```
> #OK to post
> #Julian Herman, Assignment 21, November 15th 2021
> read `/Users/julianherman/Documents/Rutgers/Fall 2021/Dynamical Models In Biology/HW/DMB.txt`
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

```
> HelpCDM( )
```

The procedures giving the underlying transformations, followed by the list of variables used are:
ChemoStat, GeneNet, Lotka, RandNice, SIRS , SIRSdemo, Volterra, VolterraM **(2)**

```
> Help(ChemoStat)
```

ChemoStat(N,C,a1,a2): The Chemostat continuous-time dynamical system with N=Bacterial poplulation densitty, and C=nutient Concentration in growth chamber (see Table 4.1 of Edelstein-Keshet, p. 122) *with paramerts a1, a2, Equations (19a_, (19b) in Edelestein-Keshet p. 127 (section 4.5, where they are called alpha1, alpha2). a1 and a2 can be symbolic or numeric. Try:*

ChemoStat(N,C,a1,a2);

ChemoStat(N,C,2,3); **(3)**

> *print(ChemoStat)*

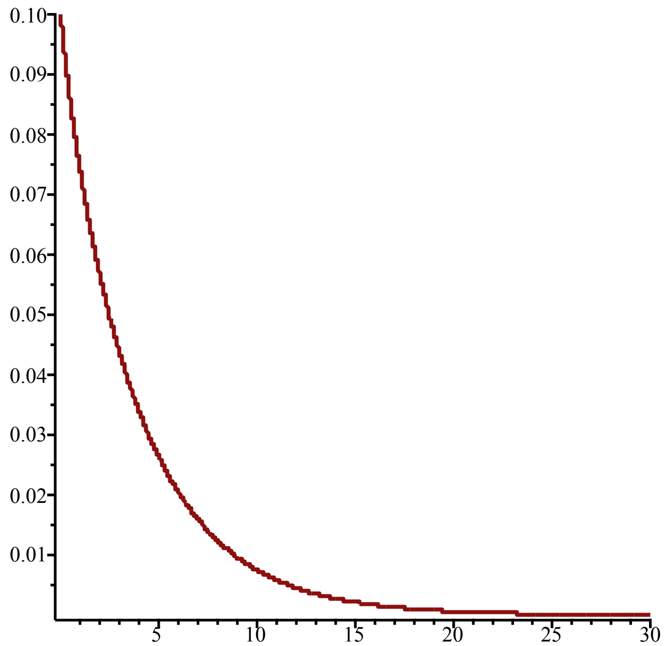
proc(N, C, a1, a2) [a1 * C * N / (C + 1) − N, − C * N / (C + 1) − C + a2] **end proc** **(4)**

> *#With parameters a1=1 and a2=3:*

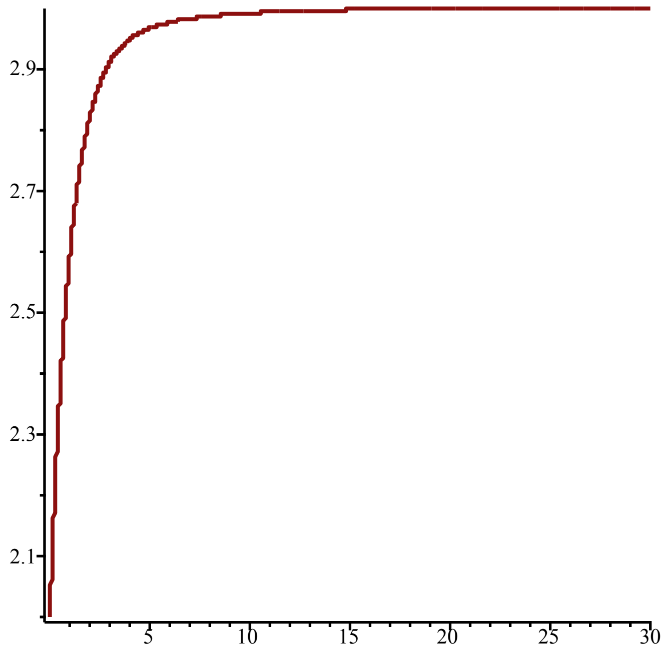
> $F := ChemoStat(N, C, 1, 3)$

$$F := \left[\frac{C\,N}{C+1} - N, -\frac{C\,N}{C+1} - C + 3\right] \quad \textbf{(5)}$$

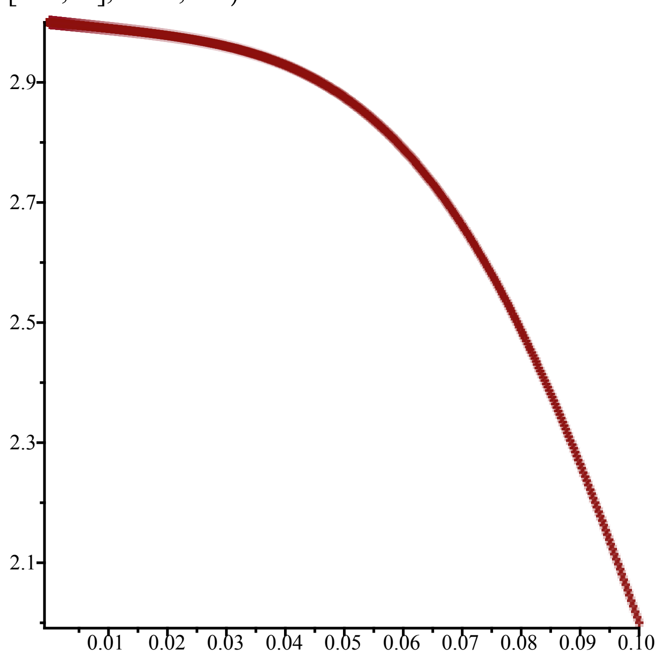
> $TimeSeries(F, [N, C], [0.1, 2], 0.01, 30, 1)$



> $TimeSeries(F, [N, C], [0.1, 2], 0.01, 30, 2)$



> *PhaseDiag*(*F*, [*N*, *C*], [0.1, 2], 0.01, 30)



> *SEquP*(*F*, [*N*, *C*])

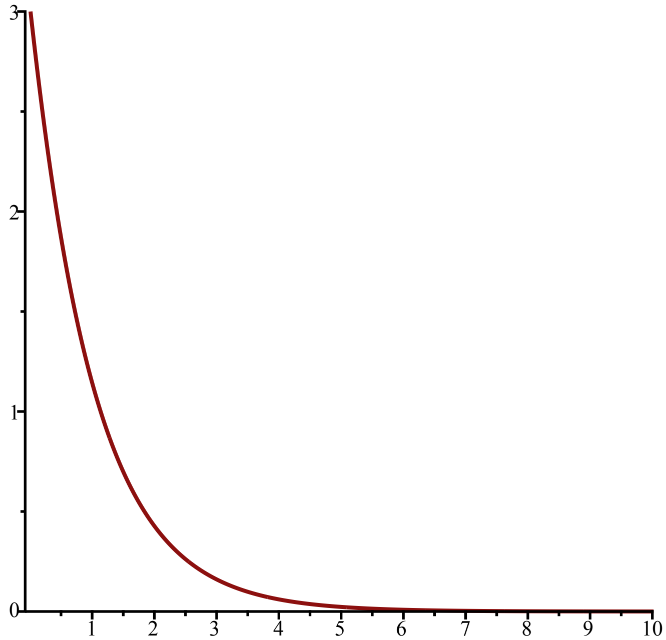
$$\{[0., 3.]\} \quad (6)$$

> *#With parameters a1=0.4 and a2=0.1:*

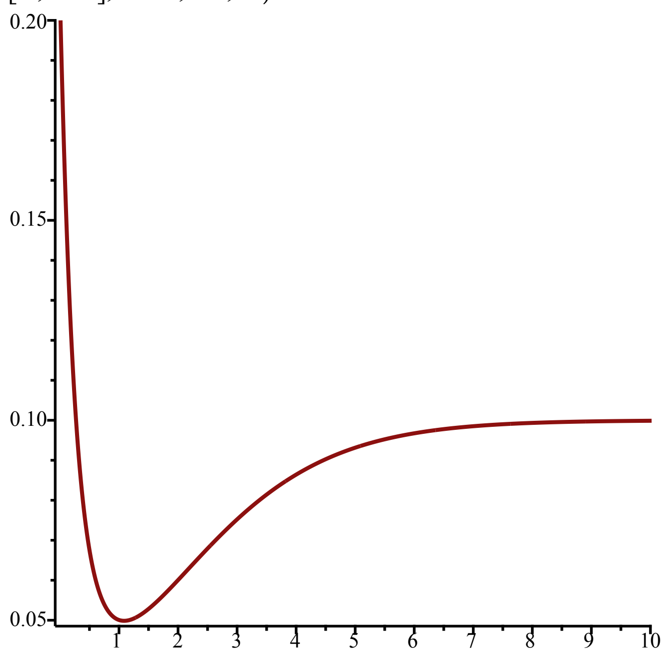
> *F* := *ChemoStat*(*N*, *C*, 0.4, 0.1)

$$F := \left[\frac{0.4\, C\, N}{C+1} - N, -\frac{C\, N}{C+1} - C + 0.1 \right] \quad (7)$$

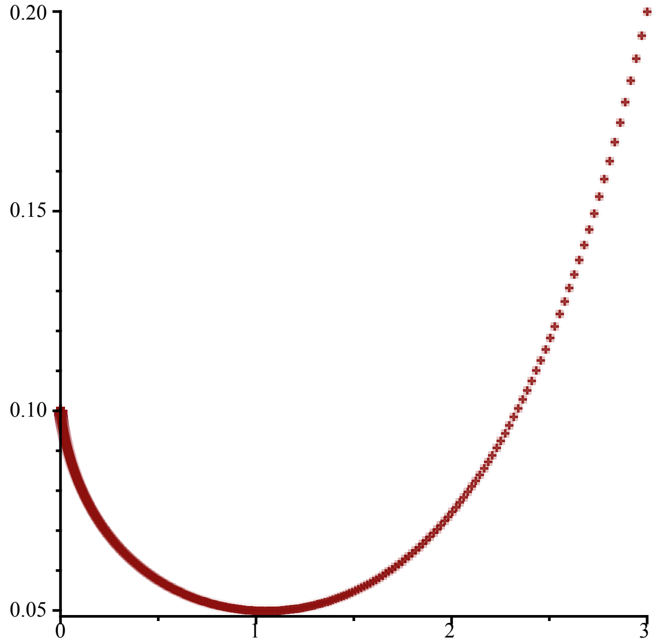
> *TimeSeries*(*F*, [*N*, *C*], [3, 0.2], 0.01, 10, 1)



> $TimeSeries(F, [N, C], [3, 0.2], 0.01, 10, 2)$



> $PhaseDiag(F, [N, C], [3, 0.2], 0.01, 30)$



> $SEquP(F, [N, C])$

$$\{[0., 0.1000000000], [0.7066666667, -1.666666667]\} \quad (8)$$

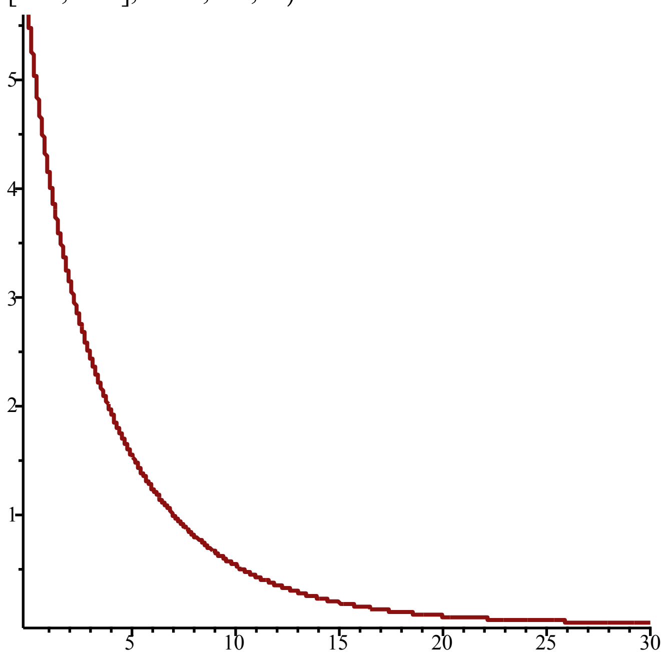
> #The above timeseries show the first eq point where N=0 and C=0.1

> #With parameters a1=0.9 and a2=8.1:

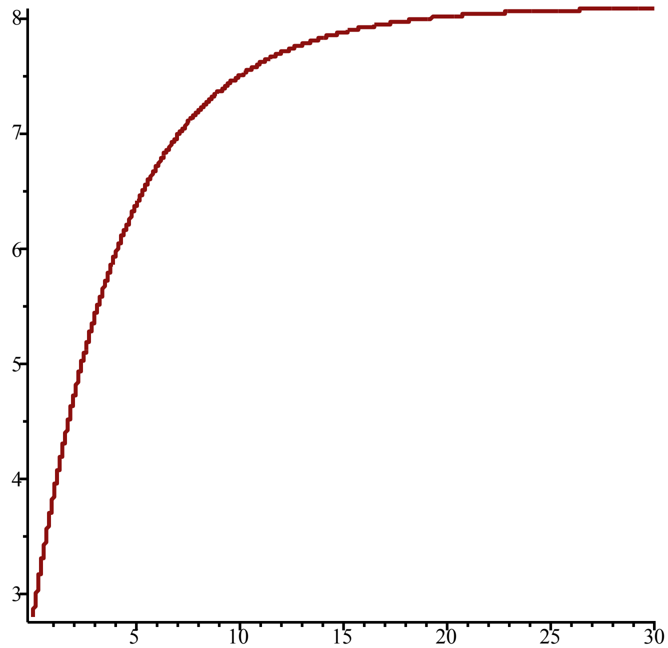
> $F := ChemoStat(N, C, 0.9, 8.1)$

$$F := \left[\frac{0.9\,C\,N}{C+1} - N, -\frac{C\,N}{C+1} - C + 8.1\right] \quad (9)$$

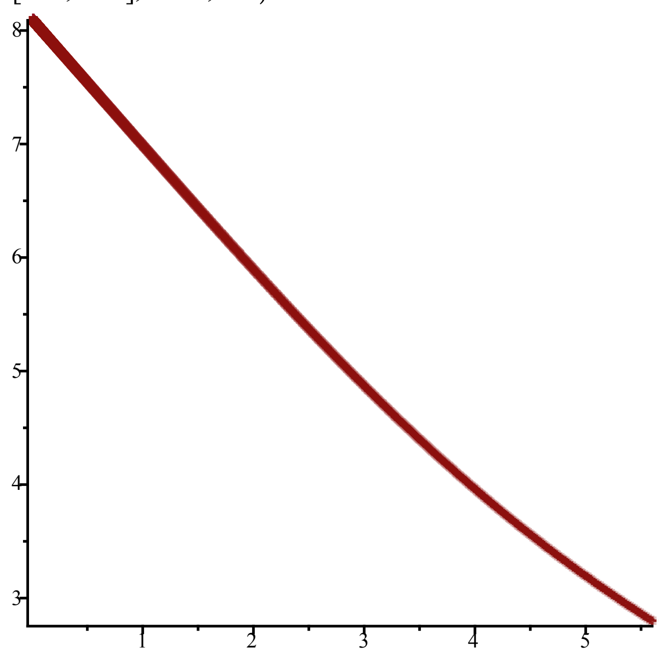
> $TimeSeries(F, [N, C], [5.6, 2.8], 0.01, 30, 1)$



> $TimeSeries(F, [N, C], [5.6, 2.8], 0.01, 30, 2)$



> $PhaseDiag(F, [N, C], [5.6, 2.8], 0.01, 30)$



> $SEquP(F, [N, C])$

$$\{[0., 8.100000000], [16.29000000, -10.]\} \quad (10)$$

> *#The above timeseries show the first eq point where N=0 and C=8.1*

>

>

>

> $Help(GeneNet)$

GeneNet(a0,a,b,n,m1,m2,m3,p1,p2,p3): The contiuous-time dynamical system, with quantities m1, m2,m3,p1,p2,p3, due to M. Elowitz and S. Leibler

described in the Ellner-Guckenheimer book, Eq. (4.1) (chapter 4, p. 112) *and parameers a0 (called alpha_0 there),a (called alpha there), b (called beta there) and n. Try:*

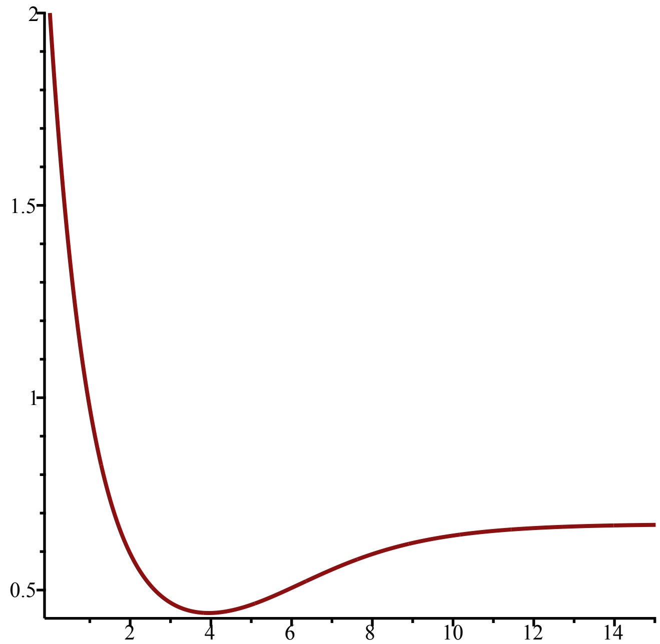
GeneNet(0,0.5,0.2,2,m1,m2,m3,p1,p2,p3); **(11)**

```
> print(GeneNet)
proc(a0, a, b, n, m1, m2, m3, p1, p2, p3)
    [ - m1 + a/(1 + p3^n) + a0, - m2 + a/(1 + p1^n) + a0, - m3 + a/(1 + p2^n)
    + a0, - b*(p1 - m1), - b*(p2 - m2), - b*(p3 - m3)]
end proc
```
(12)

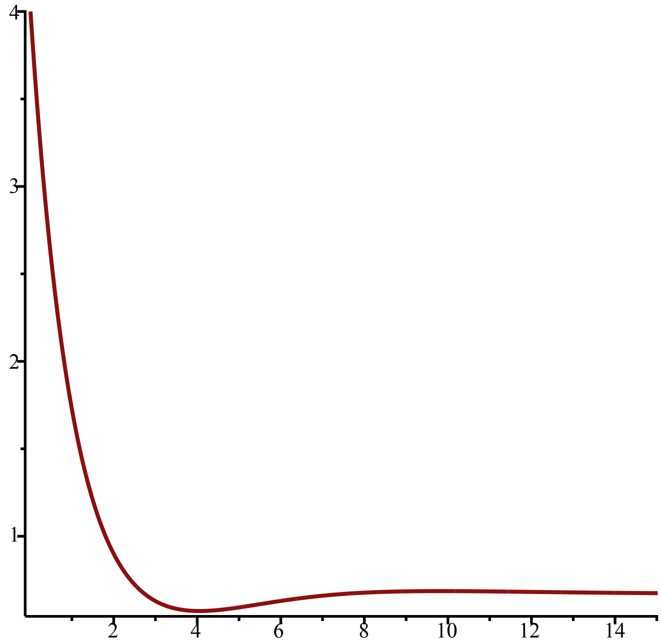
```
> #With a0=0.36, a=0.45, b=0.51:
> F := GeneNet(0.36, 0.45, 0.51, 2, m1, m2, m3, p1, p2, p3)
```

$$F := \left[-m1 + \frac{0.45}{p3^2 + 1} + 0.36, -m2 + \frac{0.45}{p1^2 + 1} + 0.36, -m3 + \frac{0.45}{p2^2 + 1} + 0.36, -0.51\, p1 + 0.51\, m1, -0.51\, p2 + 0.51\, m2, -0.51\, p3 + 0.51\, m3\right] \quad (13)$$

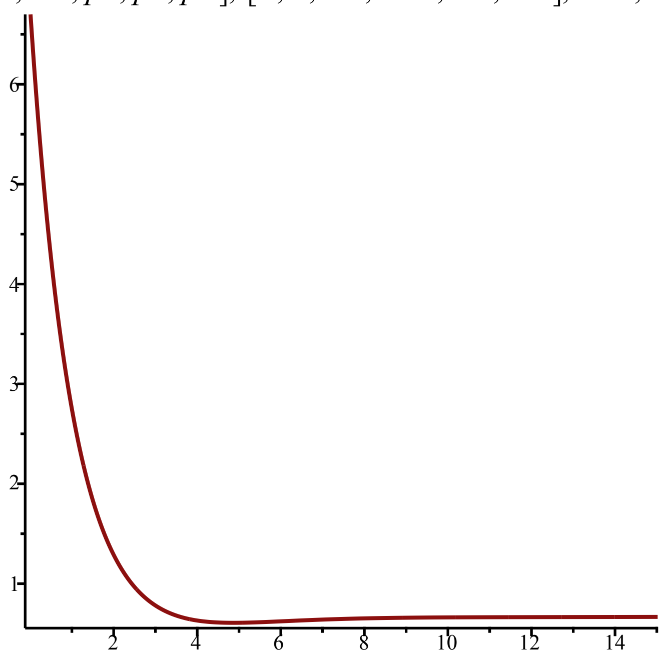
```
> TimeSeries(F, [m1, m2, m3, p1, p2, p3], [2, 4, 6.7, 4.21, 2.1, 7.4], 0.01, 15, 1)
```



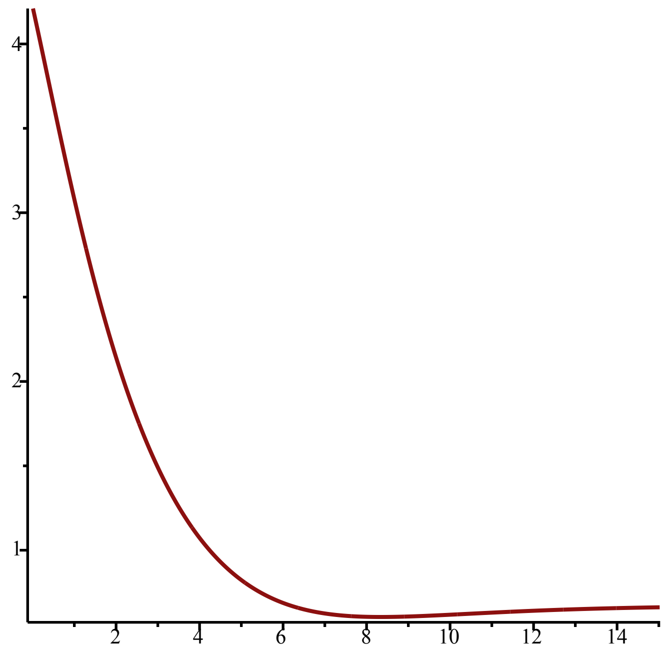

```
> TimeSeries(F, [m1, m2, m3, p1, p2, p3], [2, 4, 6.7, 4.21, 2.1, 7.4], 0.01, 15, 2)
```



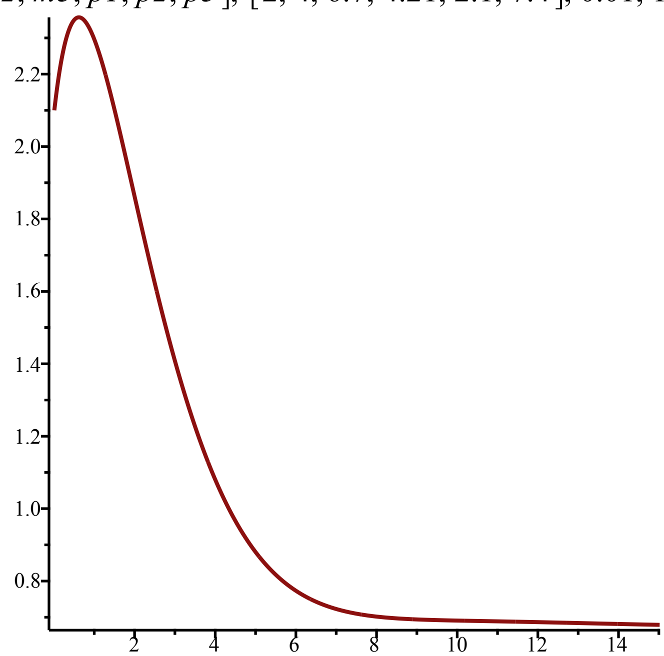
> *TimeSeries*(*F*, [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [2, 4, 6.7, 4.21, 2.1, 7.4], 0.01, 15, 3)



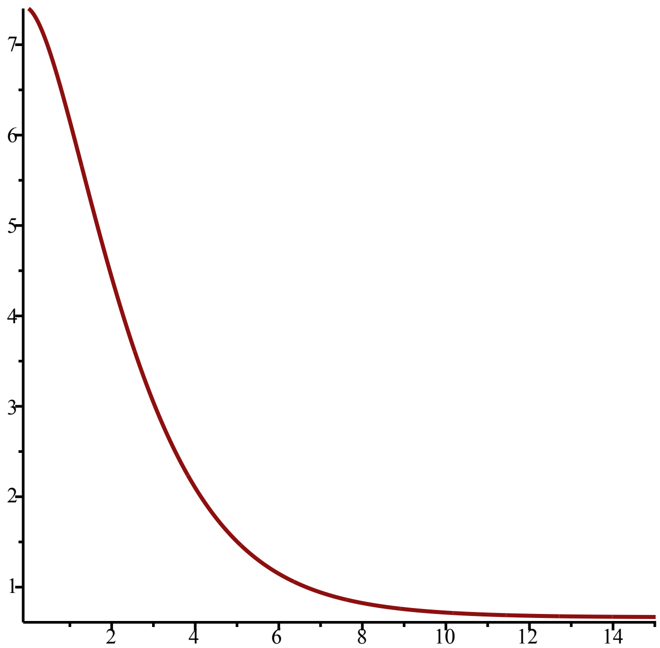
> *TimeSeries*(*F*, [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [2, 4, 6.7, 4.21, 2.1, 7.4], 0.01, 15, 4)



> *TimeSeries*(*F*, [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [2, 4, 6.7, 4.21, 2.1, 7.4], 0.01, 15, 5)



> *TimeSeries*(*F*, [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [2, 4, 6.7, 4.21, 2.1, 7.4], 0.01, 15, 6)



> *SEquP*(*F*, [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*])

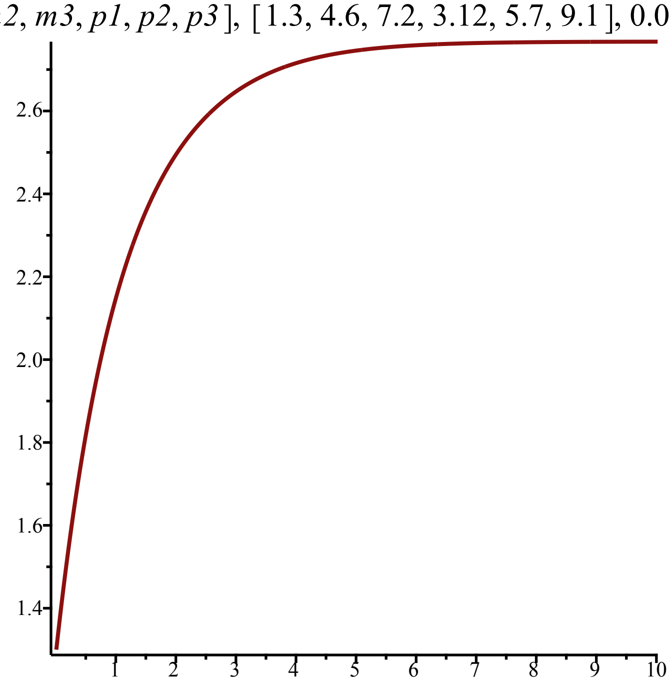
{[0.6704509276, 0.6704509276, 0.6704509276, 0.6704509276, 0.6704509276, 0.6704509276]} **(14)**

> *#With a0=2.6, a=1.45, b=7.51:*

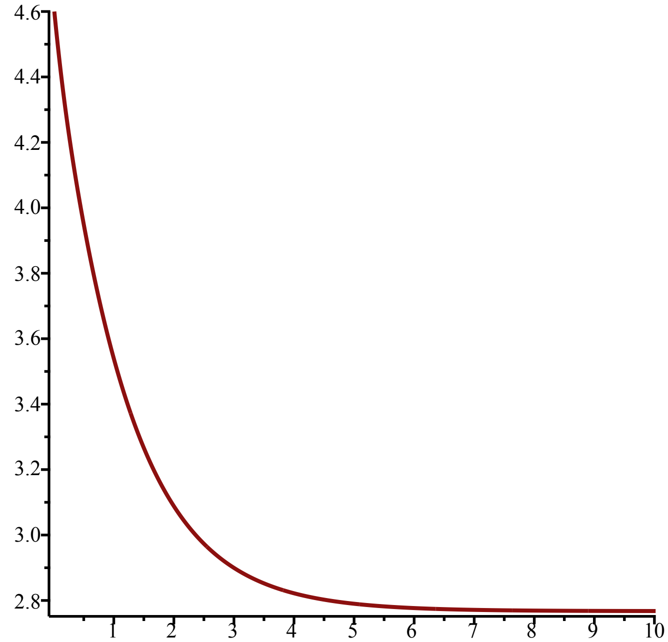
> *F* := *GeneNet*(2.6, 1.45, 7.51, 2, *m1*, *m2*, *m3*, *p1*, *p2*, *p3*)

$$F := \left[-m1 + \frac{1.45}{p3^2 + 1} + 2.6, -m2 + \frac{1.45}{p1^2 + 1} + 2.6, -m3 + \frac{1.45}{p2^2 + 1} + 2.6, -7.51\, p1 + 7.51\, m1, -7.51\, p2 + 7.51\, m2, -7.51\, p3 + 7.51\, m3\right] \quad (15)$$

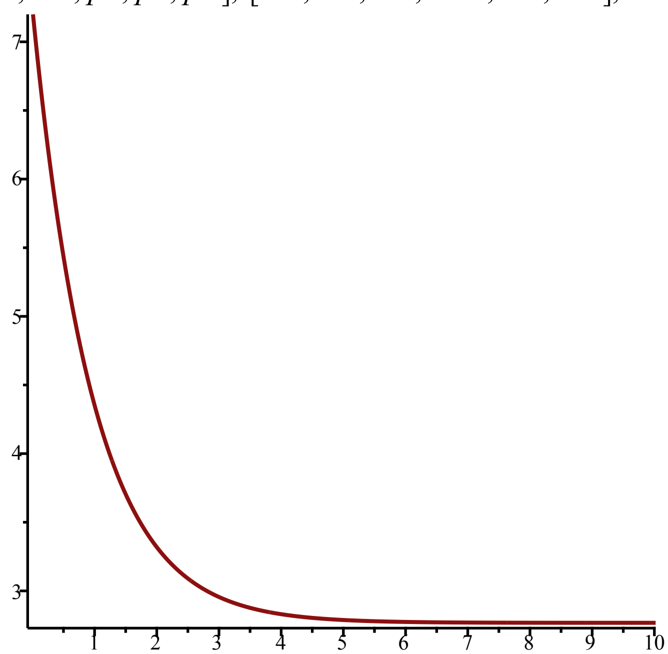
> *TimeSeries*(*F*, [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [1.3, 4.6, 7.2, 3.12, 5.7, 9.1], 0.01, 10, 1)



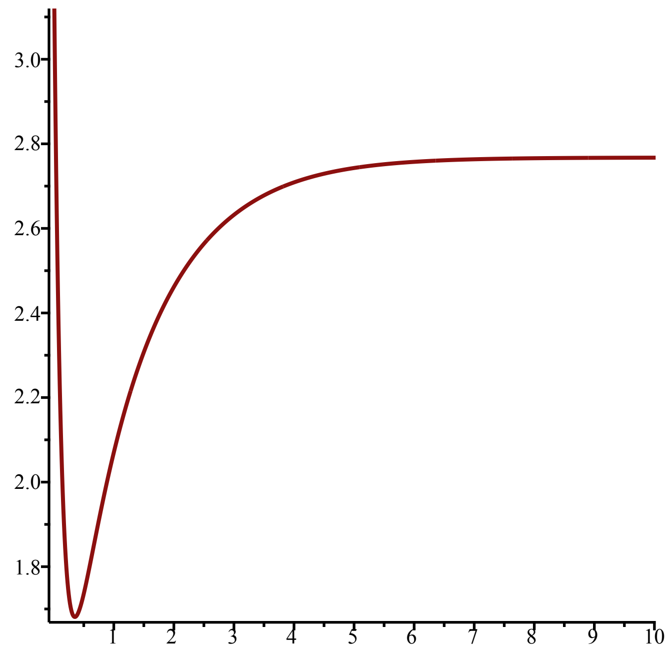
> *TimeSeries*(*F*, [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [1.3, 4.6, 7.2, 3.12, 5.7, 9.1], 0.01, 10, 2)



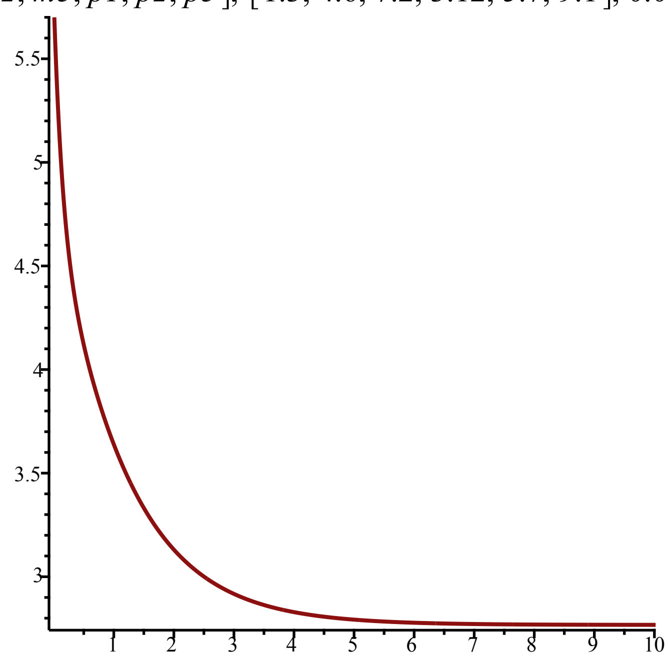
> *TimeSeries*(*F*, [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [1.3, 4.6, 7.2, 3.12, 5.7, 9.1], 0.01, 10, 3)



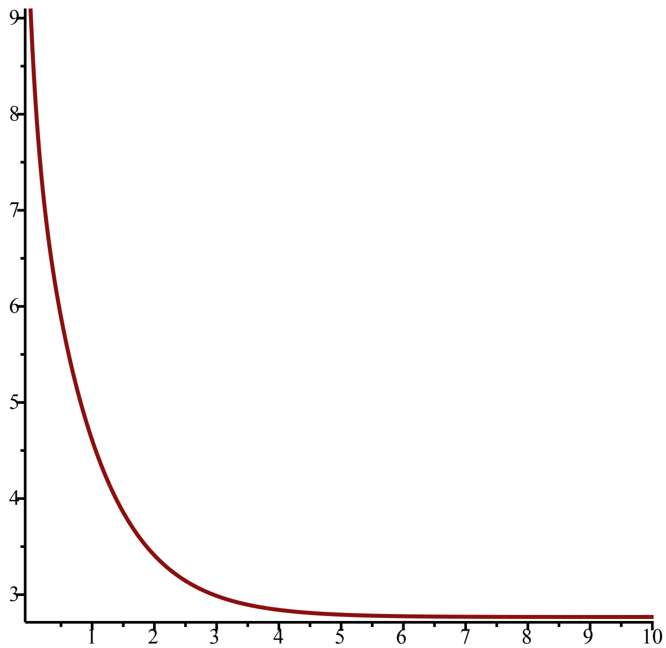
> *TimeSeries*(*F*, [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [1.3, 4.6, 7.2, 3.12, 5.7, 9.1], 0.01, 10, 4)



> *TimeSeries*(*F*, [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [1.3, 4.6, 7.2, 3.12, 5.7, 9.1], 0.01, 10, 5)



> *TimeSeries*(*F*, [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [1.3, 4.6, 7.2, 3.12, 5.7, 9.1], 0.01, 10, 6)



> $SEquP(F, [m1, m2, m3, p1, p2, p3])$

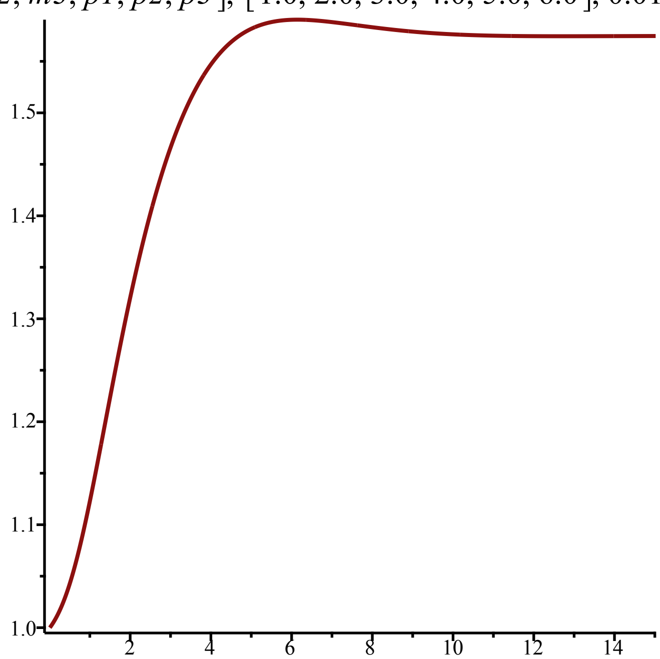
$$\{[2.767459115, 2.767459115, 2.767459115, 2.767459115, 2.767459115, 2.767459115]\} \quad \textbf{(16)}$$

> #With a0=1.0, a=2.0, b=3.0:

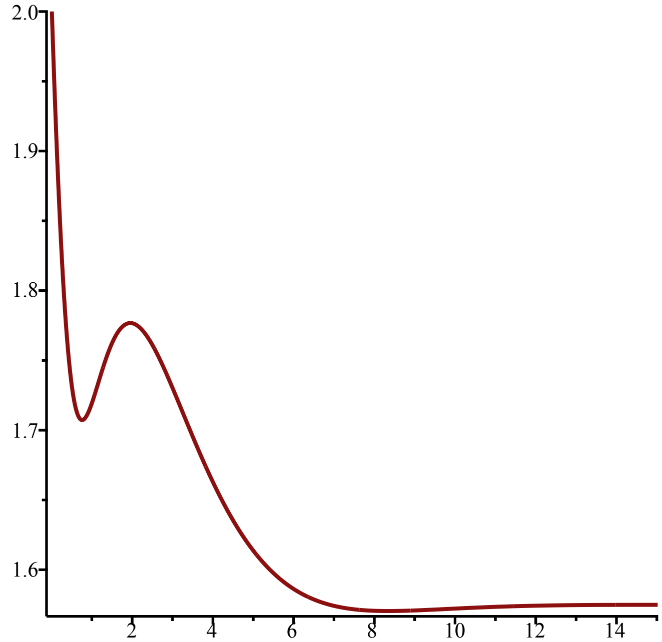
> $F := GeneNet(1.0, 2.0, 3.0, 2, m1, m2, m3, p1, p2, p3)$

$$F := \left[-m1 + \frac{2.0}{p3^2+1} + 1.0, -m2 + \frac{2.0}{p1^2+1} + 1.0, -m3 + \frac{2.0}{p2^2+1} + 1.0, -3.0\,p1 + 3.0\,m1, -3.0\,p2 + 3.0\,m2, -3.0\,p3 + 3.0\,m3\right] \quad \textbf{(17)}$$

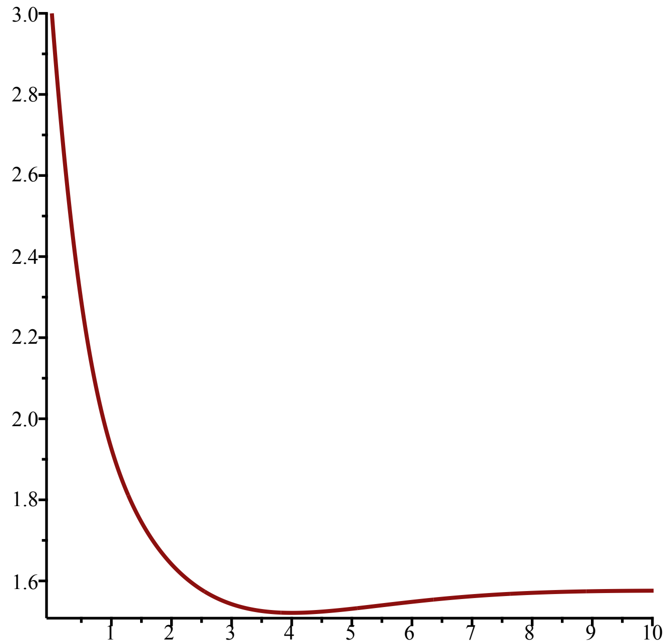
> $TimeSeries(F, [m1, m2, m3, p1, p2, p3], [1.0, 2.0, 3.0, 4.0, 5.0, 6.0], 0.01, 15, 1)$



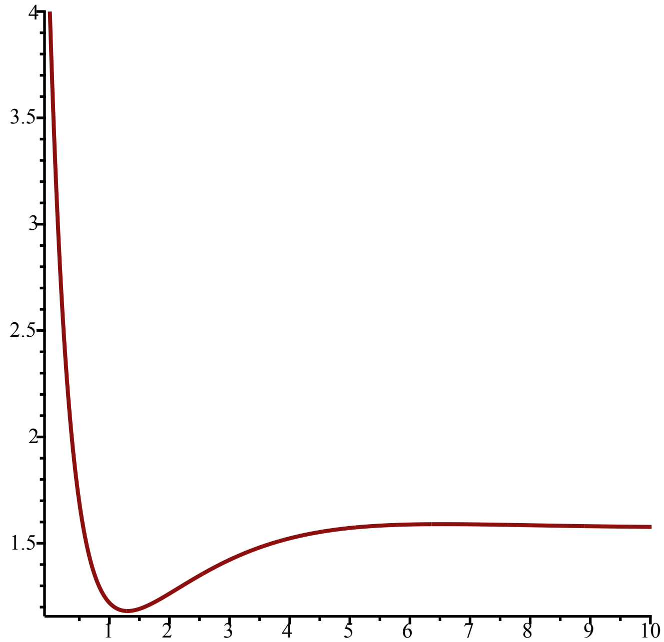
> $TimeSeries(F, [m1, m2, m3, p1, p2, p3], [1.0, 2.0, 3.0, 4.0, 5.0, 6.0], 0.01, 15, 2)$



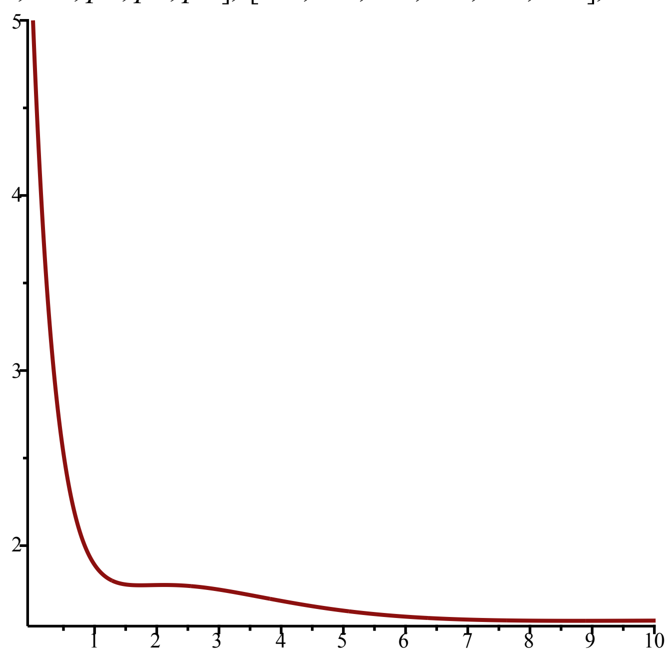
> *TimeSeries*(*F*, [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [1.0, 2.0, 3.0, 4.0, 5.0, 6.0], 0.01, 10, 3)



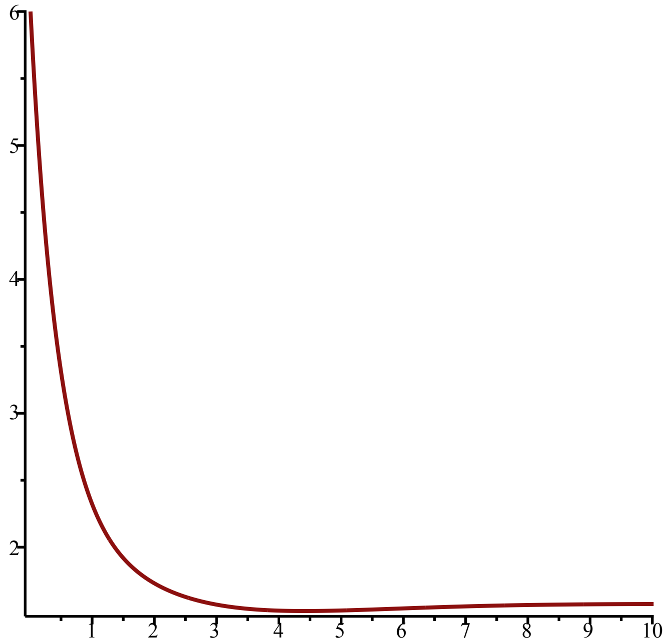
> *TimeSeries*(*F*, [*m1*, *m2*, *m3*, *p1*, *p2*, *p3*], [1.0, 2.0, 3.0, 4.0, 5.0, 6.0], 0.01, 10, 4)



> $TimeSeries(F, [m1, m2, m3, p1, p2, p3], [1.0, 2.0, 3.0, 4.0, 5.0, 6.0], 0.01, 10, 5)$



> $TimeSeries(F, [m1, m2, m3, p1, p2, p3], [1.0, 2.0, 3.0, 4.0, 5.0, 6.0], 0.01, 10, 6)$



> SEquP(F, [m1, m2, m3, p1, p2, p3])

{[1.574743074, 1.574743074, 1.574743074, 1.574743074, 1.574743074, 1.574743074]} **(18)**

>

>

>

> Help(Lotka)

Lotka(r1,k1,r2,k2,b12,b21,N1,N2): The Lotka-Volterra continuous-time dynamical system, Eqs. (9a),(9b) (p. 224, section 6.3) of Edelstein-Keshet

with popoluations N1, N2, and parameters r1,r2,k1,k2, b12, b21 (called there beta_12 and beta_21)

Try:

Lotka(r1,k1,r2,k2,b12,b21,N1,N2);

Lotka(1,2,2,3,1,2,N1,N2); **(19)**

> print(Lotka)

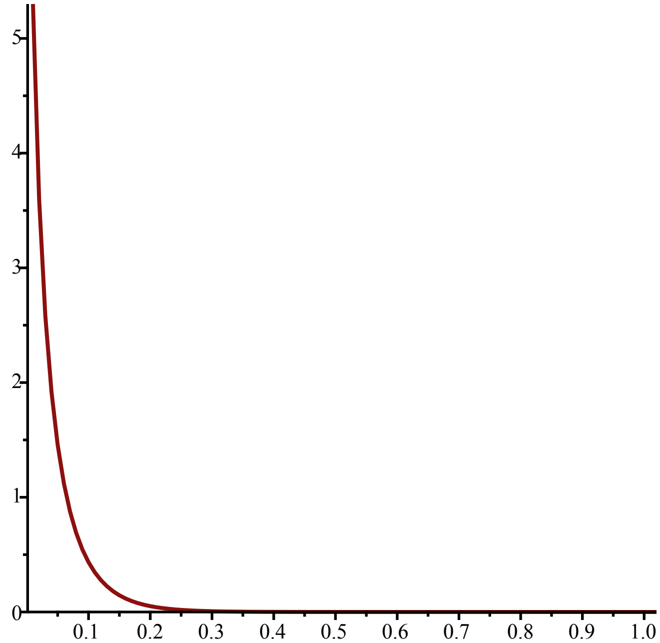
```
proc(r1, k1, r2, k2, b12, b21, N1, N2)
    [r1 * N1 * (k1 - N1 - b12 * N2) / k1, r2 * N2 * (k2 - N2 - b21 * N1) / k2]
end proc
```
(20)

> #With r1=4.5, r2=1.3, k1=2.3, k2=1.6, b12=6.4, b21=0.5:

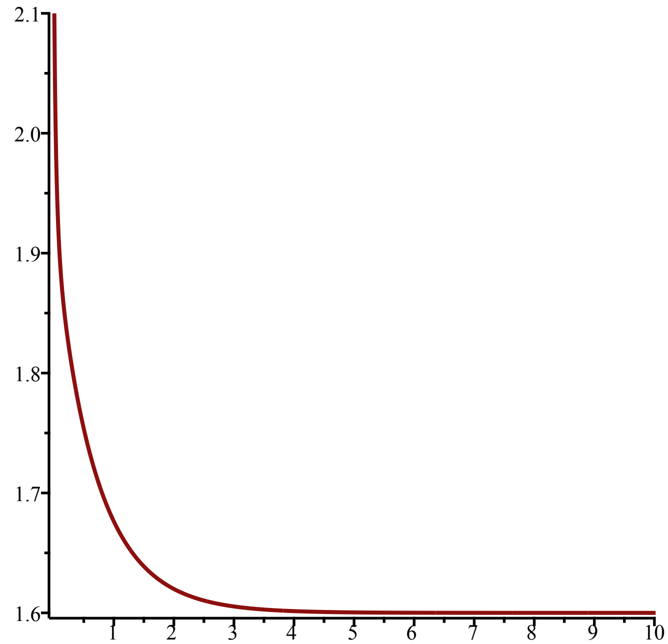
> F := Lotka(4.5, 2.3, 1.3, 1.6, 6.4, 0.5, N1, N2)

$F := [1.956521739\, N1\, (2.3 - N1 - 6.4\, N2), 0.8125000000\, N2\, (1.6 - N2 - 0.5\, N1)]$ **(21)**

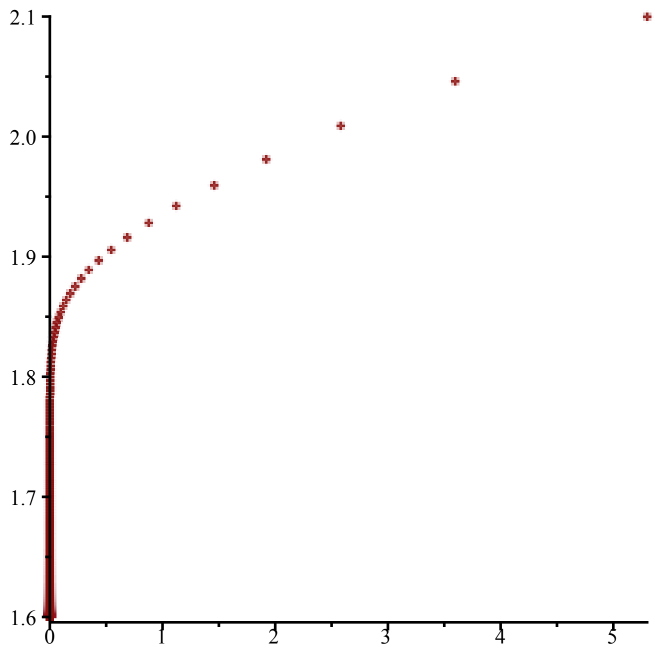
> TimeSeries(F, [N1, N2], [5.3, 2.1], 0.01, 1, 1)



> *TimeSeries*(*F*, [*N1*, *N2*], [5.3, 2.1], 0.01, 10, 2)



> *PhaseDiag*(*F*, [*N1*, *N2*], [5.3, 2.1], 0.01, 10)



> SEquP(F, [N1, N2])

$$\{[0., 1.600000000], [3.609090909, -0.2045454545]\} \quad (22)$$

> #The above timeseries are showing the first EQ point where N1=0, N2=1.6

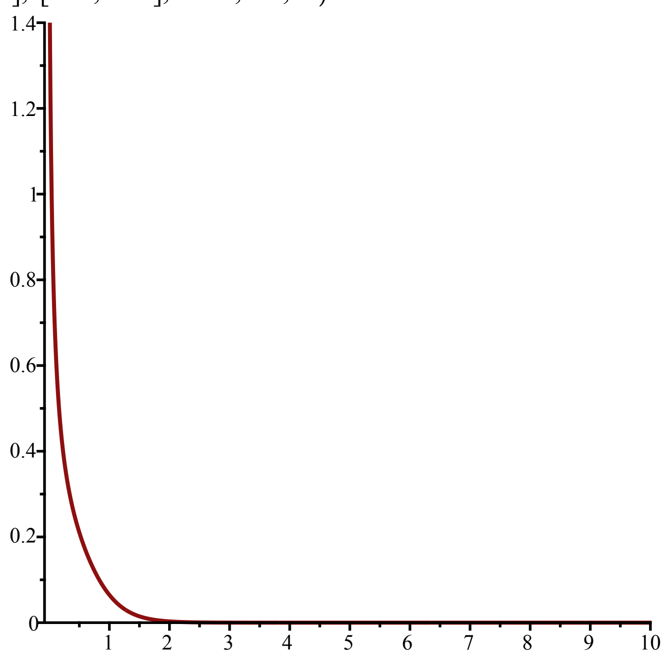
>

> #With r1=1.8, r2=6.2, k1=0.3, k2=0.6, b12=1.4, b21=0.9:

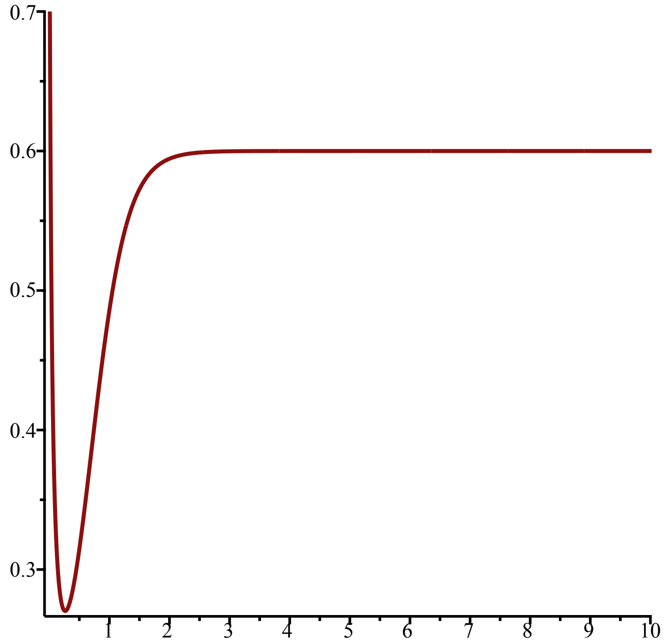
> F := Lotka(1.8, 0.3, 6.2, 0.6, 1.4, 0.9, N1, N2)

$$F := [6.000000000\, N1\, (0.3 - N1 - 1.4\, N2), 10.33333333\, N2\, (0.6 - N2 - 0.9\, N1)] \quad (23)$$

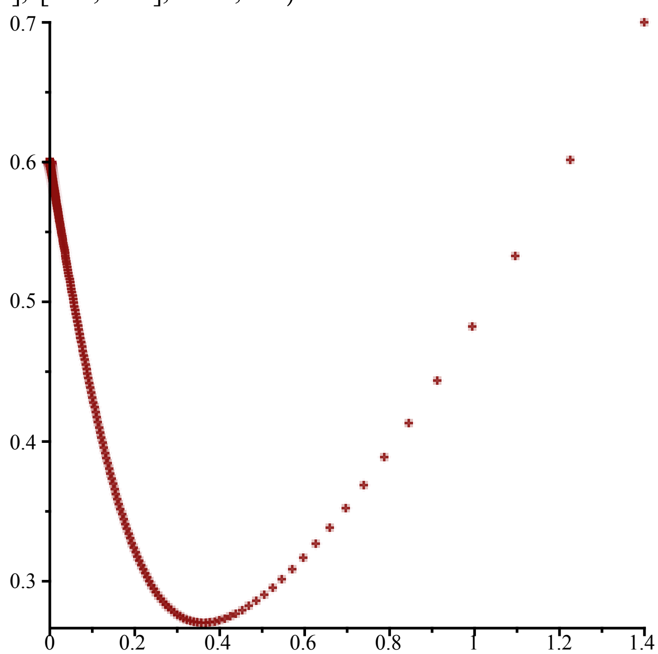
> TimeSeries(F, [N1, N2], [1.4, 0.7], 0.01, 10, 1)



> TimeSeries(F, [N1, N2], [1.4, 0.7], 0.01, 10, 2)



> *PhaseDiag*(*F*, [*N1*, *N2*], [1.4, 0.7], 0.01, 10)



> *SEquP*(*F*, [*N1*, *N2*])

{[0., 0.6000000000]} **(24)**

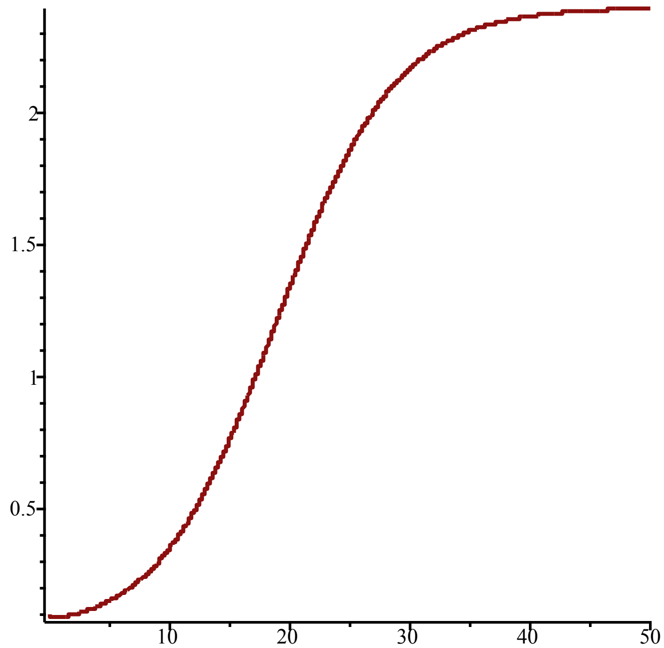
>

> *#With r1=0.2, r2=0.63, k1=2.4, k2=0.26, b12=4.1, b21=1.9:*

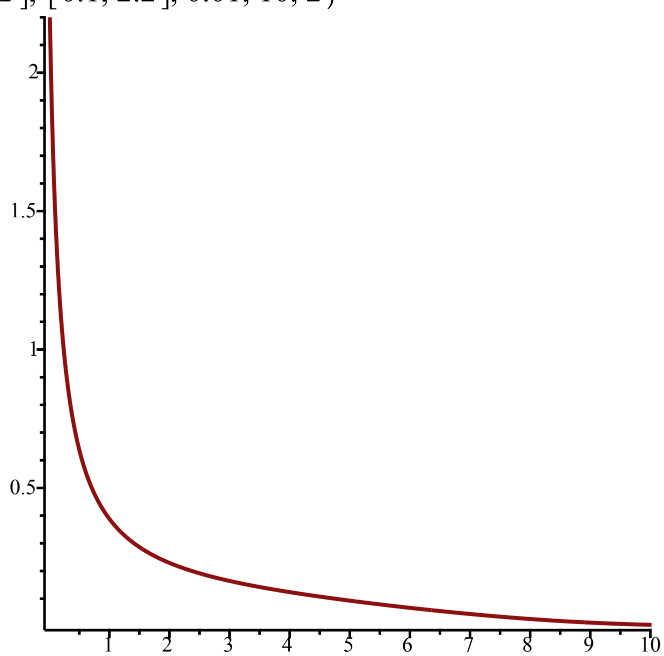
> $F := Lotka(0.2, 2.4, 0.63, 0.26, 4.1, 1.9, N1, N2)$

$F := [0.08333333333\ N1\ (2.4 - N1 - 4.1\ N2),\ 2.423076923\ N2\ (0.26 - N2 - 1.9\ N1)]$ **(25)**

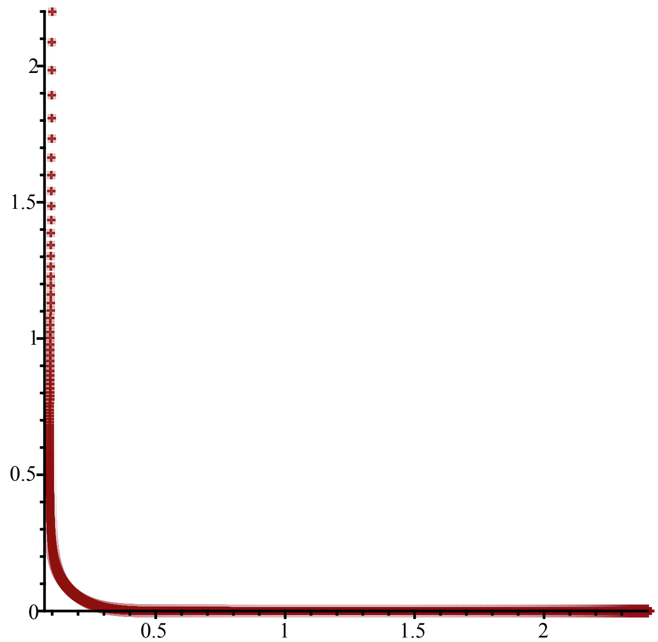
> *TimeSeries*(*F*, [*N1*, *N2*], [0.1, 2.2], 0.01, 50, 1)



> $TimeSeries(F, [N1, N2], [0.1, 2.2], 0.01, 10, 2)$



> $PhaseDiag(F, [N1, N2], [0.1, 2.2], 0.01, 100)$



> SEquP(F, [N1, N2])

$$\{[-0.1964653903, 0.6332842415], [2.400000000, 0.]\}$$ **(26)**

> #The above timeseries are showing the second EQ point where N1=2.4, N2=0.0

>

>

>

> Help(Volterra)

Volterra(a,b,c,d,x,y): The (simple, original) Volterra predator-prey continuous-time dynamical system with parameters a,b,c,d

Given by Eqs. (7a) (7b) in Edelstein-Keshet p. 219 (section 6.2).

a,b,c,d may be symbolic or numeric

Try:

Volterra(a,b,c,d,x,y);

Volterra(1,2,3,4,x,y); **(27)**

> print(Volterra)

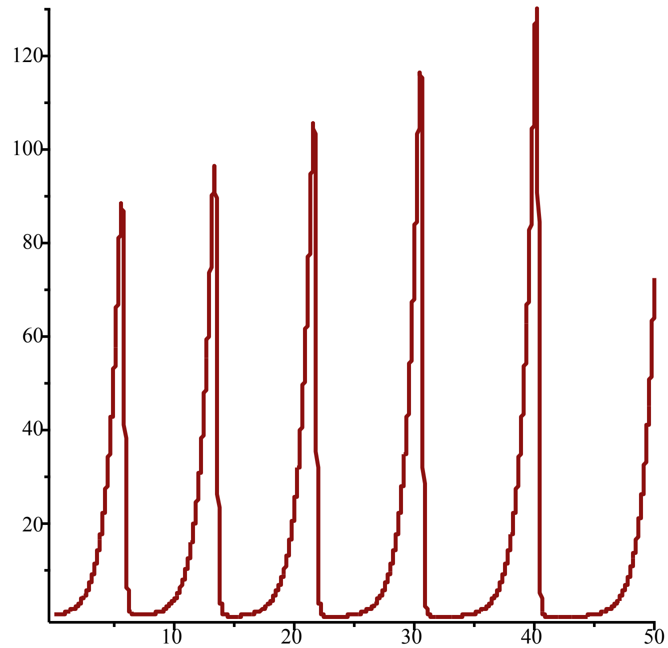
proc(a, b, c, d, x, y) [a*x − b*x*y, − c*y + d*x*y] **end proc** **(28)**

> #With a=1.0,b=0.33,c=2.3,d=0.14

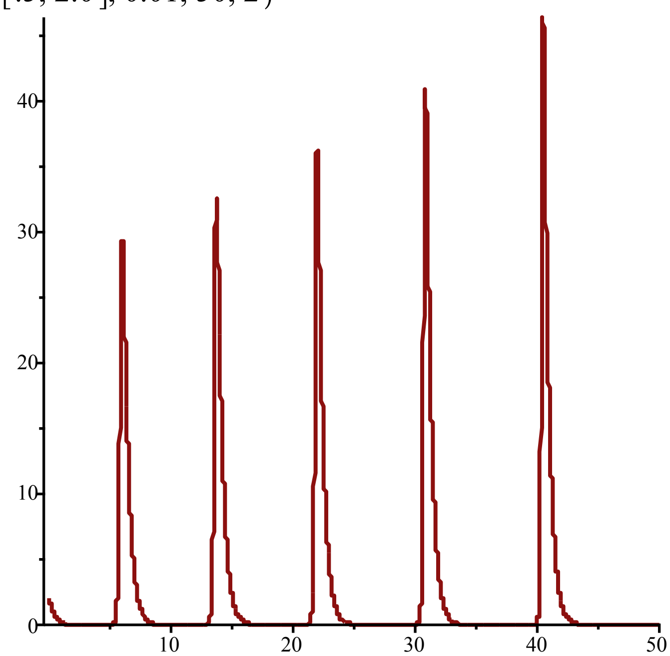
> F := Volterra(1.0, 0.33, 2.3, 0.14, x, y)

$$F := [1.0\,x - 0.33\,x\,y, -2.3\,y + 0.14\,x\,y]$$ **(29)**

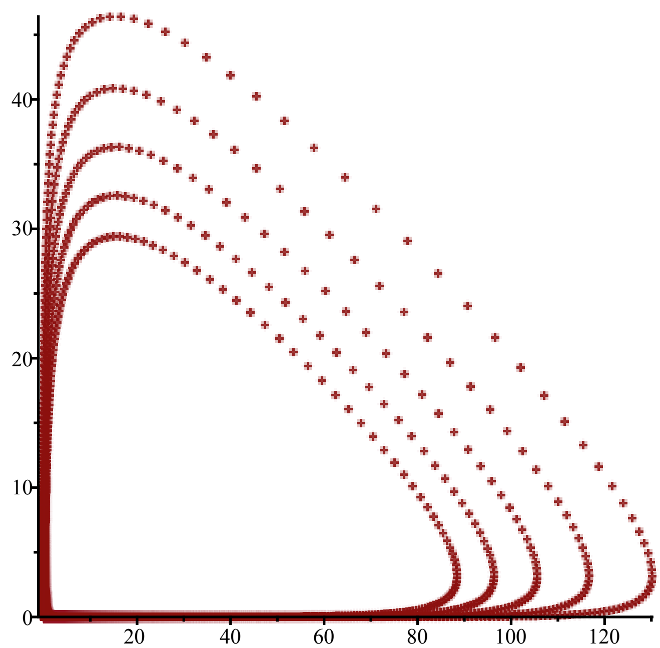
> TimeSeries(F, [x, y], [.5, 2.0], 0.01, 50, 1)



> *TimeSeries*(*F*, [*x*, *y*], [.5, 2.0], 0.01, 50, 2)



> *PhaseDiag*(*F*, [*x*, *y*], [.5, 2.0], 0.01, 50)



> $SEquP(F, [x, y])$

$$\varnothing \tag{30}$$

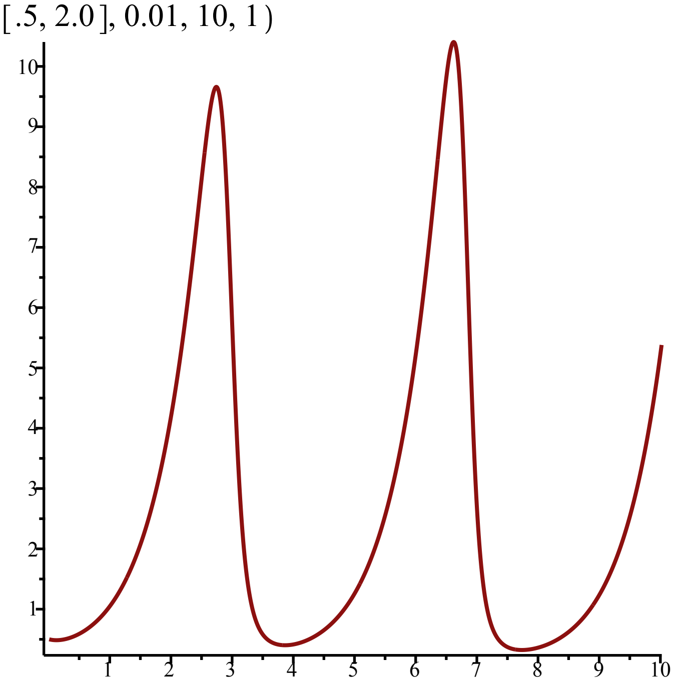
>

> #With a=1.5,b=1.0,c=3.0,d=1.0

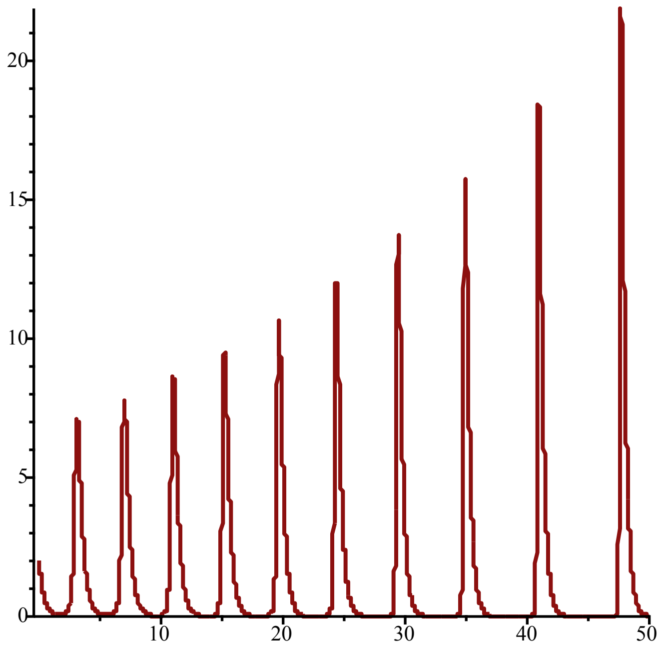
> $F := Volterra(1.5, 1.0, 3.0, 1.0, x, y)$

$$F := [1.5\,x - 1.0\,x\,y,\ -3.0\,y + 1.0\,x\,y] \tag{31}$$

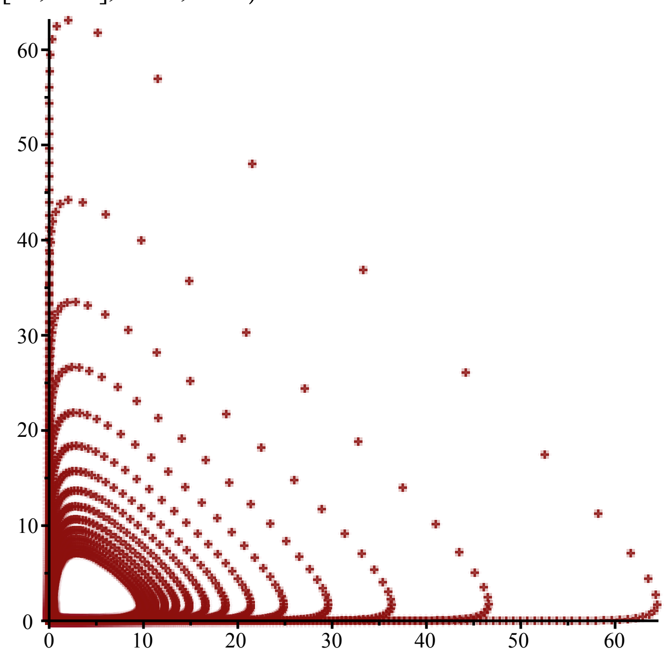
> $TimeSeries(F, [x, y], [.5, 2.0], 0.01, 10, 1)$



> $TimeSeries(F, [x, y], [.5, 2.0], 0.01, 50, 2)$



> *PhaseDiag*(*F*, [*x*, *y*], [.5, 2.0], 0.01, 100)



> *SEquP*(*F*, [*x*, *y*])

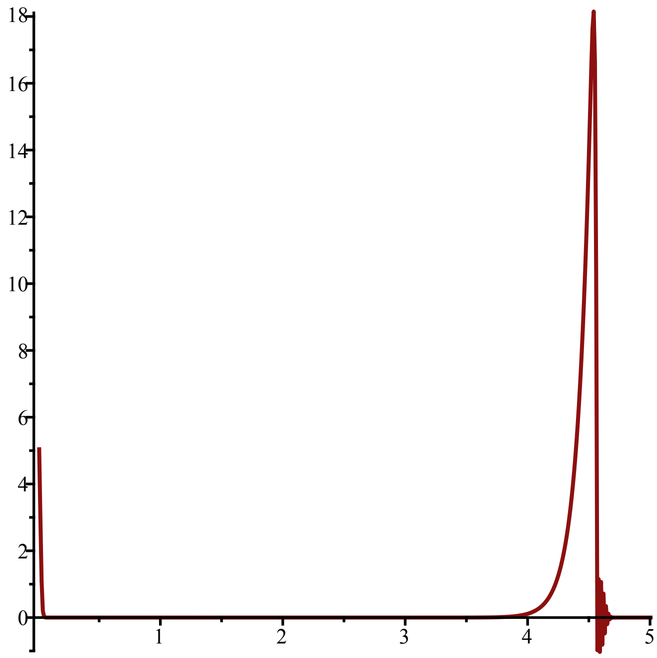
$$\varnothing \tag{32}$$

> *#With a=10.1, b=5.6, c=3.4, d=9.1*

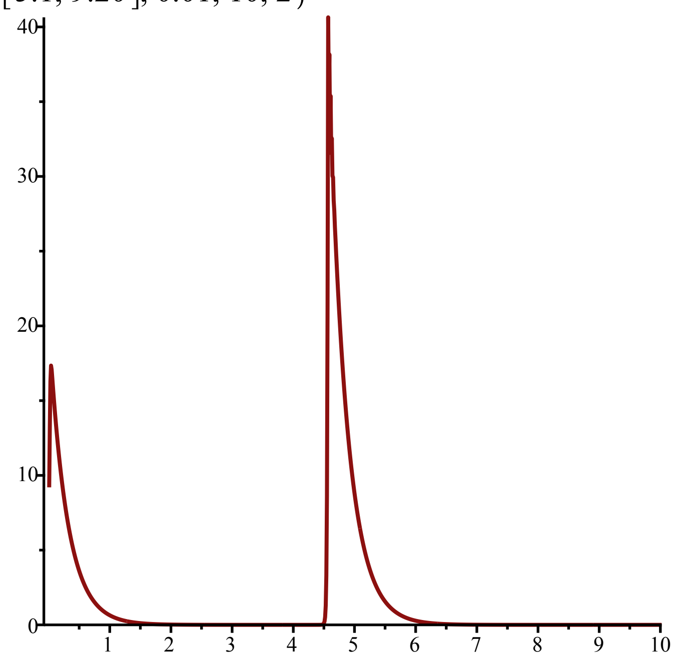
> *F* := *Volterra*(10.1, 5.6, 3.4, 9.1, *x*, *y*)

$$F := [10.1\,x - 5.6\,x\,y,\ \text{-}3.4\,y + 9.1\,x\,y] \tag{33}$$

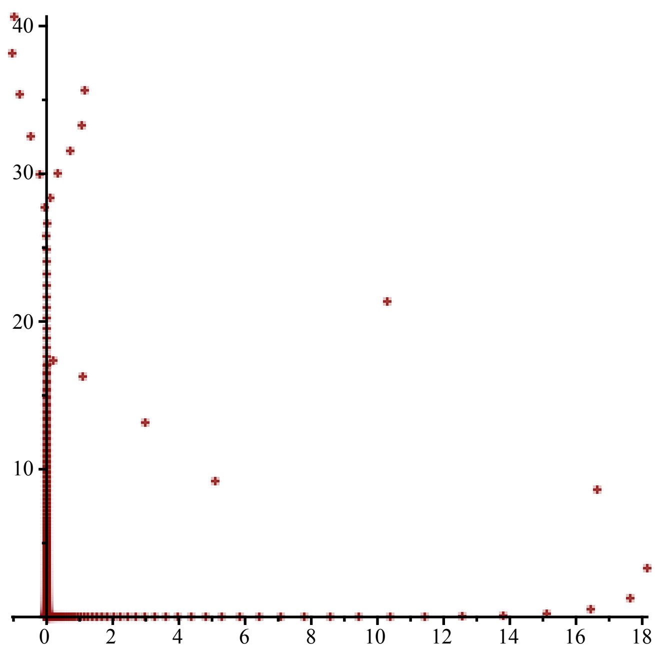
> *TimeSeries*(*F*, [*x*, *y*], [5.1, 9.20], 0.01, 5, 1)



> $TimeSeries(F, [x, y], [5.1, 9.20], 0.01, 10, 2)$



> $PhaseDiag(F, [x, y], [5.1, 9.20], 0.01, 10)$



> $SEquP(F, [x, y])$

$$\varnothing \quad (34)$$

> *#No stable equilibrium points and therefore, no horitzontal asymptotes on the timeseries!*

>

>

> $Help(VolterraM)$

VolterraM(a,b,c,d,x,K,y): The MODIFIED Volterra predator-prey continuous-time dynamical system with parameters a,b,c,d,K

Given by Eqs. (8a) (8b) in Edelstein-Keshet p. 220 (section 6.2).

a,b,c,d ,Kmay be symbolic or numeric

Try:

VolterraM(a,b,c,d,K,x,y);

VolterraM(1,2,3,4,3,x,y); **(35)**

> $print(VolterraM)$

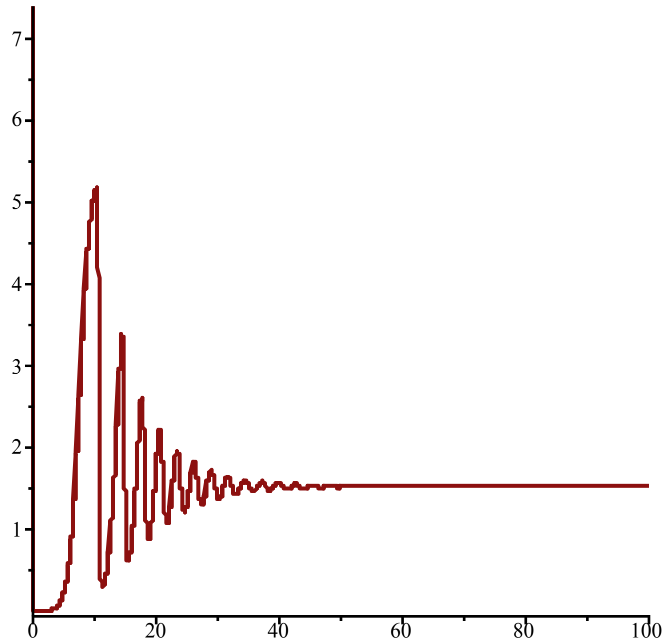
proc(a, b, c, K, d, x, y) $[a*x*(1 - x/K) - b*x*y, -c*y + d*x*y]$ **end proc** **(36)**

> *#With a=1.2, b=3.2, c=6.1, d=5.4, K=4*

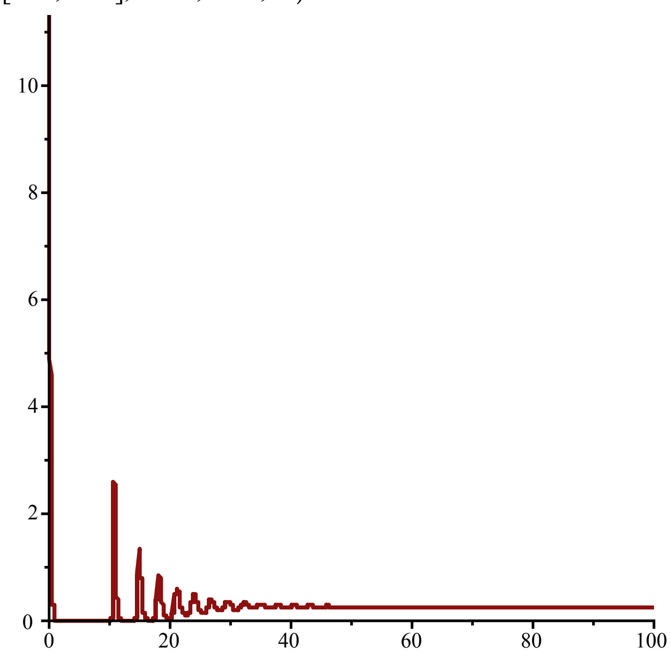
> $F := VolterraM(1.2, 3.2, 6.1, 5.4, 4, x, y)$

$$F := [1.2\,x\,(1 - 0.1851851852\,x) - 3.2\,x\,y, -6.1\,y + 4\,x\,y] \quad (37)$$

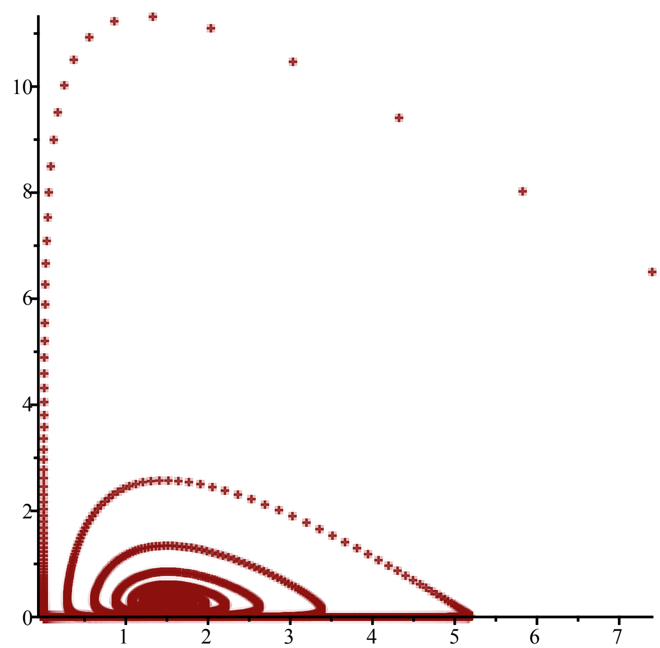
> $TimeSeries(F, [x, y], [7.4, 6.5], 0.01, 100, 1)$



> *TimeSeries*(*F*, [*x*, *y*], [7.4, 6.5], 0.01, 100, 2)



> *PhaseDiag*(*F*, [*x*, *y*], [7.4, 6.5], 0.01, 75)



> *SEquP*(*F*, [*x*, *y*])

$$\{[1.525000000, 0.2690972222]\}$$ **(38)**

> *#The timeseries show the EQ point where x=1.525 and y=0.269... because their asymptotes correspond to these values!*

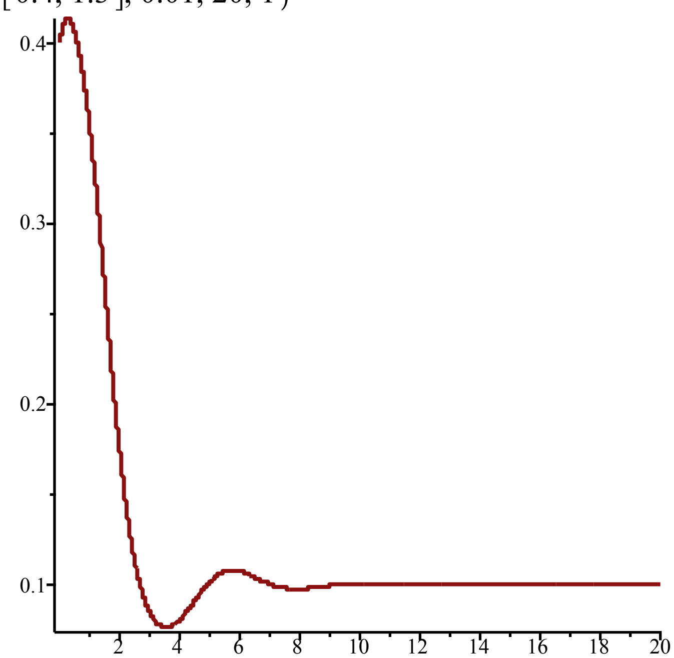
>

> *#With a=5.1, b=0.2, c=0.6, d=0.45, K=6*

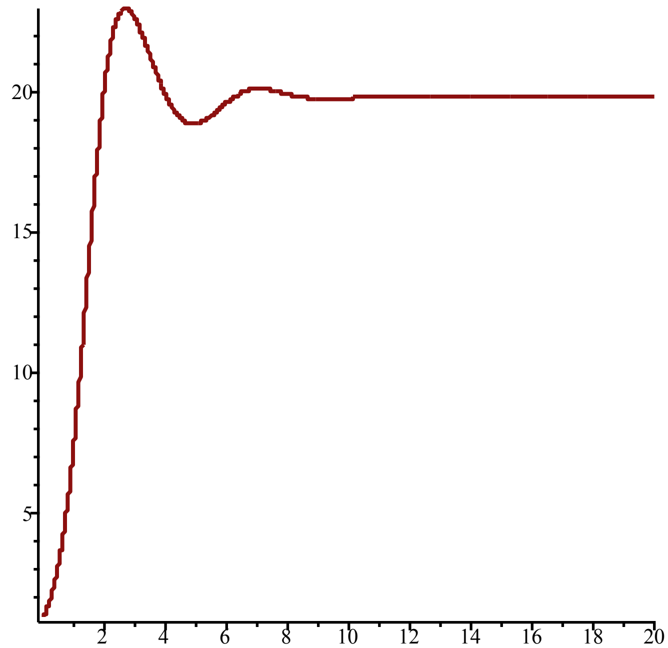
> $F := VolterraM(5.1, 0.2, 0.6, 0.45, 6, x, y)$

$$F := [5.1\, x\,(1 - 2.222222222\, x) - 0.2\, x\, y, -0.6\, y + 6\, x\, y]$$ **(39)**

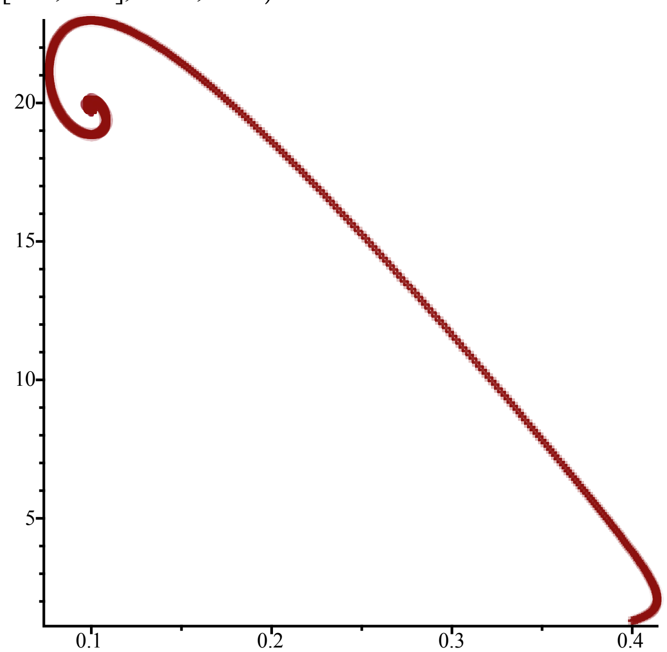
> $TimeSeries(F, [x, y], [0.4, 1.3], 0.01, 20, 1)$



> $TimeSeries(F, [x, y], [0.4, 1.3], 0.01, 20, 2)$



> $PhaseDiag(F, [x, y], [0.4, 1.3], 0.01, 100)$



> $SEquP(F, [x, y])$

$$\{[0.1000000000, 19.83333333]\} \quad (40)$$

> *#The timeseries show the EQ point where x=0.1 and y=19.8333... because their asymptotes correspond to these values!*

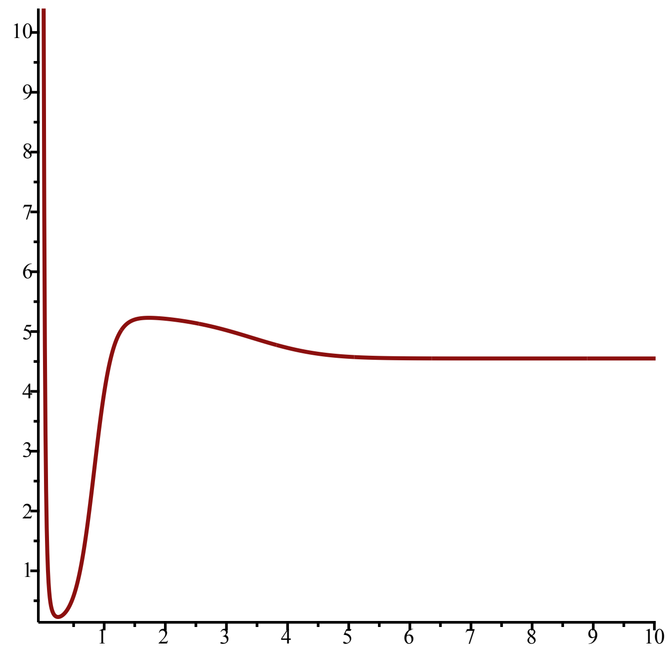
>

> *#With a=6.7, b=3.2, c=9.1, d=5.3, K=2*

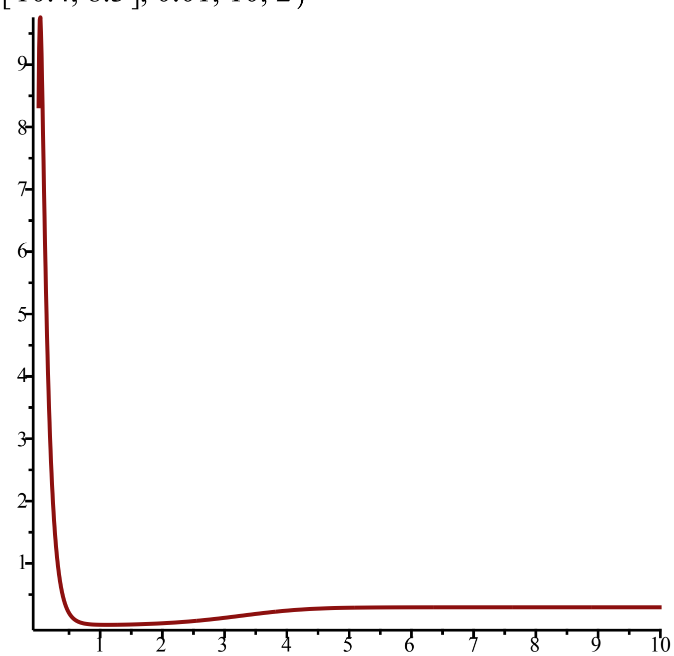
> $F := VolterraM(6.7, 3.2, 9.1, 5.3, 2, x, y)$

$$F := [6.7\,x\,(1 - 0.1886792453\,x) - 3.2\,x\,y, -9.1\,y + 2\,x\,y] \quad (41)$$

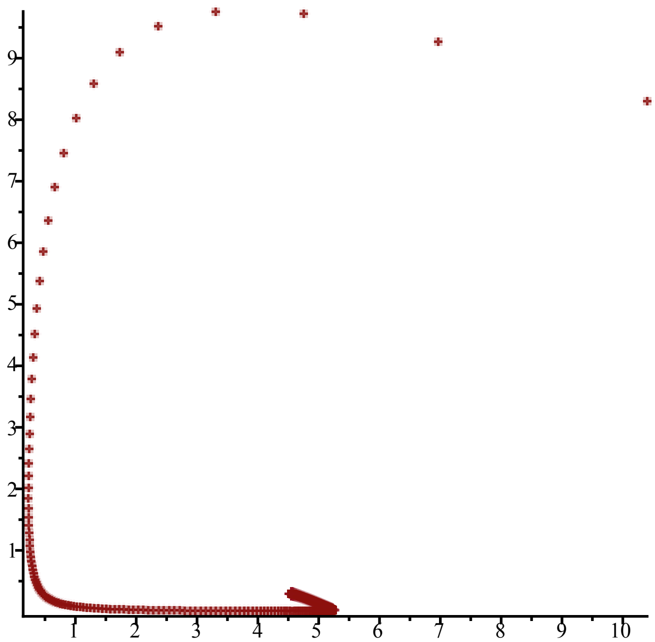
> $TimeSeries(F, [x, y], [10.4, 8.3], 0.01, 10, 1)$



> *TimeSeries*(*F*, [*x*, *y*], [10.4, 8.3], 0.01, 10, 2)



> *PhaseDiag*(*F*, [*x*, *y*], [10.4, 8.3], 0.01, 20)



> $SEquP(F, [x, y])$

$$\{[4.550000000, 0.2962853772]\} \quad (42)$$

> *#The timeseries show the EQ point where x=4.55 and y=0.29628... because their asymptotes correspond to these values!*