OK to post

Anne Somalwar, hw22, 11/22/2021

6) <u>Proof</u>: First we will show that,

if $x(0) = x_0 \neq -10$, then,

$$x(n) = \frac{x_0}{10^n + \left(\frac{1-10^{n+1}}{1-10}\right) x_0}.$$

With this equation for $x(n)$, we

have $x(n-1) = \dfrac{x_0}{10^{n-1} + \left(\frac{1-10^{n}}{1-10}\right) x_0}$.

Then, $$\frac{x(n-1)}{10 + x(n-1)} = \frac{\dfrac{x_0}{10^{n-1} + \left(\frac{1-10^{n}}{1-10}\right) x_0}}{10 + \dfrac{x_0}{10^{n-1} + \left(\frac{1-10^{n}}{1-10}\right) x_0}}$$

$$= \frac{x_0}{10\left(10^{n-1} + \left(\frac{1-10^{n}}{1-10}\right) x_0\right) + x_0}$$

$$= \frac{x_0}{10^n + \frac{10-10^{n+1}}{1-10} x_0 + x_0}$$

$$= \frac{x_0}{10^n + \frac{1-10^{n+1}}{1-10} x_0} = x(n) .$$

Thus, $x(n) = \dfrac{x_0}{10^n + \left(\frac{1-10^{n+1}}{1-10}\right) x_0}$.

Then, $x(n) = \dfrac{x_0}{10^n + \frac{10^{n+1}-1}{9} x_0} = \dfrac{x_0}{10^n + \frac{10^{n+1}}{9} x_0 - \frac{x_0}{9}}$

$$= \frac{x_0}{10^n \left(1 + \frac{x_0}{9} - \frac{x_0}{9 \cdot 10^n}\right)} .$$

Then, $\lim \left\{ \frac{x_0}{10^n\left(1+\frac{x_0}{9}-\frac{x_0}{9\cdot 10^n}\right)} \right\}_{n=0}^{\infty}$

$= \left(\lim \left\{ \frac{1}{10^n} \right\}_{n=0}^{\infty} \right) \left(\lim \left\{ \frac{x_0}{1+\frac{x_0}{9}-\frac{x_0}{9\cdot 10^n}} \right\}_{n=0}^{\infty} \right)$

$= (0)\left(\frac{x_0}{1+\frac{x_0}{9}} \right) = 0.$

Thus,

$\{x(n)\}_{n=0}^{\infty}$ always approaches 0.

Since 0 is a stable fixed point,

there exists some neighborhood

$(-\delta, \delta)$ such that if $x(n) \in (-\delta, \delta)$, than $x(n)$ will end up fixed at 0. Since $\lim \{x(n)\}_{n=0}^{\infty} = 0$, There exists some N such that for $n \geq N$, $|x(n) - 0| < \delta$, i.e. $x(n) \in (-\delta, \delta)$. Thus, $x(n)$ will end up fixed at 0. Thus, 0 is a global attractor. //