,x,x0,K1,K2): Inputs a transformation F in the list of variables x with initial point pt, outputs the trajectory of

of the discrete dynamical system (i.e. solutions of the difference equation): x(n)=F(x(n-1)) with x(0)=x0 from n=K1 to n=K2.

For the full trajectory (from n=0 to n=K2), use K1=0. Try:

*Orb(5/2*x*(1-x),[x], [0.5], 1000,1010);*

*Orb([(1+x+y)/(2+x+y),(6+x+y)/(2+4*x+5*y)],[x,y], [2.,3.], 1000,1010);* **(13)**

> $with(RealDomain):$

> $OrbF([x\cdot(1 - b\_1 - c\_1) + y\cdot(1 - \exp(-a\_1\cdot x)),\, (1 - y)\cdot b\_1 + y\cdot\exp(-a\_1\cdot x)],\, [x, y],\, [0.5, 0.5],\, 1000,\, 1010)$

[[0.4582189210, 0.1898165177], [0.4582189210, 0.1898165177], [0.4582189210, 0.1898165177], [0.4582189210, 0.1898165177], [0.4582189210, 0.1898165177], [0.4582189210, 0.1898165177], [0.4582189210, 0.1898165177], [0.4582189210, 0.1898165177], [0.4582189210, 0.1898165177], [0.4582189210, 0.1898165177], [0.4582189210, 0.1898165177]] **(14)**

> $solve\left(\left\{(b\_1 + c\_1)\cdot x\_star = y\_star\cdot(1 - \exp(-a\_1\cdot x\_star)),\, y\_star = 1 - x\_star\cdot\left(1 + \frac{c\_1}{b\_1}\right)\right\},\, \{x\_star, y\_star\}\right)$

$$\{x\_star = 0.4582189208,\, y\_star = 0.1898165177\},\, \{x\_star = 0.,\, y\_star = 1.\} \quad (15)$$

> *#Here, we see that x_star and y_star eventually become the limit of x_n and y_n as n approaches infinity*

> *#Let's experiment with other numbers*

> $b\_2 := b\_c(\,)$

$$b\_2 := 0.4705674223 \quad (16)$$

> $c\_2 := b\_c(\,)$

$$c\_2 := 0.2557169579 \quad (17)$$

> $b\_2 + c\_2$

$$0.7262843802 \quad (18)$$

> $a\_2 := a\_rand(\,)$

$$a\_2 := 44.93818573 \quad (19)$$

> $OrbF([x\cdot(1-b\_2-c\_2)+y\cdot(1-\exp(-a\_2\cdot x)), (1-y)\cdot b\_2+y\cdot\exp(-a\_2\cdot x)], [x, y], [0.5, 0.5], 1000, 1010)$

[[0.4405855122, 0.3199903765], [0.4405855122, 0.3199903765], [0.4405855122, 0.3199903765], [0.4405855122, 0.3199903765], [0.4405855122, 0.3199903765], [0.4405855122, 0.3199903765], [0.4405855122, 0.3199903765], [0.4405855122, 0.3199903765], [0.4405855122, 0.3199903765], [0.4405855122, 0.3199903765], [0.4405855122, 0.3199903765]] **(20)**

> $solve\left(\left\{(b\_2+c\_2)\cdot x\_star = y\_star\cdot(1-\exp(-a\_2\cdot x\_star)), y\_star = 1 - x\_star\cdot\left(1+\frac{c\_2}{b\_2}\right)\right\}, \{x\_star, y\_star\}\right)$

$\{x\_star = 0.4405855122, y\_star = 0.3199903765\}$ **(21)**

> *#Here, we see that x_star and y_star eventually become the limit of x_n and y_n as n approaches infinity as before*

> *#Let's try one more random number set*

> $b\_3 := b\_c()$

$b\_3 := 0.02195544416$ **(22)**

> $c\_3 := b\_c()$

$c\_3 := 0.8658728984$ **(23)**

> $b\_3 + c\_3$

0.8878283426 **(24)**

> $a\_3 := a\_rand()$

$a\_3 := 92.86320783$ **(25)**

> $OrbF([x\cdot(1-b\_3-c\_3)+y\cdot(1-\exp(-a\_3\cdot x)), (1-y)\cdot b\_3+y\cdot\exp(-a\_3\cdot x)], [x, y], [0.5, 0.5], 1000, 1010)$

[[0.02413641521, 0.02397813728], [0.02413641521, 0.02397813728], [0.02413641521, 0.02397813728], [0.02413641521, 0.02397813728], [0.02413641521, 0.02397813728], [0.02413641521, 0.02397813728], [0.02413641521, 0.02397813728], [0.02413641521, 0.02397813728], [0.02413641521, 0.02397813728], [0.02413641521, 0.02397813728], [0.02413641521, 0.02397813728]] **(26)**

> $solve\left(\left\{(b\_3+c\_3)\cdot x\_star = y\_star\cdot(1-\exp(-a\_3\cdot x\_star)), y\_star = 1 - x\_star\cdot\left(1+\frac{c\_3}{b\_3}\right)\right\}, \{x\_star, y\_star\}\right)$

$\{x\_star = 0., y\_star = 1.\}, \{x\_star = 0.02413641521, y\_star = 0.02397813728\}$ **(27)**

> *#This satisfies the conjecture numerically*

>

> $with(Domains):$

```
----------------------- Domains version 1.0 ---------------------
Initially defined domains are Z and Q the integers and rationals
Abbreviations, e.g. DUP for DenseUnivariatePolynomial, also made
```

> *#Problem 4*

> $b\_4 := b\_c()$

$b\_4 := 0.4850963949$ **(28)**

> $a\_4 := 0.5$

$$a\_4 := 0.5 \qquad (29)$$

```
> Help(OrbkF)
```

OrbkF(k,z,f,INI,K1,K2): Same as Orbk(k,z,f,INI,K1,K2), but in floating-point (to get around Maple's annoying habit not to automatically convert to floating point exp(floatingpoint)
To investigate the long-term behavior Linda Allen's Conjecture 2 of
https://sites.math.rutgers.edu/~zeilberg/Bio21/AllenSIR.pdf
with initial conditions x(0)=0.3, x(1)=0.4, a=3, b=2 Type:
a:=0.3; b:=0.2; OrbkF(2,z,z[1]*(1-b)+ (1-z[1])*(1-exp(-a*z[2])),[0.3,0.4],1000,1010);
then type
solve(b*y-(1-y)*(1-exp(-a*y)),y); **(30)**

```
> OrbkF(2, z, ( z[1]·(1 − b_4) + (1 − z[1])·(1 − exp(−a_4·z[2]))), [0.5, 0.5], 1000, 1010);
> OrbkF(2, z, z[1]* (1 − b_4) + (1-z[1]) * (1-exp(-a_4*z[2])), [0.3, 0.4], 1000, 1010)
> #I'm not sure why but I cannot get the two above lines of code to run (was stuck for over 5 minutes before I stopped it). I even tried copying the exact line from the DMB.txt file, but something seems to have a bug, so I can't properly test conjecture 2. I'll use solve to see what it should be
> solve(b_4·x = (1 − x)·(1 − exp(−a_4·x)), x)
```

Warning, solutions may have been lost

$$RootOf\left(-\_Z\,e^{-a\_4\,\_Z} + b\_4\,\_Z + e^{-a\_4\,\_Z} + \_Z - 1\right) \qquad (31)$$

```
> #I genuinely don't know why these are not working, but I hope that writing out the code demonstrates knowledge of how I would approach this problem. To numerically solve, I would define and try more values of a and b
>
>
> #Problem 5
>
>
>
>
>
```