```
> #OK to post homework
  #Shreya Ghosh, 11-15-2021,Assignment 21
> read "/Users/shreyaghosh/Documents/DMB.txt"
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

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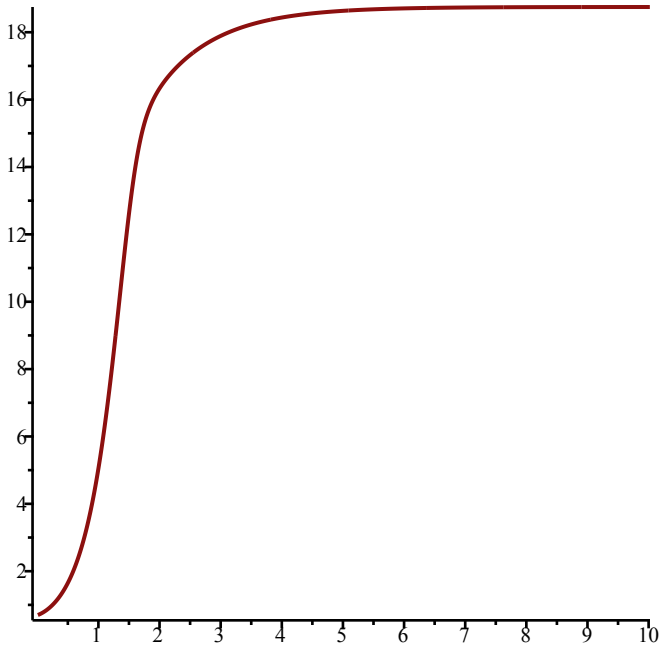
(1)

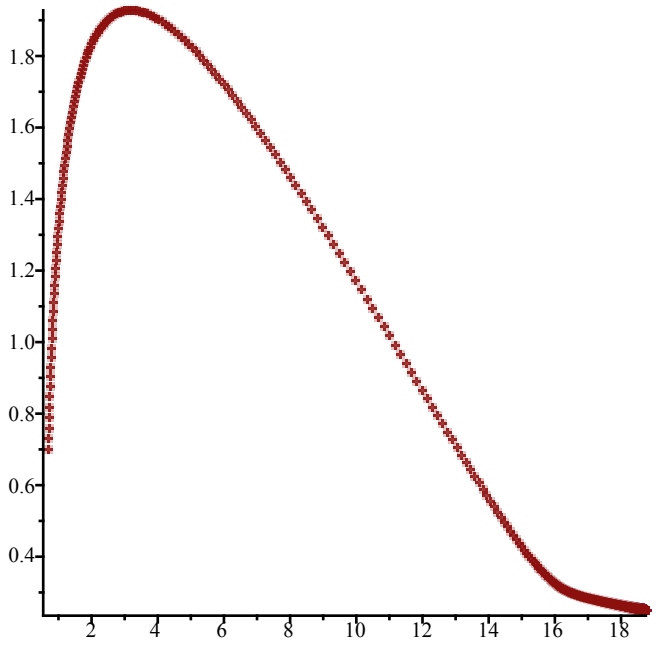
```
> #ChemoStat
> F := ChemoStat(N, C, 5, 4)
```

$$F := \left[\frac{5\,C\,N}{C+1} - N, -\frac{C\,N}{C+1} - C + 4 \right] \tag{2}$$

```
> TimeSeries(F, [N, C], [0.7, 0.7], 0.01, 10, 1)
```



> $PhaseDiag(F, [N, C], [0.7, 0.7], 0.01, 10)$



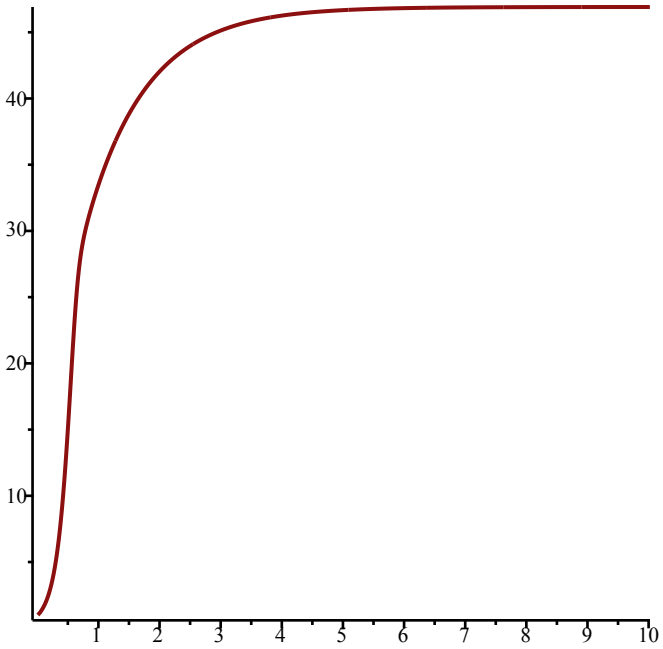
> $SEquP(F, [N, C])$

$$\{[18.75000000, 0.2500000000]\} \quad \textbf{(3)}$$

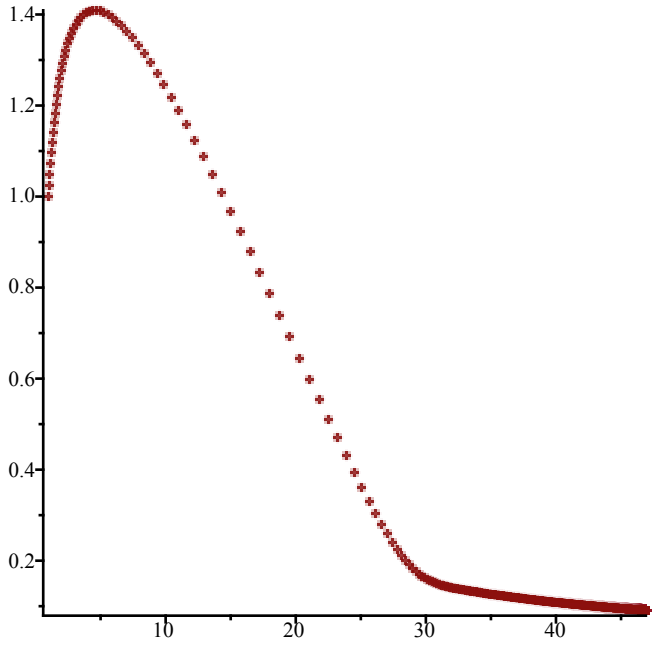
> $F := ChemoStat(N, C, 12, 4)$

$$F := \left[\frac{12\,C\,N}{C+1} - N, -\frac{C\,N}{C+1} - C + 4\right] \quad \textbf{(4)}$$

> $TimeSeries(F, [N, C], [1, 1], 0.01, 10, 1)$



> *PhaseDiag*(*F*, [*N*, *C*], [1, 1], 0.01, 10)



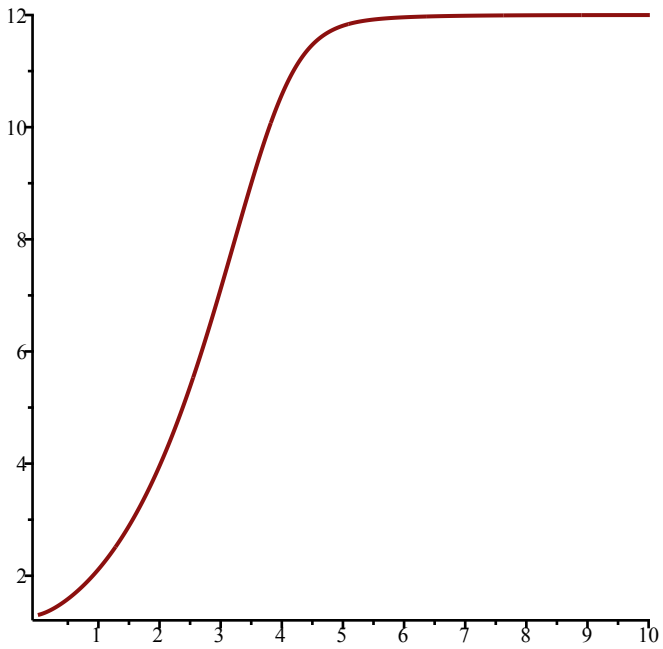
> *SEquP*(*F*, [*N*, *C*])

$$\{[46.90909091, 0.09090909091]\} \quad \textbf{(5)}$$

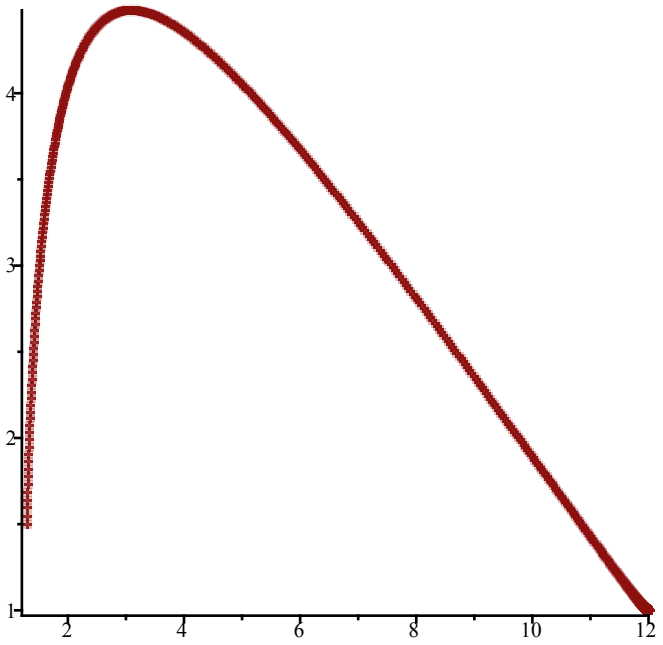
> $F := ChemoStat(N, C, 2, 7)$

$$F := \left[\frac{2\,C\,N}{C+1} - N, -\frac{C\,N}{C+1} - C + 7\right] \quad \textbf{(6)}$$

> *TimeSeries*(*F*, [*N*, *C*], [1.3, 1.5], 0.01, 10, 1)



> $PhaseDiag(F, [N, C], [1.3, 1.5], 0.01, 10)$



> $SEquP(F, [N, C])$

$$\{[12., 1.]\} \quad \textbf{(7)}$$

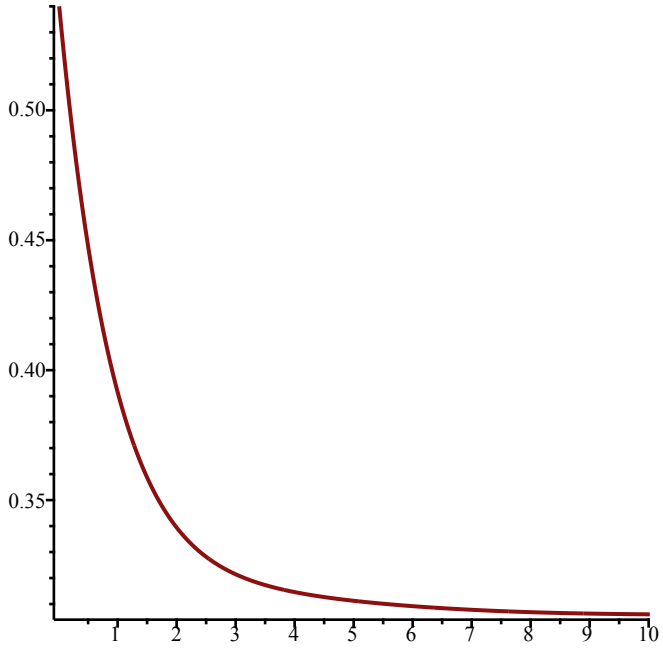
> *#The stable equilibrium points for ChemoStat coincide with the horizontal asymptotes found in the time series*

> *#GeneNet*

> $F := GeneNet(0, 0.4, 0.6, 1, m1, m2, m3, p1, p2, p3)$

$$F := \left[-m1 + \frac{0.4}{1 + p3}, -m2 + \frac{0.4}{1 + p1}, -m3 + \frac{0.4}{1 + p2}, -0.6\,p1 + 0.6\,m1, -0.6\,p2 + 0.6\,m2, -0.6\,p3 + 0.6\,m3\right] \quad \textbf{(8)}$$

```
> TimeSeries(F, [m1, m2, m3, p1, p2, p3], [0.54, 0.37, 0.28, 0.99, 0.64, 0.33], 0.01, 10, 1)
```



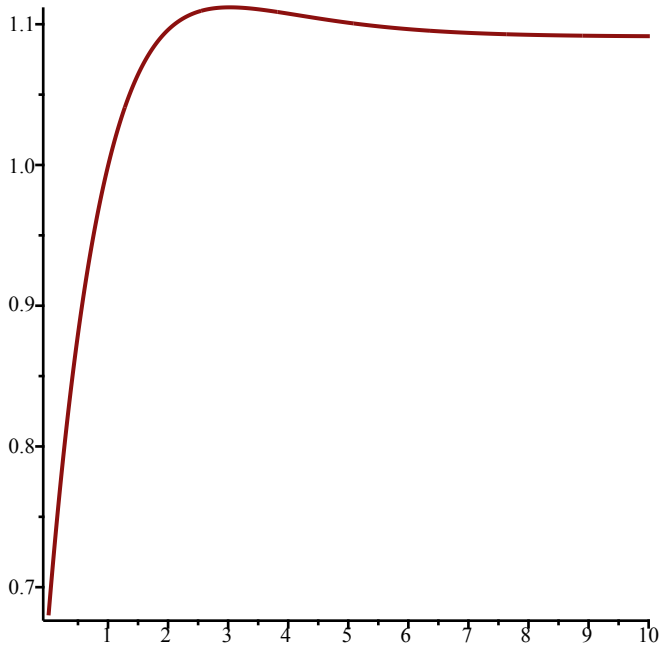

```
> SEquP(F, [m1, m2, m3, p1, p2, p3])
```

$$\{[0.3062257748, 0.3062257748, 0.3062257748, 0.3062257748, 0.3062257748, 0.3062257748]\} \quad \textbf{(9)}$$

```
> F := GeneNet(1, 0.2, 0.74, 2, m1, m2, m3, p1, p2, p3)
```

$$F := \left[-m1 + \frac{0.2}{p3^2 + 1} + 1,\ -m2 + \frac{0.2}{p1^2 + 1} + 1,\ -m3 + \frac{0.2}{p2^2 + 1} + 1,\ -0.74\,p1 + 0.74\,m1,\ -0.74\,p2 + 0.74\,m2,\ -0.74\,p3 + 0.74\,m3\right] \quad \textbf{(10)}$$

```
> TimeSeries(F, [m1, m2, m3, p1, p2, p3], [0.68, 0.44, 0.22, 0.36, 0.19, 0.14], 0.01, 10, 1)
```



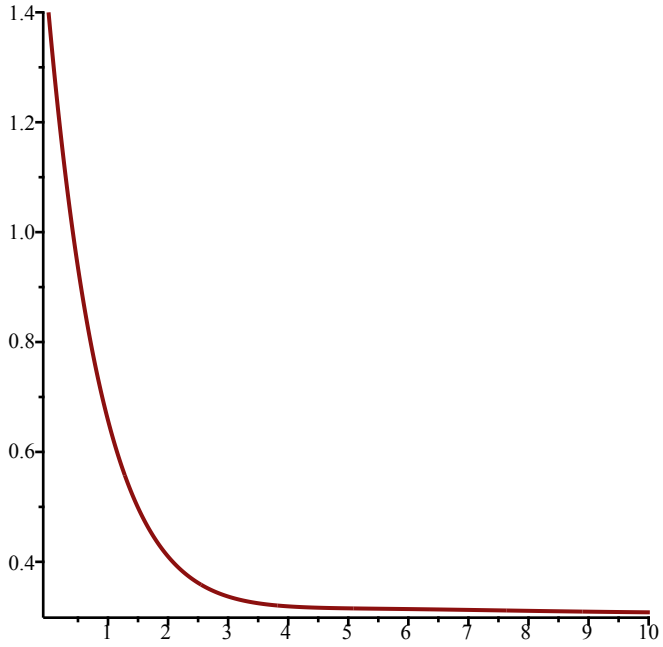

```
> SEquP(F, [m1, m2, m3, p1, p2, p3])
```

$$\{[1.091286433, 1.091286433, 1.091286433, 1.091286433, 1.091286433, 1.091286433]\} \quad \textbf{(11)}$$

```
> F := GeneNet(0, 0.4, 0.66, 1, m1, m2, m3, p1, p2, p3)
```

$$F := \left[-m1 + \frac{0.4}{1 + p3},\ -m2 + \frac{0.4}{1 + p1},\ -m3 + \frac{0.4}{1 + p2},\ -0.66\,p1 + 0.66\,m1,\ -0.66\,p2 + 0.66\,m2,\ -0.66\,p3 + 0.66\,m3 \right] \quad \textbf{(12)}$$

```
> TimeSeries(F, [m1, m2, m3, p1, p2, p3], [1.4, 1.66, 0.12, 0.72, 0.88, 1.1], 0.01, 10, 1)
```



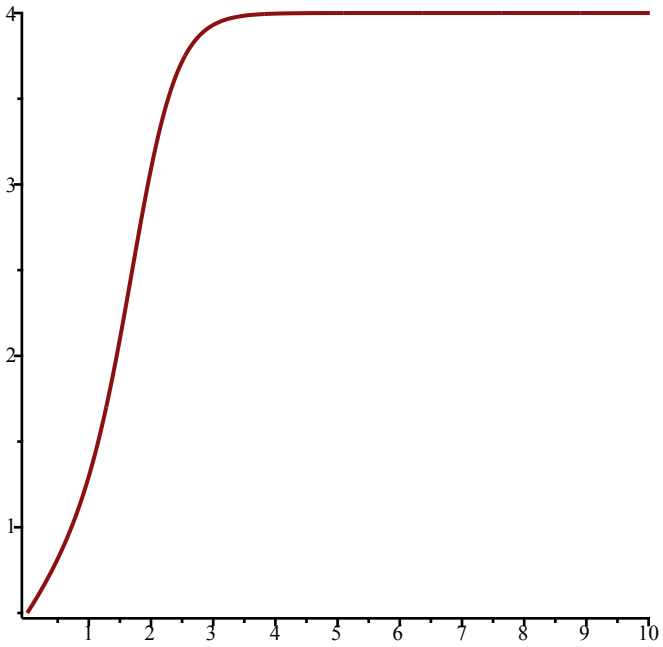

```
> SEquP(F, [m1, m2, m3, p1, p2, p3])
```

$$\{[0.3062257748, 0.3062257748, 0.3062257748, 0.3062257748, 0.3062257748, 0.3062257748]\} \quad \textbf{(13)}$$

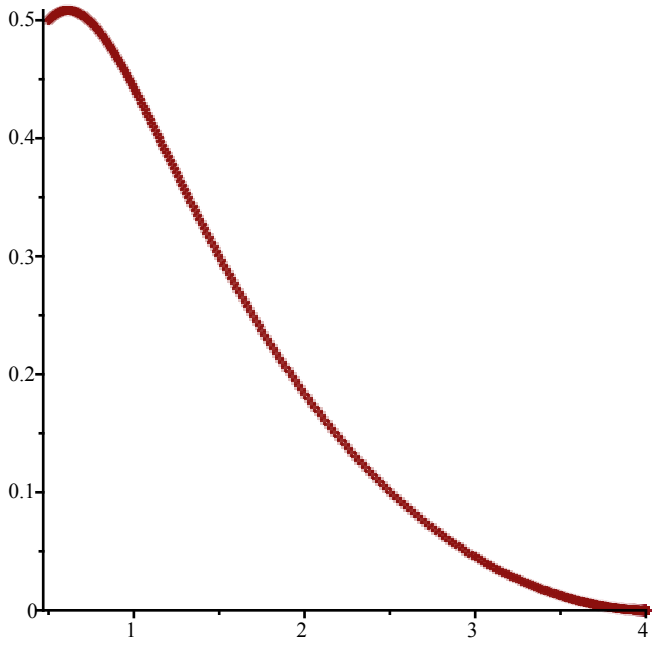
```
> #The stable equilibrium points for GeneNet coincide with the horizontal asymptotes found in the
      time series. The phase diagrams cannot be drawn because the function is not in R2
> #Lotka
> F := Lotka(3, 4, 1, 6, 4, 9, N1, N2)
```

$$F := \left[\frac{3\,N1\,(4 - N1 - 4\,N2)}{4},\ \frac{N2\,(6 - N2 - 9\,N1)}{6} \right] \quad \textbf{(14)}$$

```
> TimeSeries(F, [N1, N2], [0.5, 0.5], 0.01, 10, 1)
```



> $PhaseDiag(F, [N1, N2], [0.5, 0.5], 0.01, 10)$



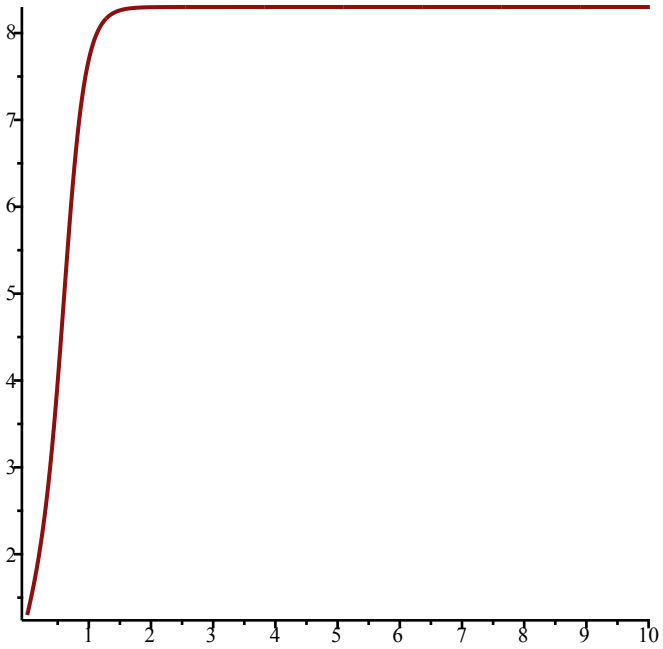
> $SEquP(F, [N1, N2])$

$$\{[0., 6.], [4., 0.]\} \quad \textbf{(15)}$$

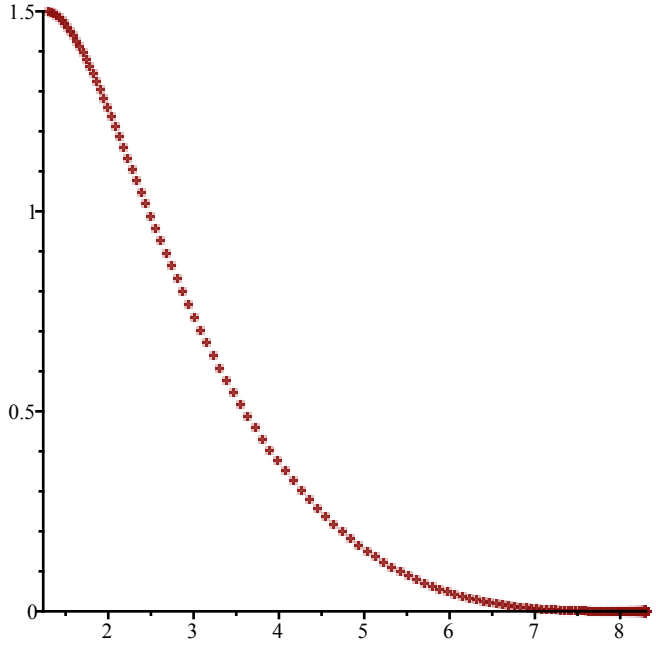
> $F := Lotka(5.5, 8.3, 4, 9, 2.3, 6, N1, N2)$

$$F := \left[0.6626506024\, N1\, (8.3 - N1 - 2.3\, N2),\ \frac{4\, N2\, (9 - N2 - 6\, N1)}{9}\right] \quad \textbf{(16)}$$

> $TimeSeries(F, [N1, N2], [1.3, 1.5], 0.01, 10, 1)$



> *PhaseDiag*(*F*, [*N1*, *N2*], [1.3, 1.5], 0.01, 10)



> *SEquP*(*F*, [*N1*, *N2*])

$$\{[0., 9.], [8.300000000, 0.]\} \quad \textbf{(17)}$$

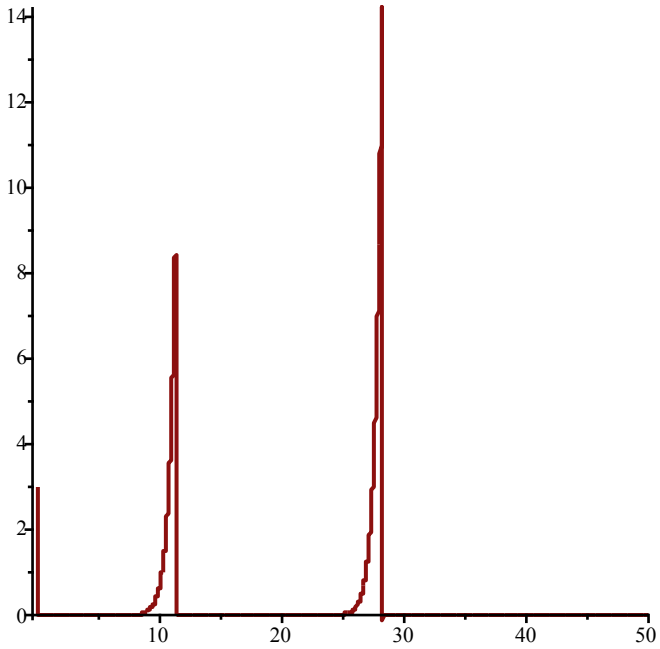
> #*The stable equilibrium points for Lotka coincide with the horizontal asymptotes found in the time series*

> #*Volterra*

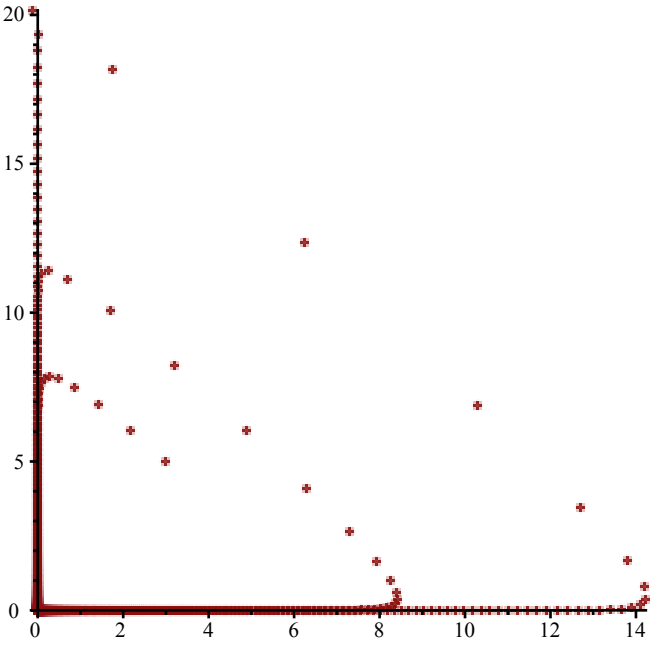
> $F := Volterra(2, 6, 3, 8, x, y)$

$$F := [-6xy + 2x, 8xy - 3y] \quad \textbf{(18)}$$

> *TimeSeries*(*F*, [*x*, *y*], [3, 5], 0.01, 50, 1)



> $PhaseDiag(F, [x, y], [3, 5], 0.01, 50)$



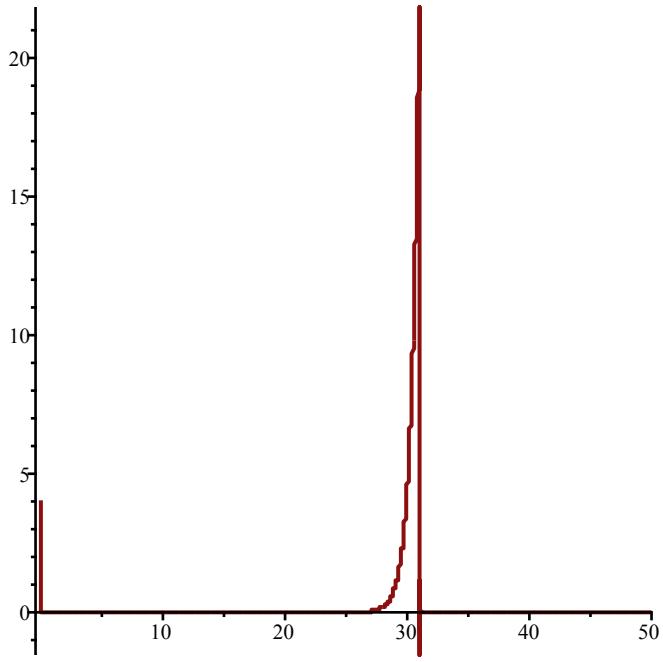
> $SEquP(F, [x, y])$

$$\varnothing \tag{19}$$

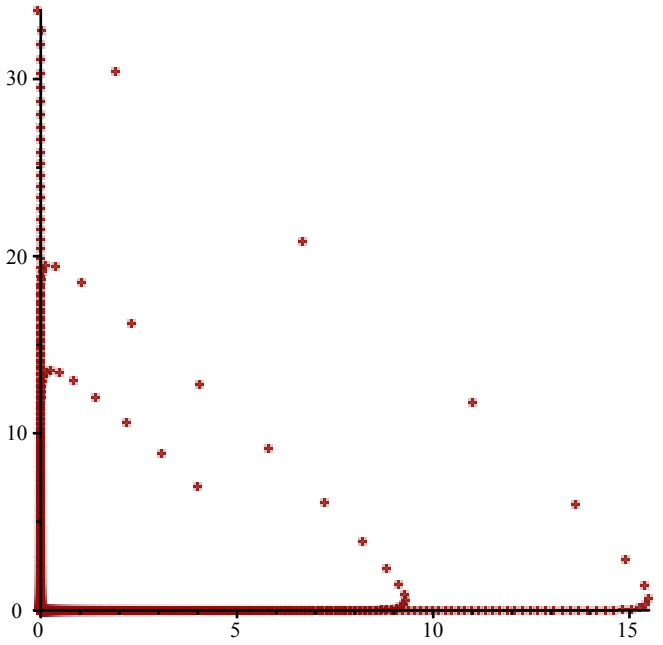
> $F := Volterra(1.6, 3.5, 2.6, 7.3, x, y)$

$$F := [1.6\,x - 3.5\,x\,y,\ -2.6\,y + 7.3\,x\,y] \tag{20}$$

> $TimeSeries(F, [x, y], [4, 17], 0.01, 50, 1)$



> *PhaseDiag*(*F*, [*x*, *y*], [4, 7], 0.01, 50, 1)



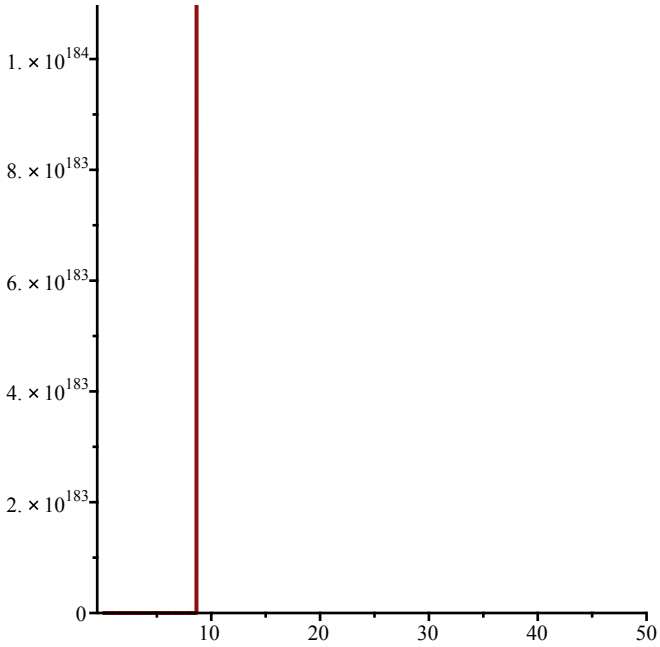
> *SEquP*(*F*, [*x*, *y*])

$$\varnothing \tag{21}$$

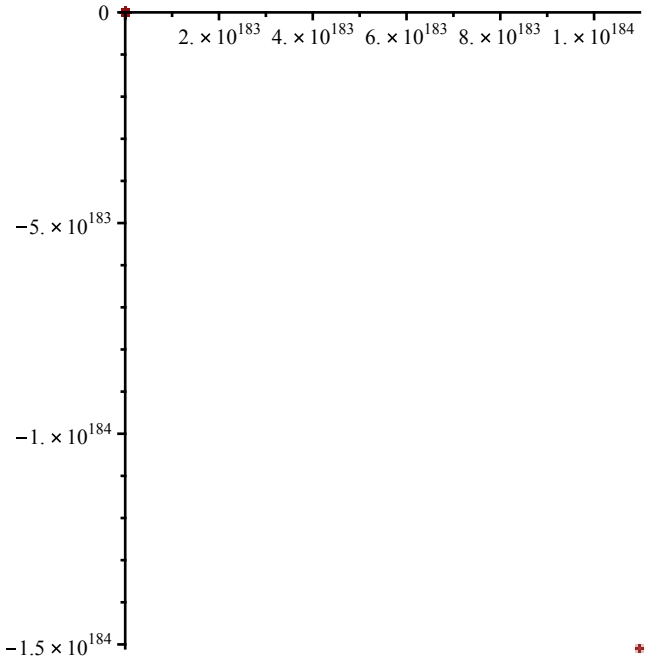
> $F := Volterra(7.4, 3.2, 8.2, 4.4, x, y)$

$$F := [7.4\,x - 3.2\,x\,y,\ -8.2\,y + 4.4\,x\,y] \tag{22}$$

> *TimeSeries*(*F*, [*x*, *y*], [14, 17], 0.01, 50, 1)



> *PhaseDiag*(*F*, [*x*, *y*], [14, 17], 0.01, 50)



> *SEquP*(*F*, [*x*, *y*])

$$\varnothing \tag{23}$$

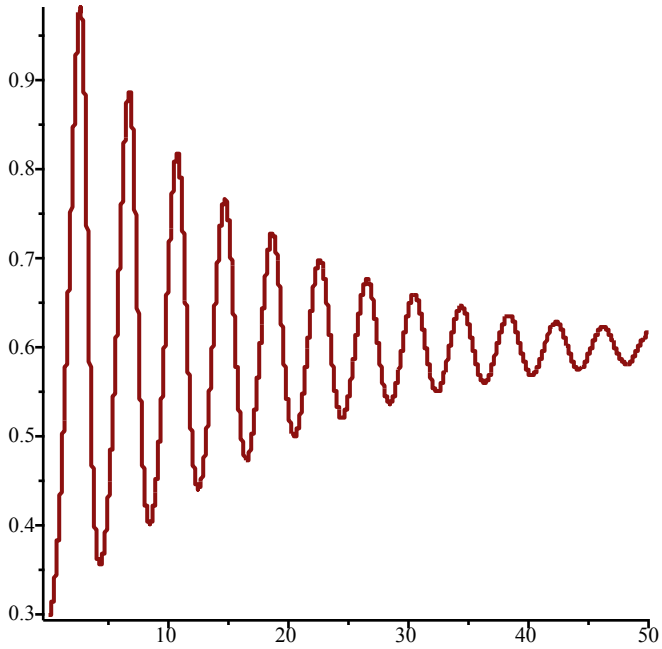
> #*The Volterra system does not have stable equilibria*

> #*VolterraM*

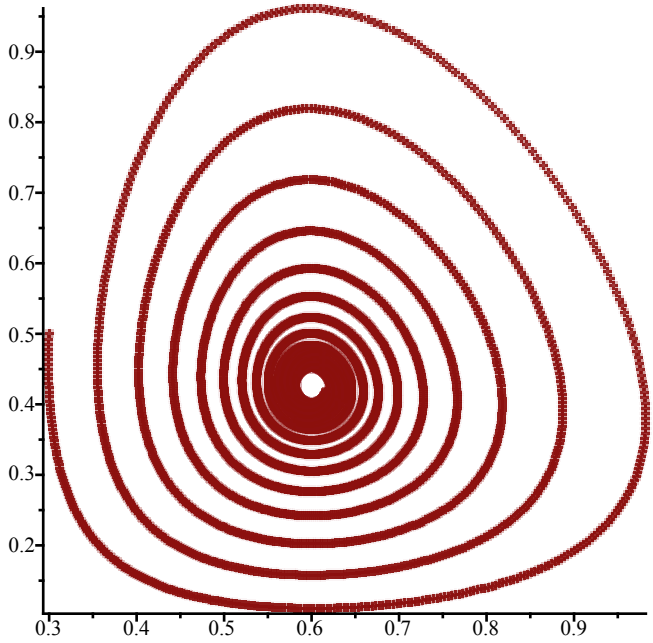
> $F := VolterraM(1, 2, 3, 4, 5, x, y)$

$$F := \left[x\left(1 - \frac{x}{4}\right) - 2\,x\,y,\ 5\,x\,y - 3\,y\right] \tag{24}$$

> *TimeSeries*(*F*, [*x*, *y*], [.3, .5], 0.01, 50, 1)



> *PhaseDiag*(*F*, [*x*, *y*], [.3, .5], 0.01, 50)



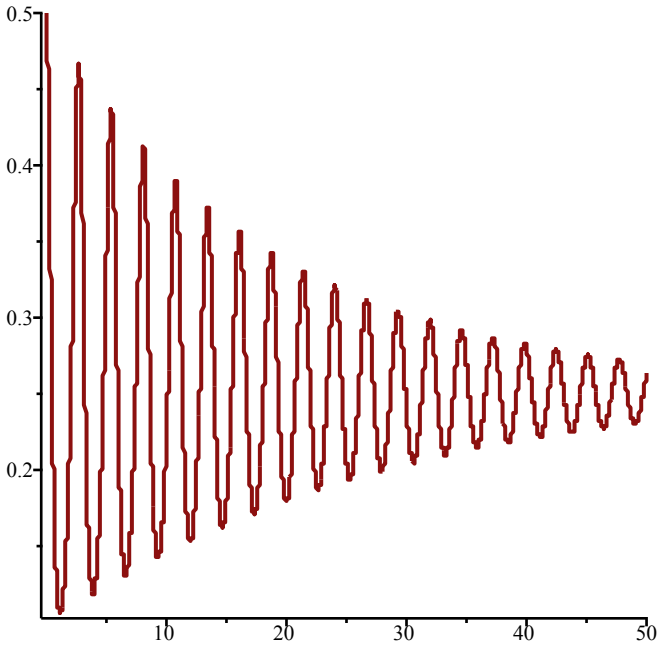
> *SEquP*(*F*, [*x*, *y*])

$$\{[0.6000000000, 0.4250000000]\} \quad \textbf{(25)}$$

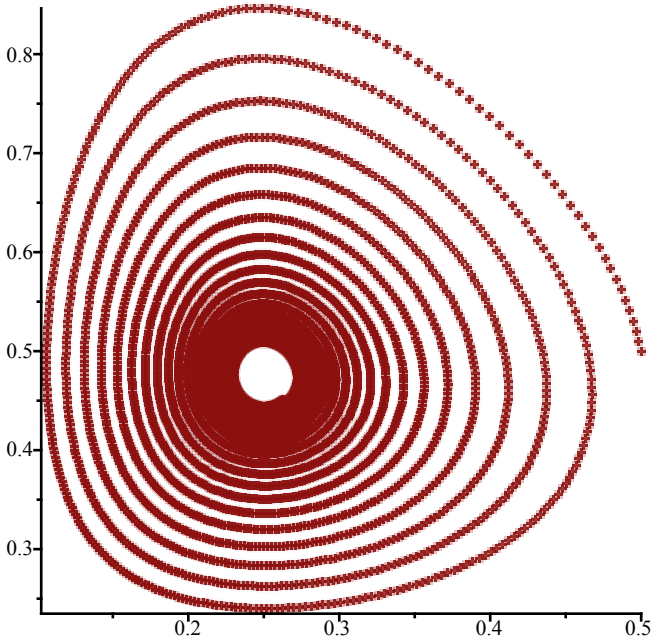
> $F := VolterraM(3, 6, 2, 5, 8, x, y)$

$$F := \left[3\,x\left(1-\frac{x}{5}\right)-6\,x\,y,\ 8\,x\,y-2\,y\right] \quad \textbf{(26)}$$

> *TimeSeries*(*F*, [*x*, *y*], [0.5, 0.5], 0.01, 50, 1)



> *PhaseDiag*(*F*, [*x*, *y*], [0.5, 0.5], 0.01, 50)



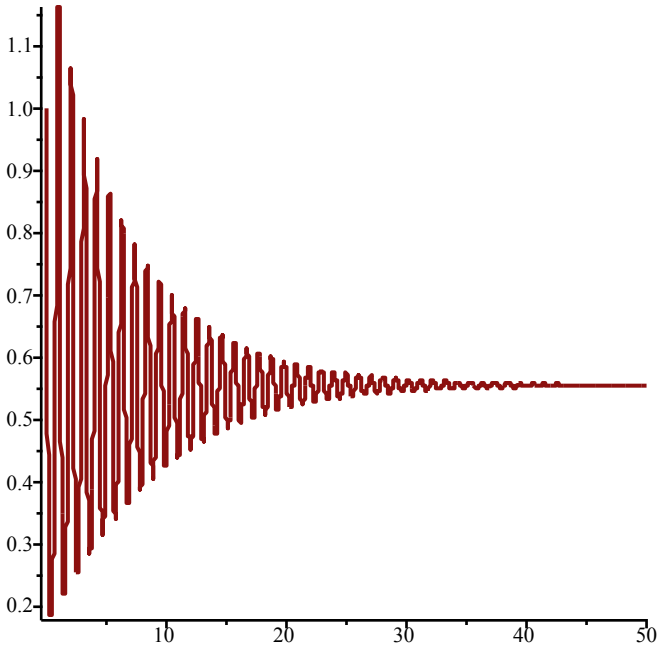
> *SEquP*(*F*, [*x*, *y*])

$$\{[0.2500000000, 0.4750000000]\} \quad \textbf{(27)}$$

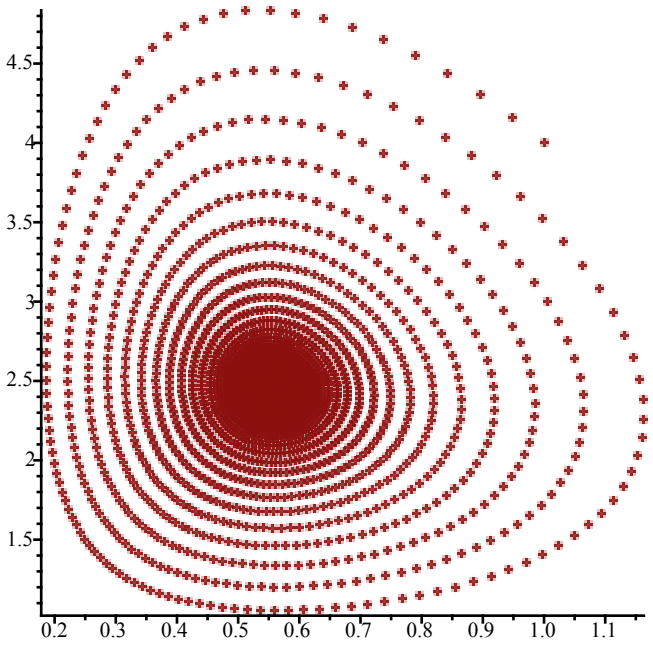
> $F := VolterraM(8, 3, 5, 7, 9, x, y)$

$$F := \left[8\,x\left(1-\frac{x}{7}\right)-3\,x\,y,\ 9\,x\,y-5\,y\right] \quad \textbf{(28)}$$

> *TimeSeries*(*F*, [*x*, *y*], [1, 4], 0.01, 50, 1)



> *PhaseDiag*(*F*, [*x*, *y*], [1, 4], 0.01, 50, 1)



> *SEquP*(*F*, [*x*, *y*])

$$\{[0.5555555556,\ 2.455026455]\} \qquad \textbf{(29)}$$

> *#The modified Volterra system does have stable equilibria that corresponds to the horizontal asymptote found in the time series*