

```
> #Okay to post
> #Anusha Nagar, Homework 21, 11.13.2021
>
> read "C://Users/an646/Documents/DMB.txt"
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)

accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

*The most current version is available on WWW at:
<http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt> .
Please report all bugs to: DoronZeil at gmail dot com .*

*For general help, and a list of the MAIN functions,
type "Help()". For specific help type "Help(procedure_name);"*

*For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);*

*For a list of the functions that give examples of Discrete-time dynamical systems (some famous),
type: HelpDDM());*

For help with any of them type: Help(ProcedureName);

*For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM());
For help with any of them type: Help(ProcedureName);*

*> Help(ChemoStat)
ChemoStat(N,C,a1,a2): The Chemostat continuous-time dynamical system with N=Bacterial
population density, and C=nutrient Concentration in growth chamber (see Table 4.1 of
Edelstein-Keshet, p. 122)*

*with paramerts a1, a2, Equations (19a_ , (19b) in Edelestein-Keshet p. 127 (section 4.5, where
they are called alpha1, alpha2). a1 and a2 can be symbolic or numeric. Try:*

```
ChemoStat(N,C,a1,a2);  
ChemoStat(N,C,2,3);
```

(2)

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>
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```
> #For chemostat
```

```
> RNG := rand(1..10) :
```

```
> a11 := RNG( )
```

$a11 := 5$

(3)

```
> a21 := RNG( )
```

$a21 := 10$

(4)

```
> a12 := RNG( )
```

$a12 := 2$

(5)

```
> a22 := RNG( )
```

$a22 := 2$

(6)

```
> a13 := RNG( )
```

$a13 := 4$

(7)

```
> a23 := RNG( )
```

$a23 := 8$

(8)

```
> Fi_1 := ChemoStat(N, C, a11, a21)
```

$$Fi_1 := \left[\frac{5 C N}{C + 1} - N, -\frac{C N}{C + 1} - C + 10 \right]$$

(9)

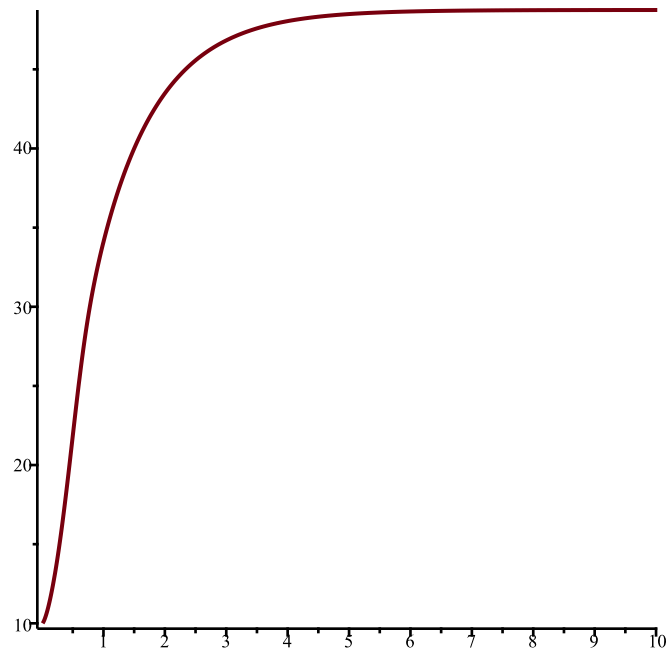
```
>
```

```
> SEquP(Fi_1, [N, C])
```

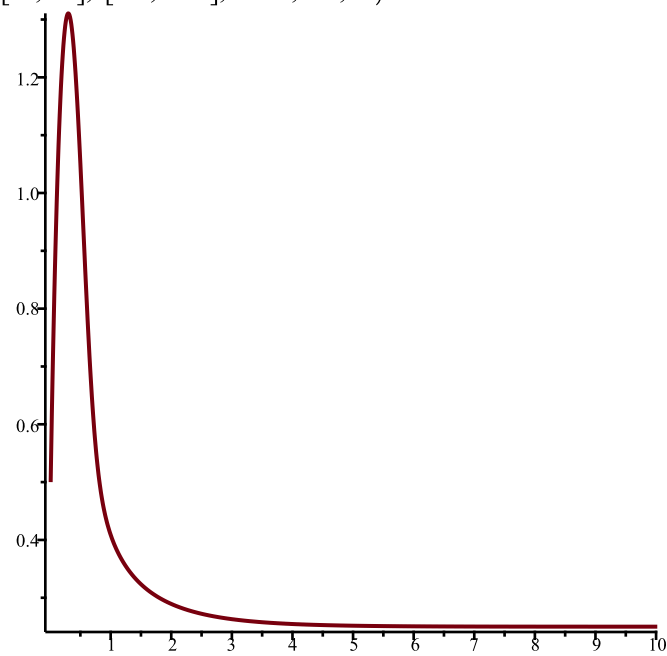
{[48.75000000, 0.2500000000]}

(10)

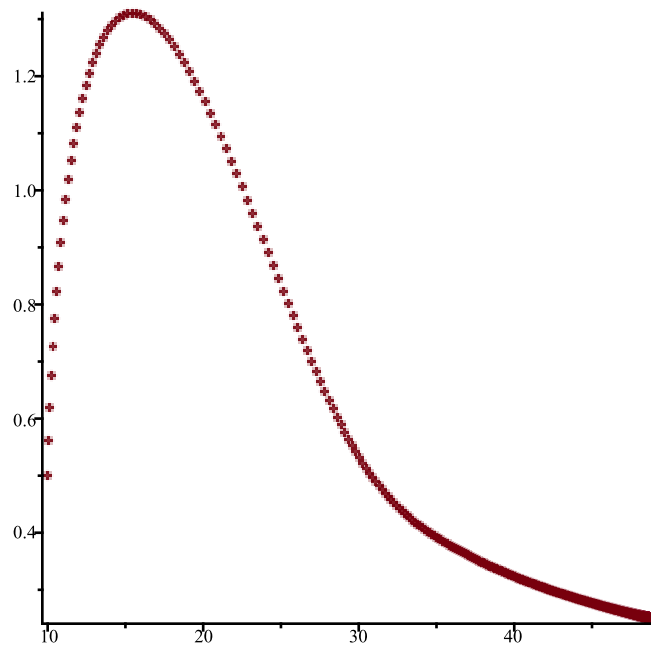
```
> TimeSeries(Fi_1, [N, C], [10, 0.5], 0.01, 10, 1)
```



> *TimeSeries*(*Fi_1*, [*N*, *C*], [10, 0.5], 0.01, 10, 2)



> *PhaseDiag*(*Fi_1*, [*N*, *C*], [10, 0.5], 0.01, 10)



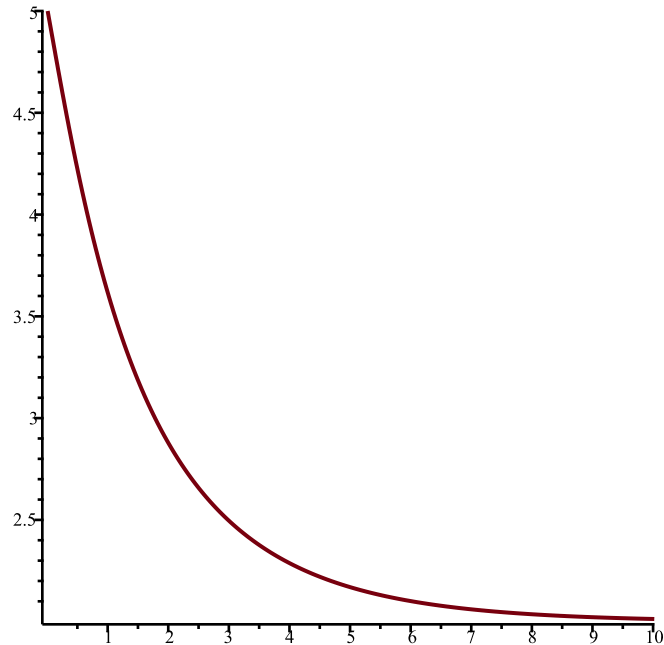
```
>
> Fi_2 := ChemoStat(N, C, a12, a22)
```

$$Fi_2 := \left[\frac{2CN}{C+1} - N, -\frac{CN}{C+1} - C + 2 \right] \quad (11)$$

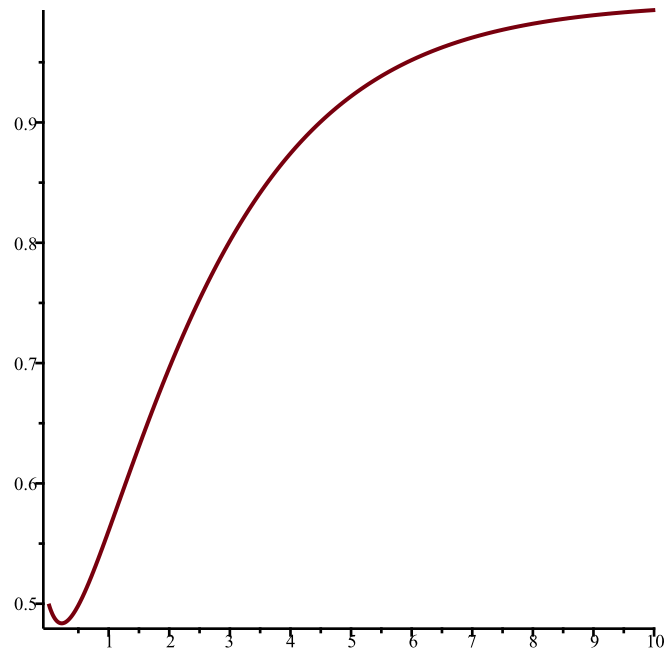
```
>
> SEquP(Fi_2, [N, C])
```

$$\{[2., 1.]\} \quad (12)$$

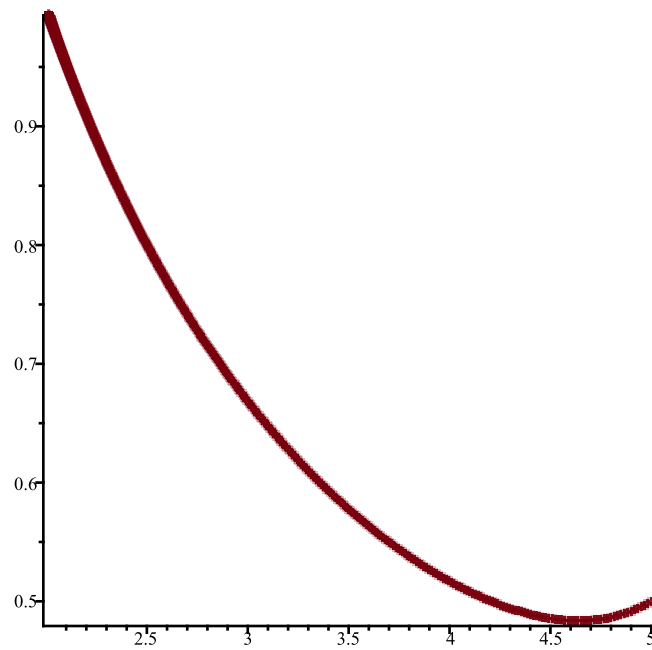
```
> TimeSeries(Fi_2, [N, C], [5, 0.5], 0.01, 10, 1)
```



```
> TimeSeries(Fi_2, [N, C], [5, 0.5], 0.01, 10, 2)
```



> PhaseDiag(Fi_2, [N, C], [5, 0.5], 0.01, 10)



> Fi_3 := ChemoStat(N, C, a13, a23)

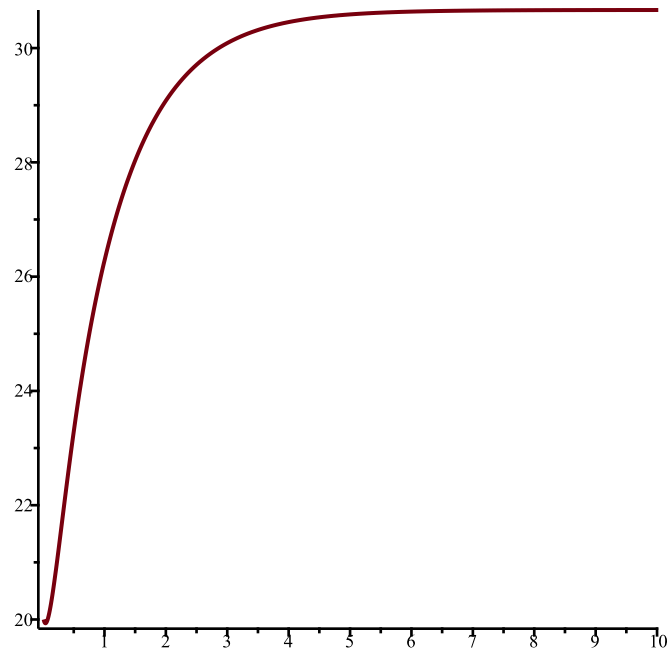
$$Fi_3 := \left[\frac{4CN}{C+1} - N, -\frac{CN}{C+1} - C + 8 \right] \quad (13)$$

>

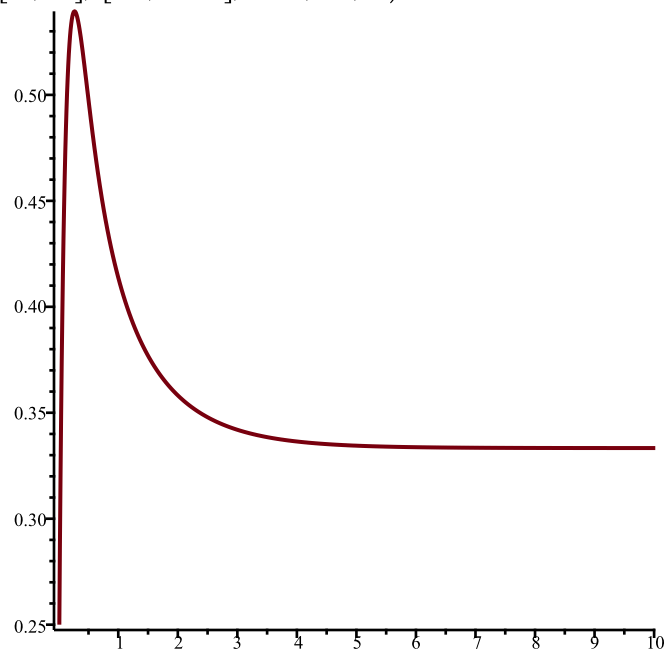
> SEquP(Fi_3, [N, C])

$$\{ [30.66666667, 0.3333333333] \} \quad (14)$$

> TimeSeries(Fi_3, [N, C], [20, 0.25], 0.01, 10, 1)



> *TimeSeries*(*Fi_3*, [*N*, *C*], [20, 0.25], 0.01, 10, 2)



> *PhaseDiag*(*Fi_3*, [*N*, *C*], [20, 0.25], 0.01, 10)

> $b_2 := \text{RNG2}()$ $b_2 := 0.8757751878$ (23)

> $n_2 := \text{RNG}()$ $n_2 := 3$ (24)

> $a_{03} := \text{RNG2}()$ $a_{03} := 0.3157837057$ (25)

> $a_3 := \text{RNG2}()$ $a_3 := 0.05872123377$ (26)

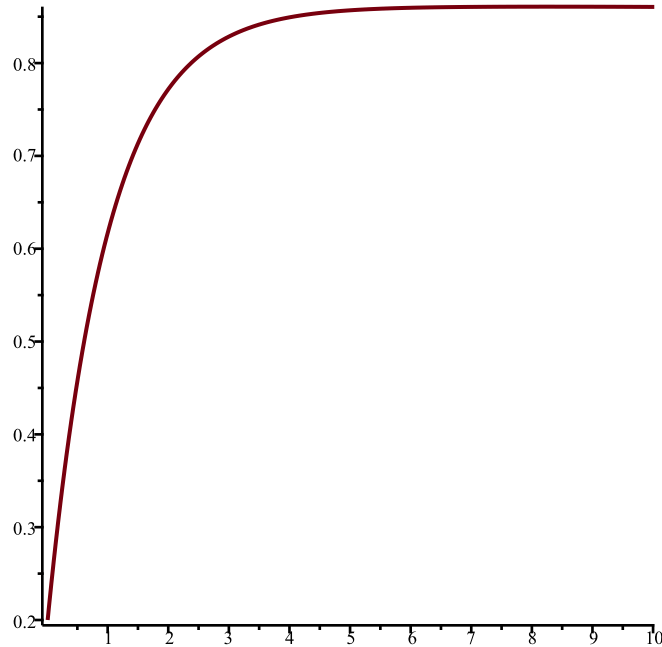
> $b_3 := \text{RNG2}()$ $b_3 := 0.5108327385$ (27)

> $n_3 := \text{RNG}()$ $n_3 := 5$ (28)

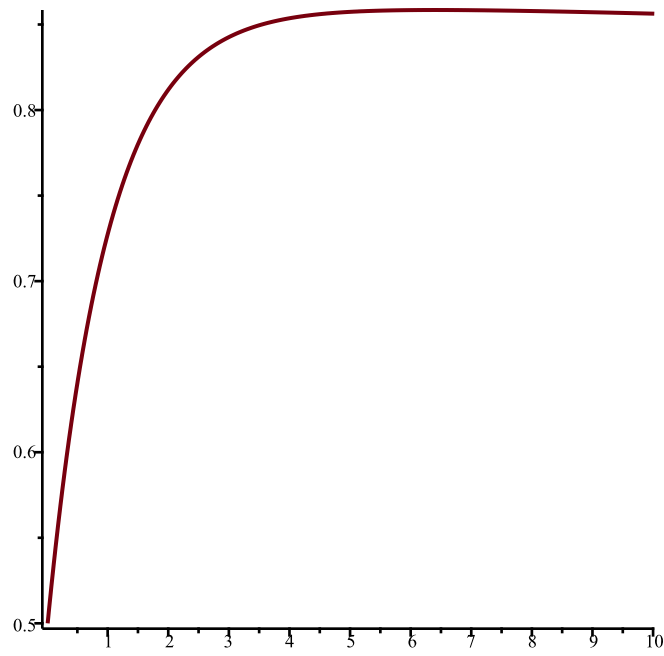
> $Fii_1 := \text{GeneNet}(a_{01}, a_1, b_1, n_1, m1, m2, m3, p1, p2, p3)$
 $Fii_1 := \left[-m1 + \frac{0.1331490761}{p3^6 + 1} + 0.7281441144, -m2 + \frac{0.1331490761}{p1^6 + 1} + 0.7281441144, \right.$ (29)
 $-m3 + \frac{0.1331490761}{p2^6 + 1} + 0.7281441144, -0.06587642124 p1 + 0.06587642124 m1,$
 $\left. -0.06587642124 p2 + 0.06587642124 m2, -0.06587642124 p3 + 0.06587642124 m3 \right]$

> $\text{SEquP}(Fii_1, [m1, m2, m3, p1, p2, p3])$

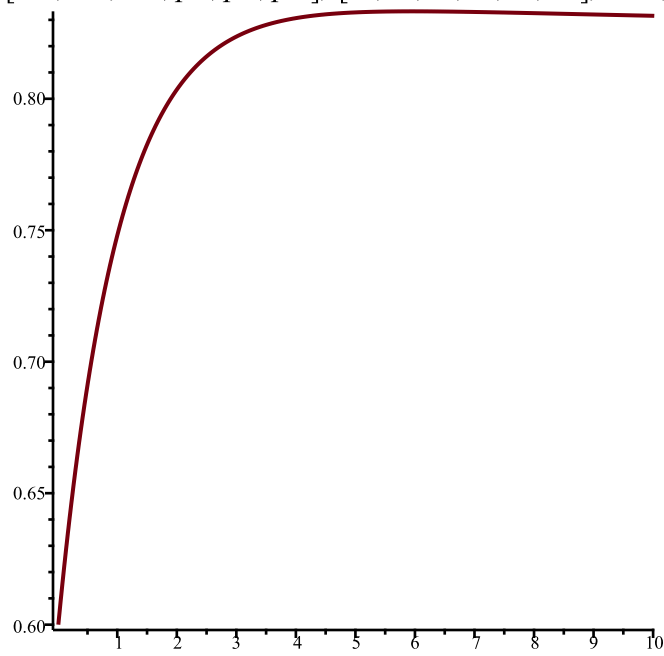
> $\text{TimeSeries}(Fii_1, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 1)$



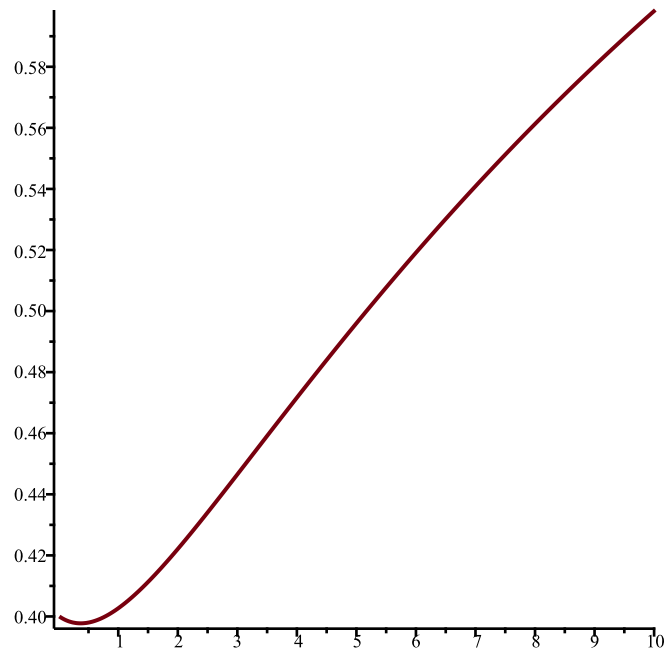
> $\text{TimeSeries}(Fii_1, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 2)$



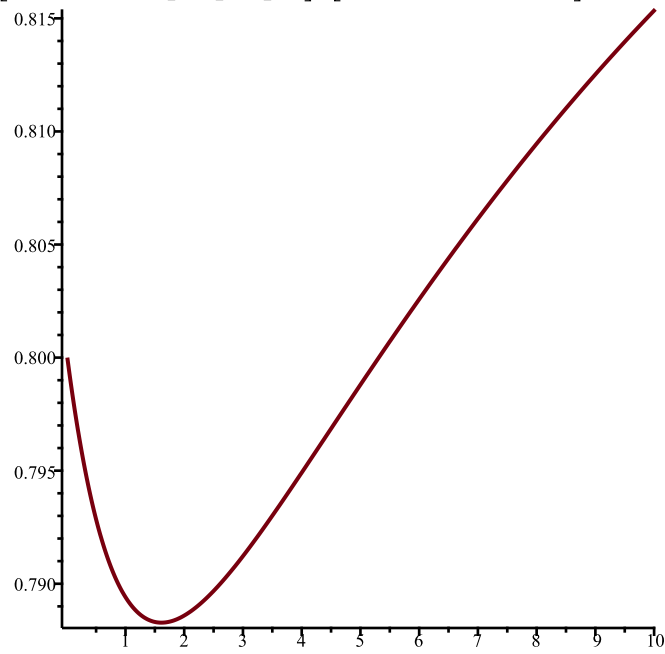
> *TimeSeries(Fii_1, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 3)*



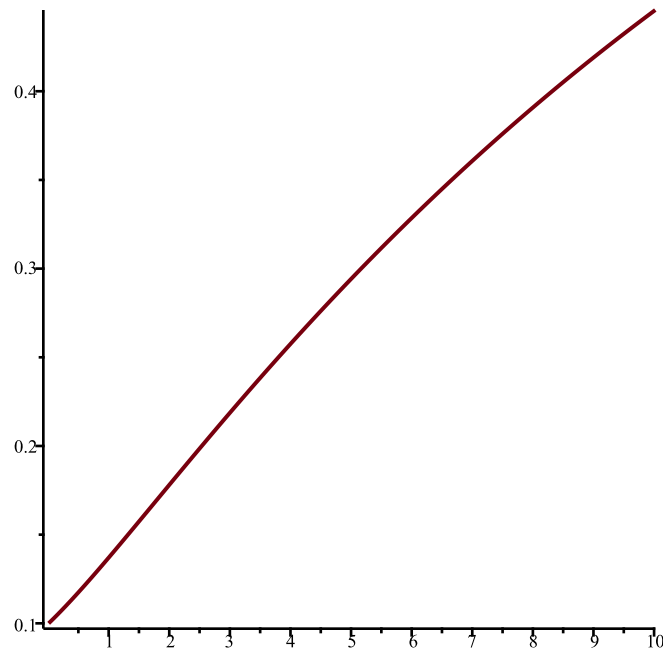
> *TimeSeries(Fii_1, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 4)*



> *TimeSeries(Fii_1, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 5)*



> *TimeSeries(Fii_1, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 6)*



>

> $Fii_2 := GeneNet(a_02, a_2, b_2, n_2, m1, m2, m3, p1, p2, p3)$

$$Fii_2 := \left[-m1 + \frac{0.7355427430}{p3^3 + 1} + 0.3909313924, -m2 + \frac{0.7355427430}{p1^3 + 1} + 0.3909313924, \right. \quad (30)$$

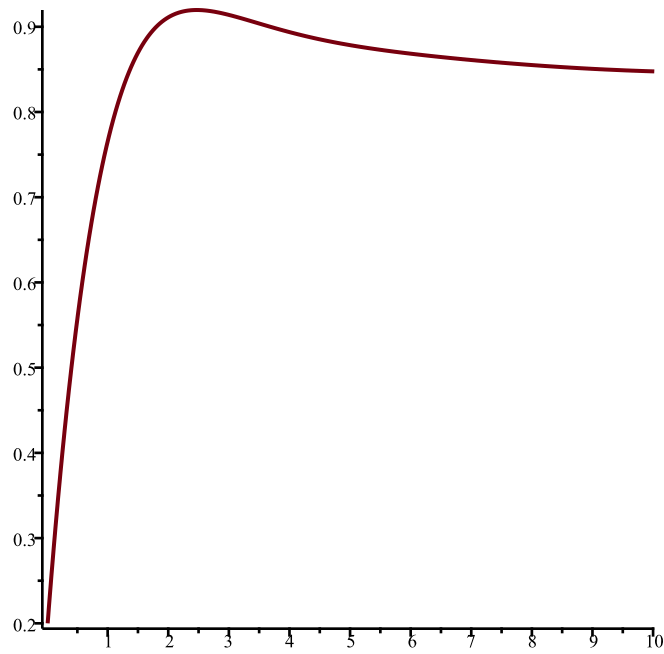
$$\left. -m3 + \frac{0.7355427430}{p2^3 + 1} + 0.3909313924, -0.8757751878 p1 + 0.8757751878 m1, \right.$$

$$\left. -0.8757751878 p2 + 0.8757751878 m2, -0.8757751878 p3 + 0.8757751878 m3 \right]$$

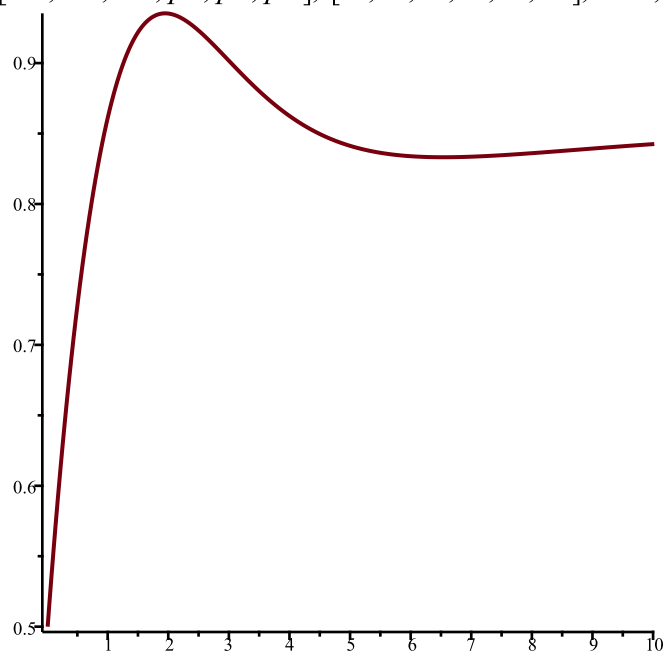
> $SEquP(Fii_1, [m1, m2, m3, p1, p2, p3])$

Interrupt with data = 0000000000000000

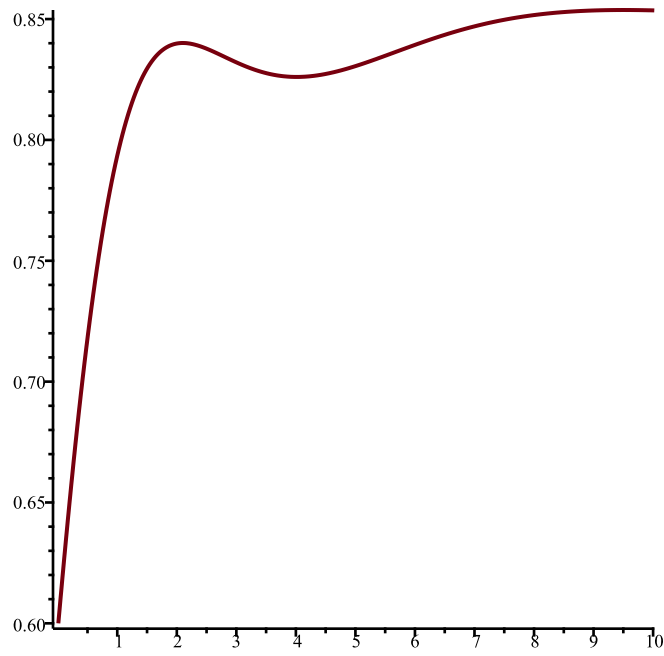
> $TimeSeries(Fii_2, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 1)$



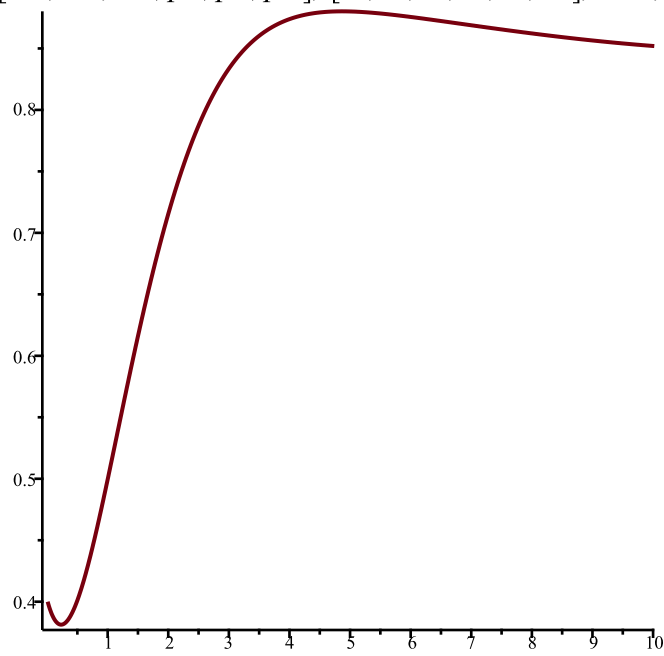
> *TimeSeries(Fii_2, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 2)*



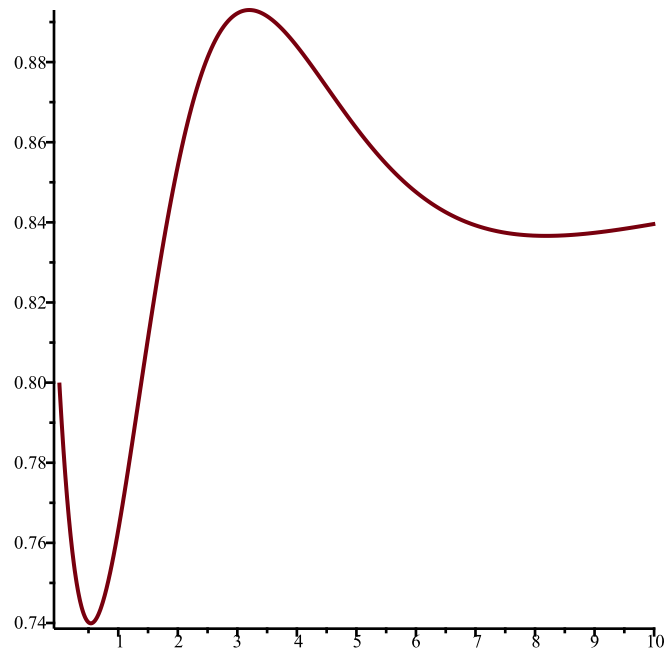
> *TimeSeries(Fii_2, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 3)*



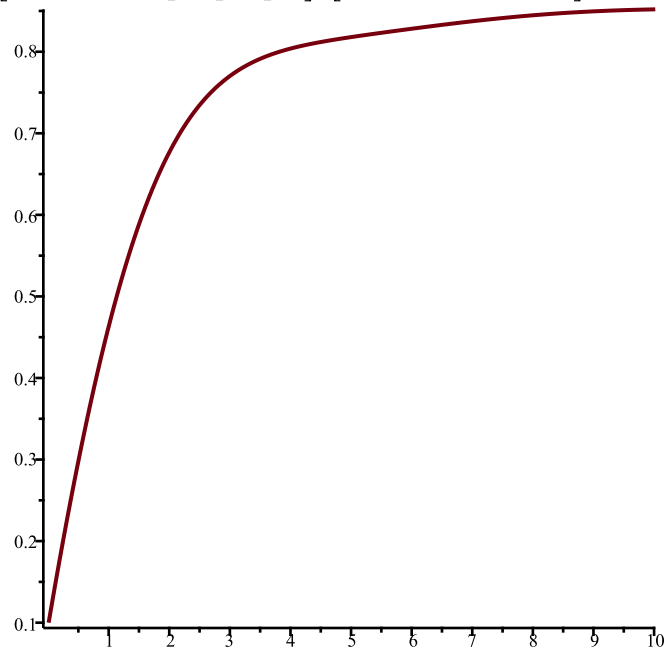
> *TimeSeries(Fii_2, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 4)*



> *TimeSeries(Fii_2, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 5)*



> TimeSeries(Fii_2, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 6)



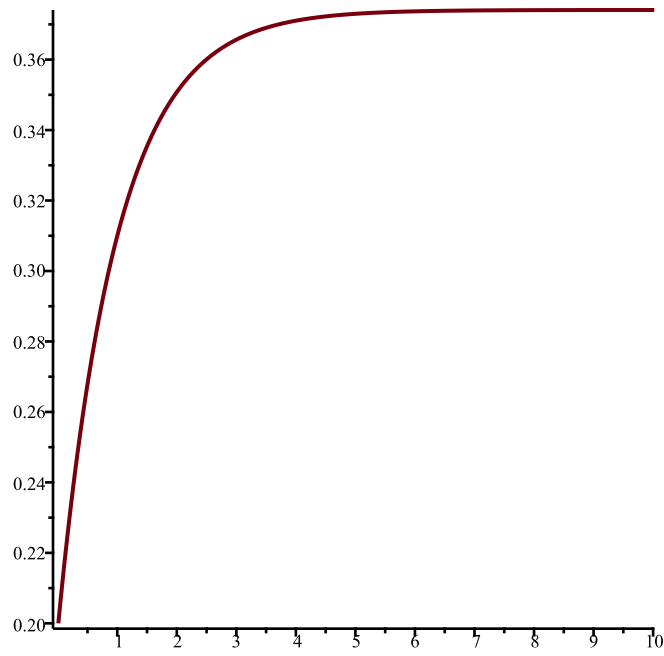
> Fii_3 := GeneNet(a_03, a_3, b_3, n_3, m1, m2, m3, p1, p2, p3)

$$\begin{aligned}
 Fii_3 := & \left[-m1 + \frac{0.05872123377}{p3^5 + 1} + 0.3157837057, -m2 + \frac{0.05872123377}{p1^5 + 1} \right. \\
 & + 0.3157837057, -m3 + \frac{0.05872123377}{p2^5 + 1} + 0.3157837057, -0.5108327385 p1 \\
 & + 0.5108327385 m1, -0.5108327385 p2 + 0.5108327385 m2, -0.5108327385 p3 \\
 & \left. + 0.5108327385 m3 \right]
 \end{aligned}$$

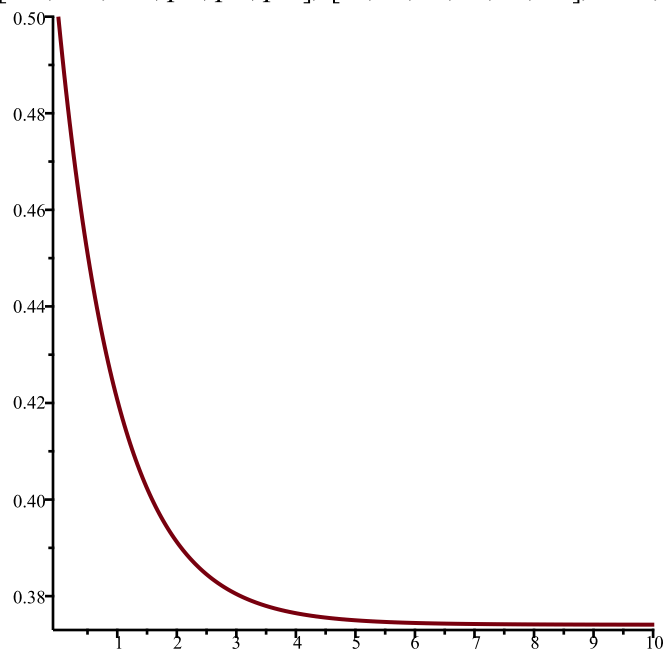
(31)

> SEquP(Fii_1, [m1, m2, m3, p1, p2, p3])

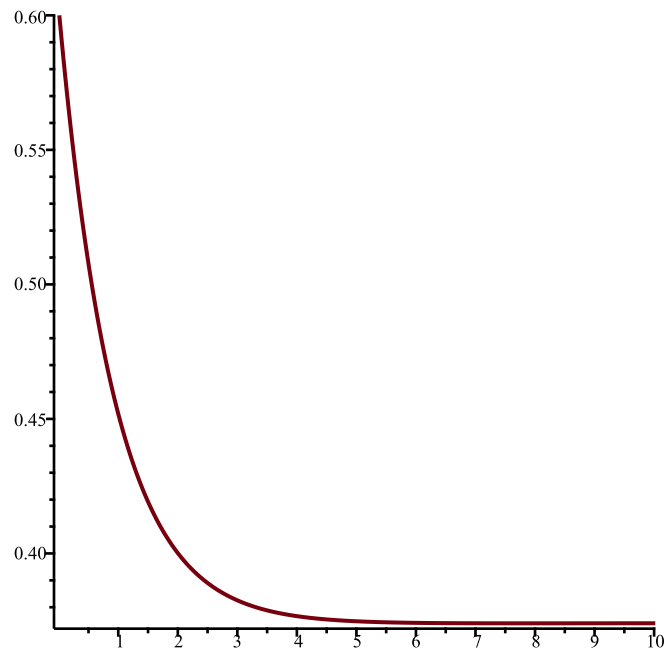
> TimeSeries(Fii_3, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 1)



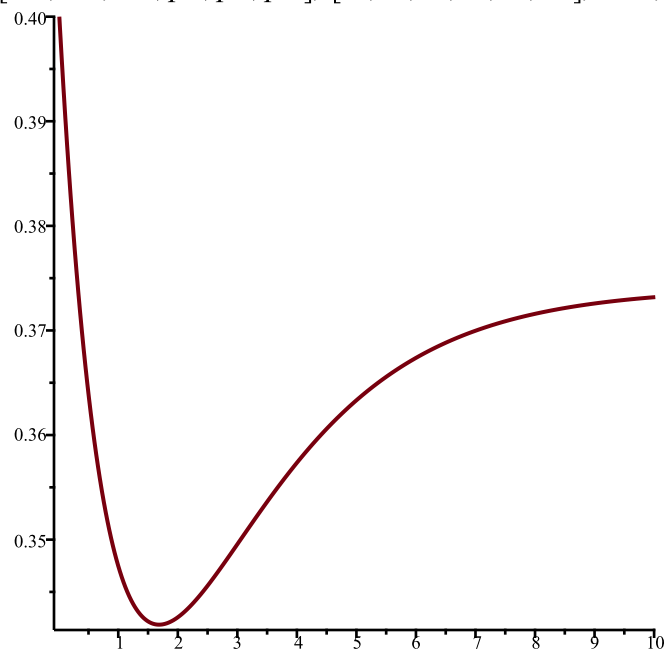
> *TimeSeries(Fii_3, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 2)*



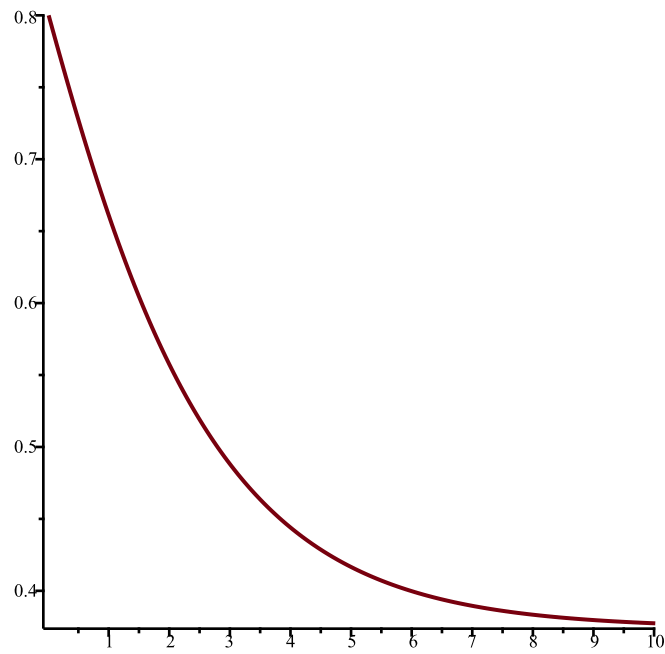
> *TimeSeries(Fii_3, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 3)*



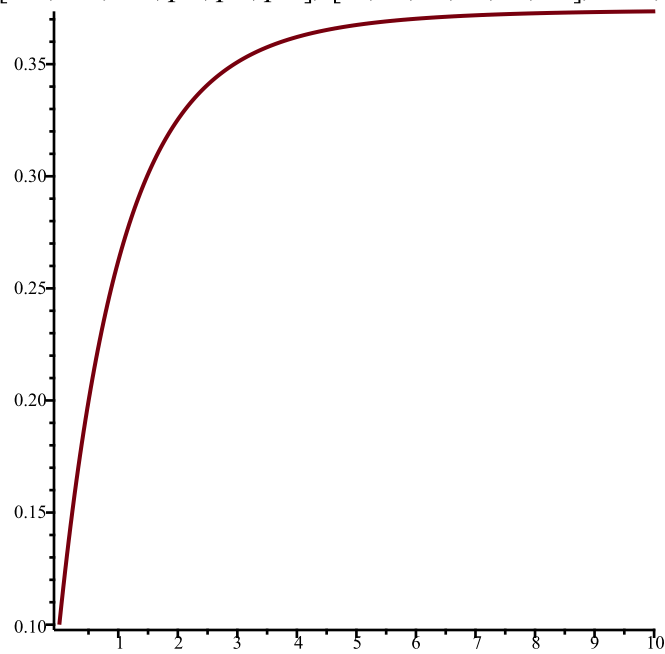
> *TimeSeries(Fii_3, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 4)*



> *TimeSeries(Fii_3, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 5)*



> *TimeSeries(Fii_3, [m1, m2, m3, p1, p2, p3], [.2, .5, .6, .4, .8, .1], 0.01, 10, 6)*



>
>

> *Help(Lotka)*

Lotka(r1,k1,r2,k2,b12,b21,N1,N2): The Lotka-Volterra continuous-time dynamical system, Eqs. (9a),(9b) (p. 224, section 6.3) of Edelstein-Keshet with popoulations N1, N2, and parameters r1,r2,k1,k2, b12, b21 (called there beta_12 and beta_21)

Try:

Lotka(r1,k1,r2,k2,b12,b21,N1,N2);

$$\text{Lotka}(1,2,2,3,1,2,N1,N2); \quad (32)$$

$$\begin{aligned} > r11 := \text{RNG}(\) \\ & r11 := 2 \end{aligned} \quad (33)$$

$$\begin{aligned} > k11 := \text{RNG}(\) \\ & k11 := 3 \end{aligned} \quad (34)$$

$$\begin{aligned} > r21 := \text{RNG}(\) \\ & r21 := 1 \end{aligned} \quad (35)$$

$$\begin{aligned} > k21 := \text{RNG}(\) \\ & k21 := 10 \end{aligned} \quad (36)$$

$$\begin{aligned} > b121 := \text{RNG}(\) \\ & b121 := 7 \end{aligned} \quad (37)$$

$$\begin{aligned} > b211 := \text{RNG}(\) \\ & b211 := 6 \end{aligned} \quad (38)$$

$$\begin{aligned} > \\ > r12 := \text{RNG}(\) \\ & r12 := 10 \end{aligned} \quad (39)$$

$$\begin{aligned} > k12 := \text{RNG}(\) \\ & k12 := 7 \end{aligned} \quad (40)$$

$$\begin{aligned} > r22 := \text{RNG}(\) \\ & r22 := 9 \end{aligned} \quad (41)$$

$$\begin{aligned} > k22 := \text{RNG}(\) \\ & k22 := 6 \end{aligned} \quad (42)$$

$$\begin{aligned} > b122 := \text{RNG}(\) \\ & b122 := 1 \end{aligned} \quad (43)$$

$$\begin{aligned} > b212 := \text{RNG}(\) \\ & b212 := 5 \end{aligned} \quad (44)$$

$$\begin{aligned} > r13 := \text{RNG}(\) \\ & r13 := 2 \end{aligned} \quad (45)$$

$$\begin{aligned} > k13 := \text{RNG}(\) \\ & k13 := 3 \end{aligned} \quad (46)$$

$$\begin{aligned} > r23 := \text{RNG}(\) \\ & r23 := 10 \end{aligned} \quad (47)$$

$$\begin{aligned} > k23 := \text{RNG}(\) \\ & k23 := 5 \end{aligned} \quad (48)$$

$$\begin{aligned} > b123 := \text{RNG}(\) \\ & b123 := 3 \end{aligned} \quad (49)$$

$$\begin{aligned} > b213 := \text{RNG}(\) \\ & b213 := 8 \end{aligned} \quad (50)$$

$$\begin{aligned} > Fiii_1 := \text{Lotka}(r11, k11, r21, k21, b121, b211, N1, N2) \end{aligned} \quad (51)$$

$$Fiii_1 := \left[\frac{2 N1 (3 - N1 - 7 N2)}{3}, \frac{N2 (10 - N2 - 6 N1)}{10} \right] \quad (51)$$

$$\text{SEquP}(Fiii_1, [N1, N2]) \quad \{[0., 10.], [3., 0.]\} \quad (52)$$

$$Fiii_2 := \text{Lotka}(r12, k12, r22, k22, b122, b212, N1, N2)$$

$$Fiii_2 := \left[\frac{10 N1 (7 - N1 - N2)}{7}, \frac{3 N2 (6 - N2 - 5 N1)}{2} \right] \quad (53)$$

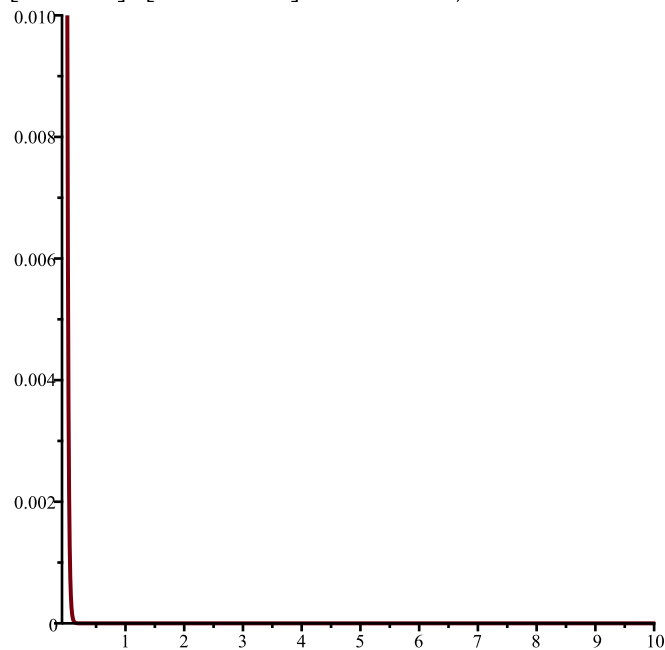
$$\text{SEquP}(Fiii_2, [N1, N2]) \quad \{[-0.2500000000, 7.2500000000], [7., 0.]\} \quad (54)$$

$$Fiii_3 := \text{Lotka}(r13, k13, r23, k23, b123, b213, N1, N2)$$

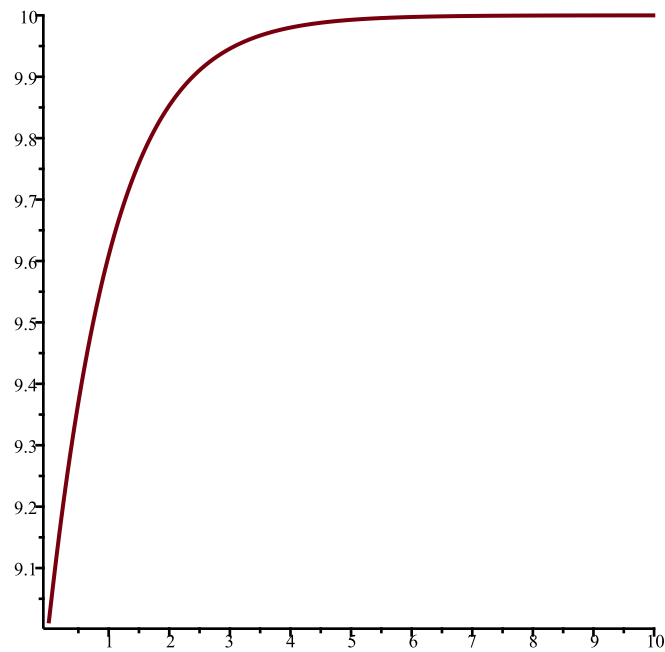
$$Fiii_3 := \left[\frac{2 N1 (3 - N1 - 3 N2)}{3}, 2 N2 (5 - N2 - 8 N1) \right] \quad (55)$$

$$\text{SEquP}(Fiii_3, [N1, N2]) \quad \{[0., 5.], [3., 0.]\} \quad (56)$$

> TimeSeries(Fiii_1, [N1, N2], [0.01, 9.01], 0.01, 10, 1)



> TimeSeries(Fiii_1, [N1, N2], [0.01, 9.01], 0.01, 10, 2)



> Help(PhaseDiag)

PhaseDiag(F,x,pt,h,A): Inputs a transformation F in the list of variables x (of length 2), i.e. a mapping from R^2 to R^2 gives the

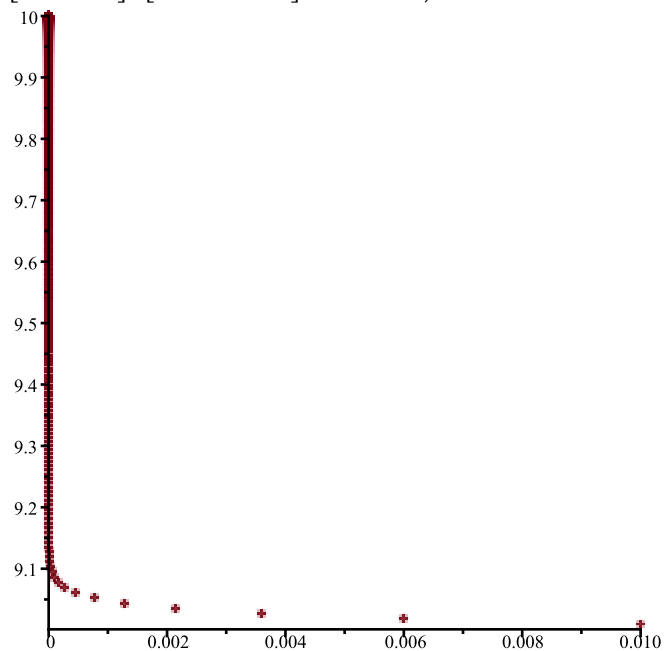
*The phase diagram of the solution with initial condition $x(0)=pt$
 $dx/dt=F[1](x(t))$ by a discrete time dynamical system with step-size h from $t=0$ to $t=A$*

Try:

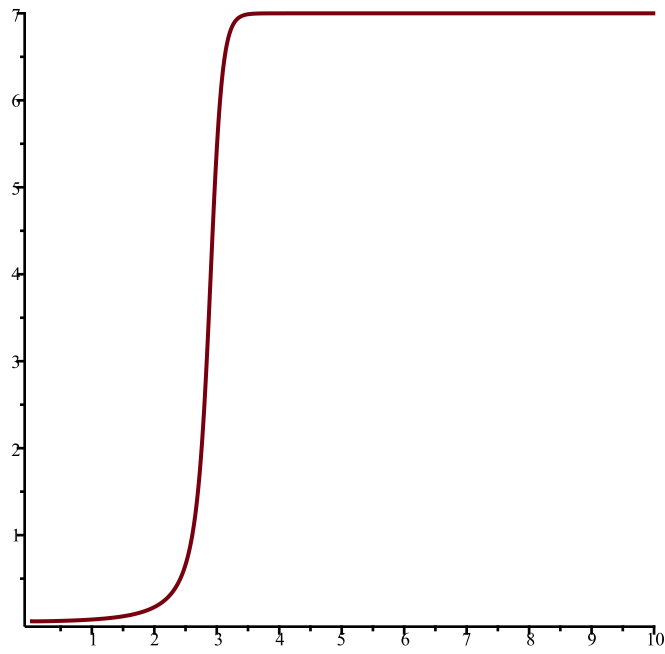
PhaseDiag([x(1-y),y*(1-x)],[x,y],[0.5,0.5], 0.01, 10);*

(57)

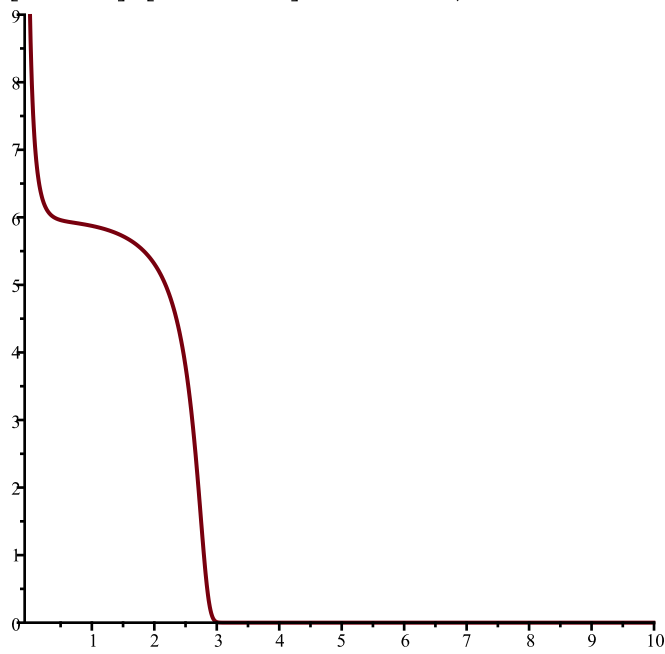
> PhaseDiag(Fiii_1, [N1, N2], [0.01, 9.01], 0.01, 10)



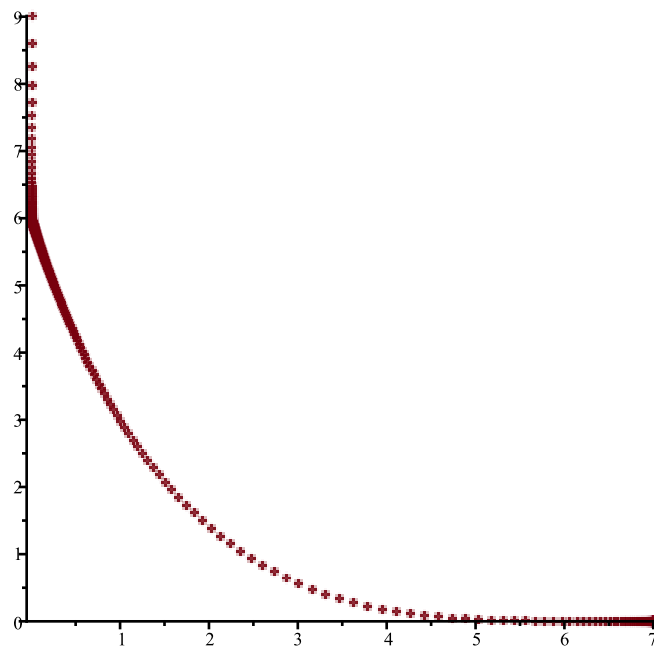
> TimeSeries(Fiii_2, [N1, N2], [0.01, 9.01], 0.01, 10, 1)



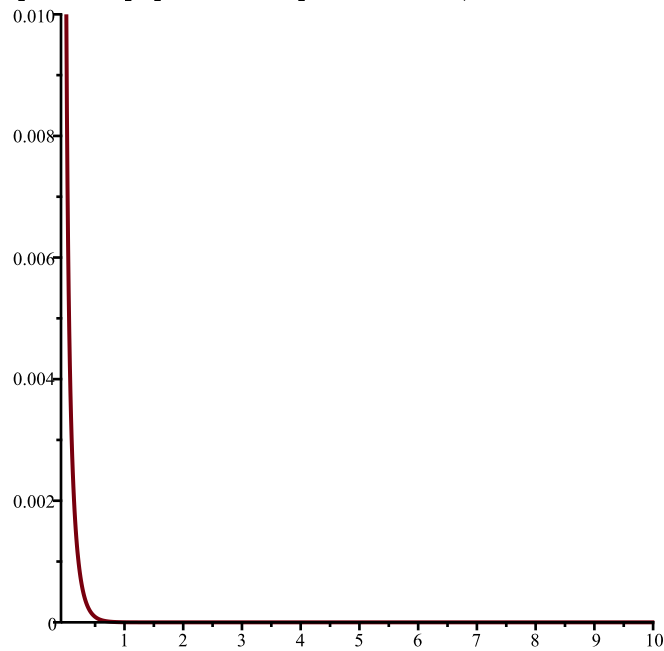
> *TimeSeries(Fiii_2, [N1, N2], [0.01, 9.01], 0.01, 10, 2)*



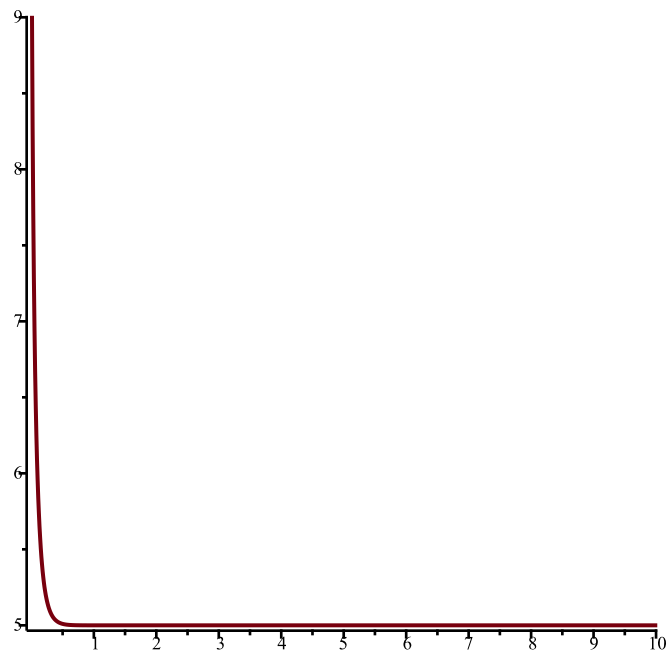
> *PhaseDiag(Fiii_2, [N1, N2], [0.01, 9.01], 0.01, 10)*



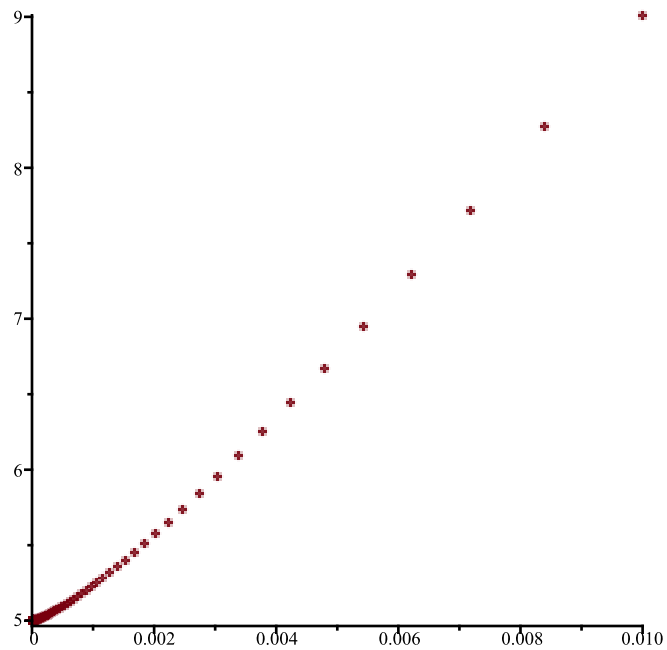
> *TimeSeries(Fiii_3, [N1, N2], [0.01, 9.01], 0.01, 10, 1)*



> *TimeSeries(Fiii_3, [N1, N2], [0.01, 9.01], 0.01, 10, 2)*



```
> PhaseDiag(Fiii_3, [N1, N2], [0.01, 9.01], 0.01, 10)
```



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>
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```
> #Volterra
```

```
> Help(Volterra)
```

Volterra(a,b,c,d,x,y): The (simple, original) Volterra predator-prey continuous-time dynamical system with parameters a,b,c,d

Given by Eqs. (7a) (7b) in Edelstein-Keshet p. 219 (section 6.2).

a,b,c,d may be symbolic or numeric

Try:

```
Volterra(a,b,c,d,x,y);
```

Volterra(1,2,3,4,x,y); (58)

> *a_1 := RNG()*

a_1 := 7 (59)

> *b_1 := RNG()*

b_1 := 5 (60)

> *c_1 := RNG()*

c_1 := 2 (61)

> *d_1 := RNG()*

d_1 := 5 (62)

>

> *a_2 := RNG()*

a_2 := 6 (63)

> *b_2 := RNG()*

b_2 := 8 (64)

> *c_2 := RNG()*

c_2 := 10 (65)

> *d_2 := RNG()*

d_2 := 9 (66)

> *a_3 := RNG()*

a_3 := 7 (67)

> *b_3 := RNG()*

b_3 := 8 (68)

> *c_3 := RNG()*

c_3 := 2 (69)

> *d_3 := RNG()*

d_3 := 10 (70)

> *Fiv_1 := Volterra(a_1, b_1, c_1, d_1, x, y)*

Fiv_1 := [-5 x y + 7 x, 5 x y - 2 y] (71)

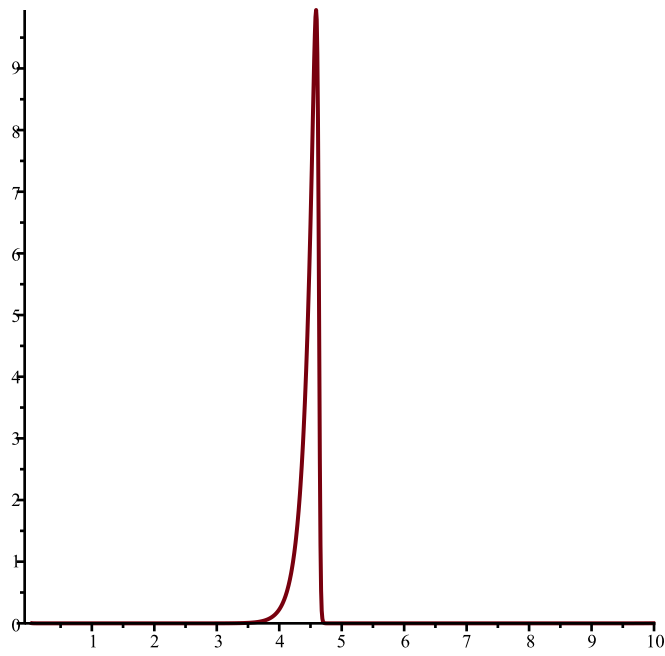
> *Fiv_2 := Volterra(a_2, b_2, c_2, d_2, x, y)*

Fiv_2 := [-8 x y + 6 x, 9 x y - 10 y] (72)

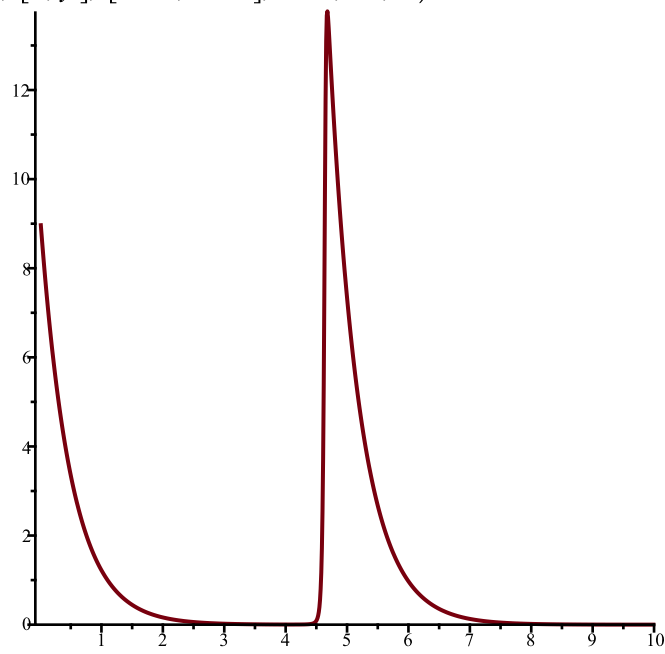
> *Fiv_3 := Volterra(a_3, b_3, c_3, d_3, x, y)*

Fiv_3 := [-8 x y + 7 x, 10 x y - 2 y] (73)

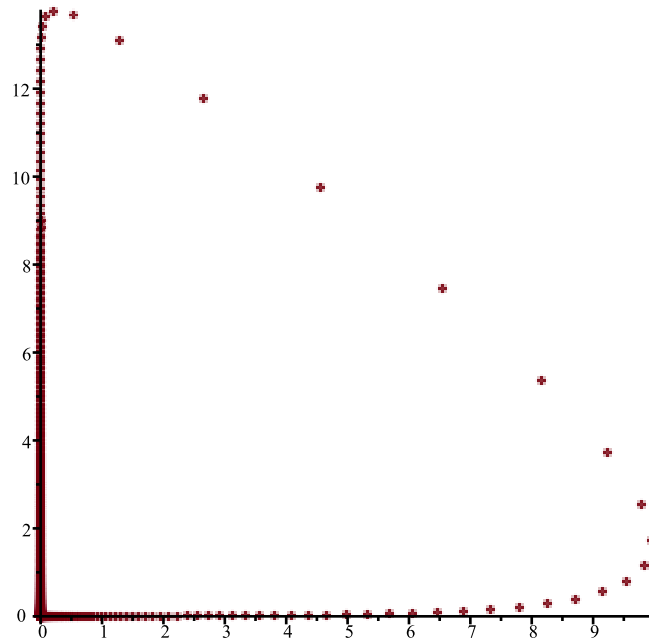
> *TimeSeries(Fiv_1, [x, y], [0.01, 9.01], 0.01, 10, 1)*



> `TimeSeries(Fiv_1, [x, y], [0.01, 9.01], 0.01, 10, 2)`



> `PhaseDiag(Fiv_1, [x, y], [0.01, 9.01], 0.01, 10)`

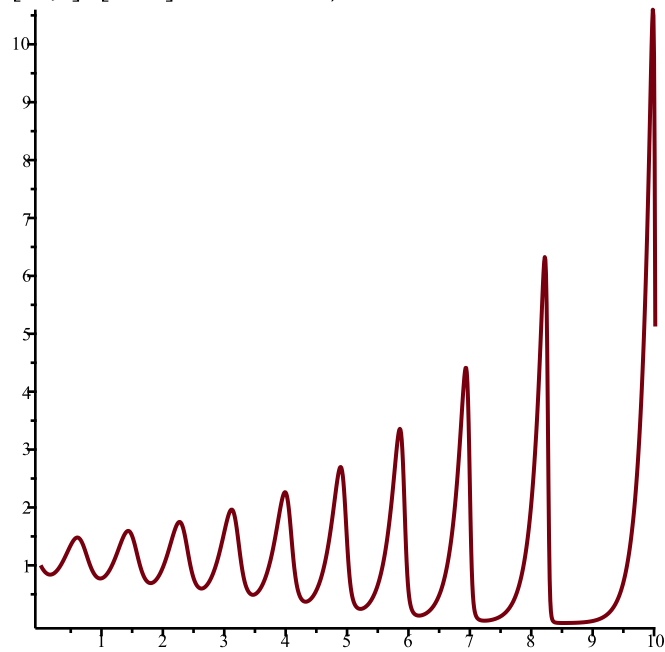


> *SEquP(Fiv_1, [x, y])*

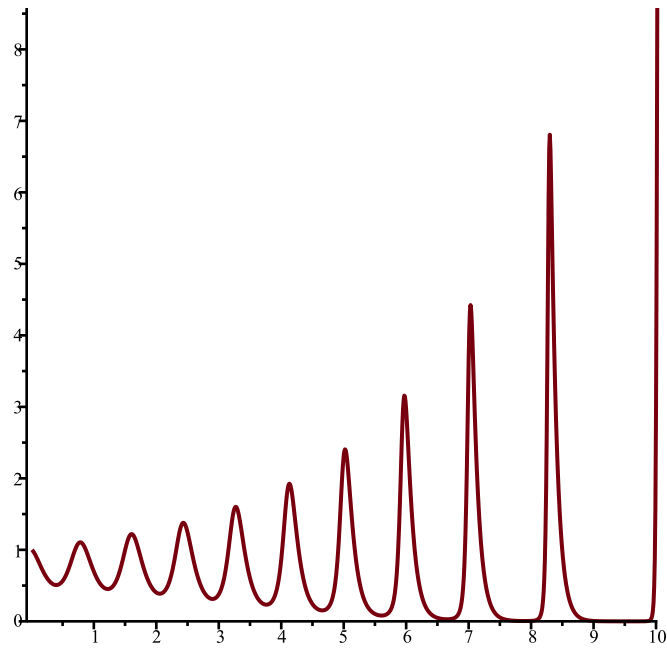
∅

(74)

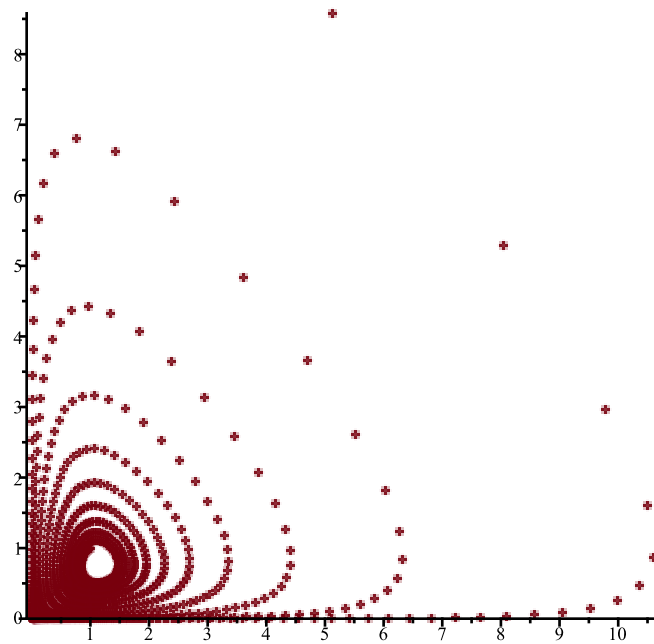
> *TimeSeries(Fiv_2, [x, y], [1, 1], 0.01, 10, 1)*



> *TimeSeries(Fiv_2, [x, y], [1, 1], 0.01, 10, 2)*



> PhaseDiag(Fiv_2, [x, y], [1, 1], 0.01, 10)

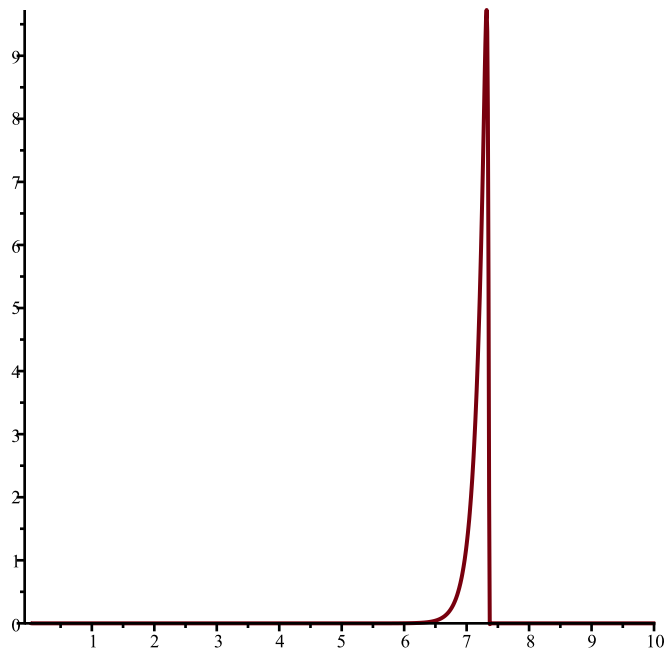


> SEquP(Fiv_2, [x, y])

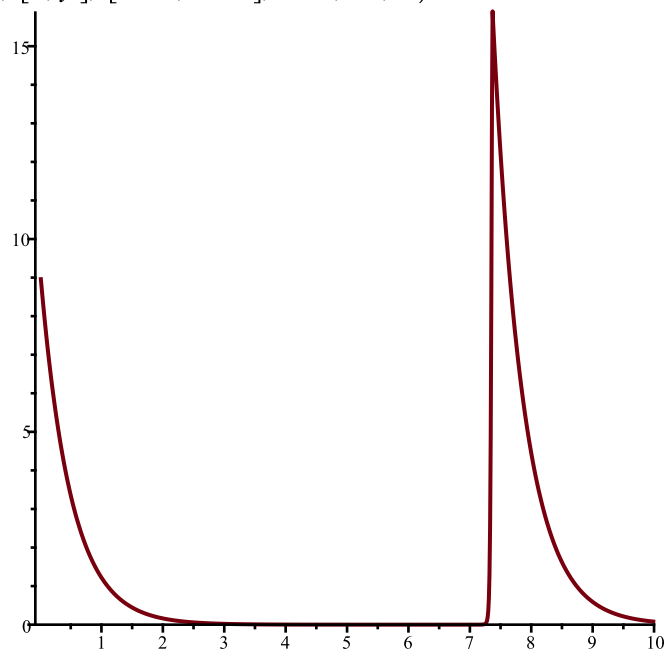
{[1.111111111, 0.750000000]}

(75)

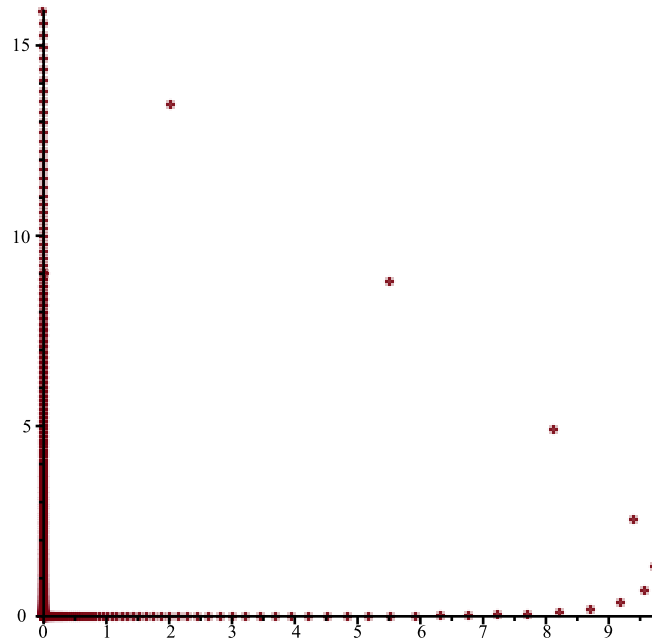
> TimeSeries(Fiv_3, [x, y], [0.01, 9.01], 0.01, 10, 1)



> *TimeSeries(Fiv_3, [x, y], [0.01, 9.01], 0.01, 10, 2)*



> *PhaseDiag(Fiv_3, [x, y], [0.01, 9.01], 0.01, 10)*



```
> SEquP(Fiv_3, [x, y])
```

\emptyset

(76)

```
>
```

```
>
```

```
> #VolterraM
```

```
> Help(VolterraM)
```

VolterraM(a,b,c,d,x,K,y): The MODIFIED Volterra predator-prey continuous-time dynamical system with parameters a,b,c,d,K

Given by Eqs. (8a) (8b) in Edelstein-Keshet p. 220 (section 6.2).

a,b,c,d ,K may be symbolic or numeric

Try:

```
VolterraM(a,b,c,d,K,x,y);
```

```
VolterraM(1,2,3,4,3,x,y);
```

(77)

```
> K1 := RNG( )
```

K1 := 6

(78)

```
> K2 := RNG( )
```

K2 := 4

(79)

```
> K3 := RNG( )
```

K3 := 1

(80)

```
> Fv_1 := VolterraM(a_1, b_1, c_1, d_1, K1, x, y)
```

$$Fv_1 := \left[7x \left(1 - \frac{x}{5} \right) - 5xy, 6xy - 2y \right]$$

(81)

```
> Fv_2 := VolterraM(a_2, b_2, c_2, d_2, K2, x, y)
```

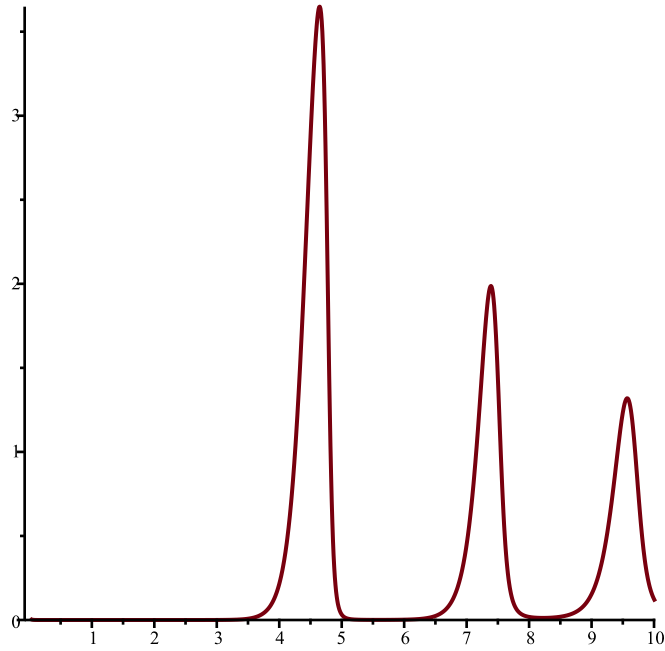
$$Fv_2 := \left[6x \left(1 - \frac{x}{9} \right) - 8xy, 4xy - 10y \right]$$

(82)

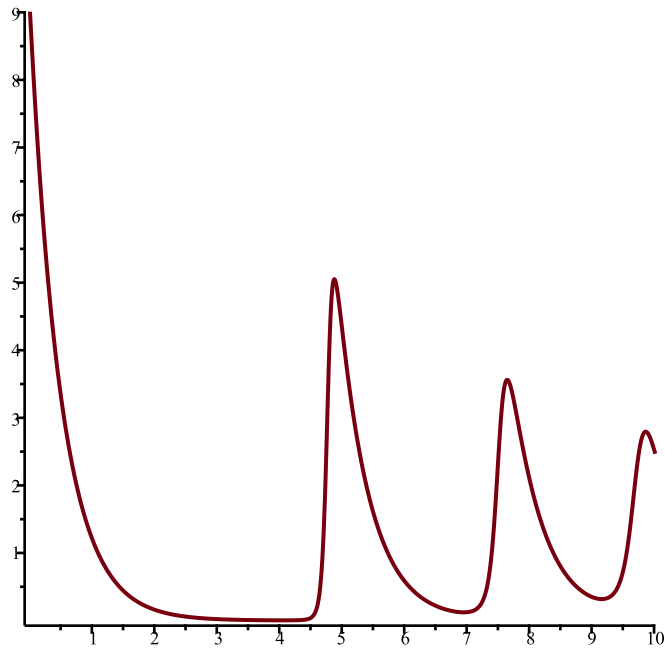
> $Fv_3 := \text{VolterraM}(a_3, b_3, c_3, d_3, K3, x, y)$

$$Fv_3 := \left[7x \left(1 - \frac{x}{10} \right) - 8xy, xy - 2y \right]$$

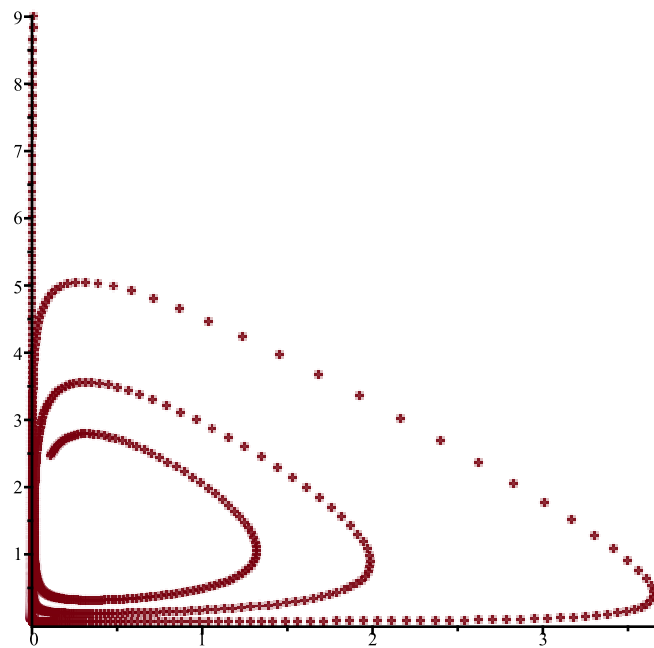
> $\text{TimeSeries}(Fv_1, [x, y], [0.01, 9.01], 0.01, 10, 1)$



> $\text{TimeSeries}(Fv_1, [x, y], [0.01, 9.01], 0.01, 10, 2)$



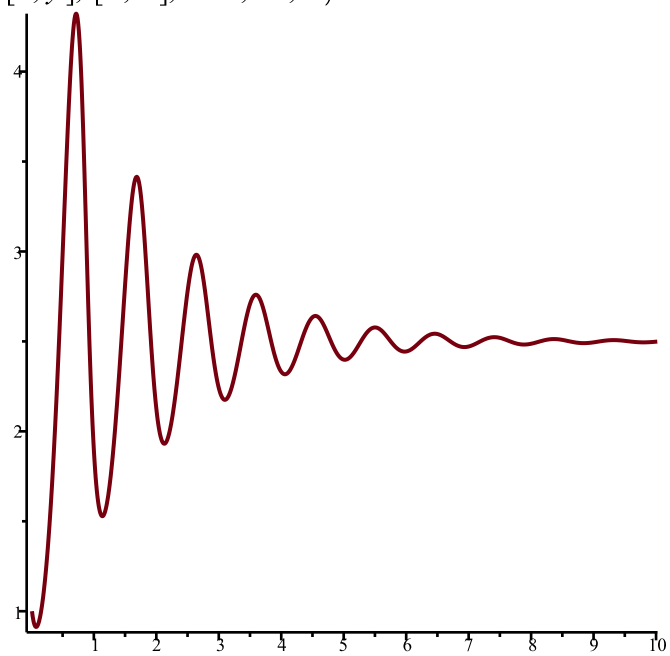
> $\text{PhaseDiag}(Fv_1, [x, y], [0.01, 9.01], 0.01, 10)$



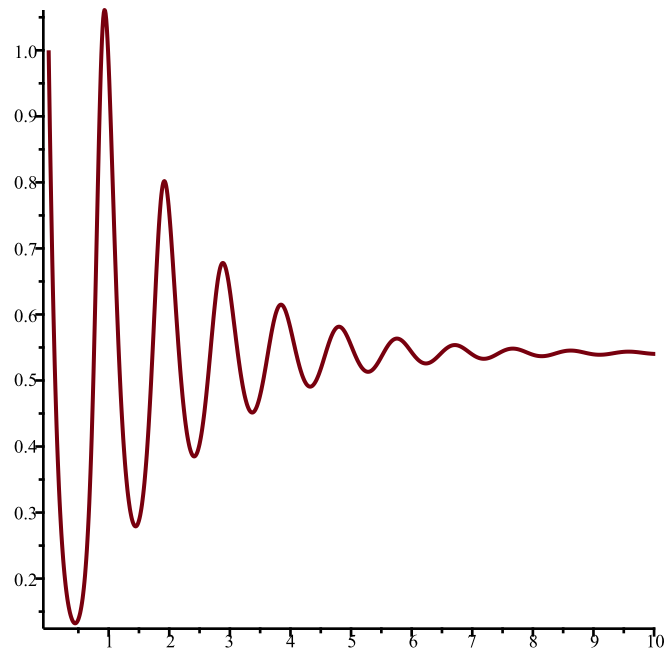
```
> SEquP(Fv_1, [x, y])
      { [0.3333333333, 1.306666667] }
```

(84)

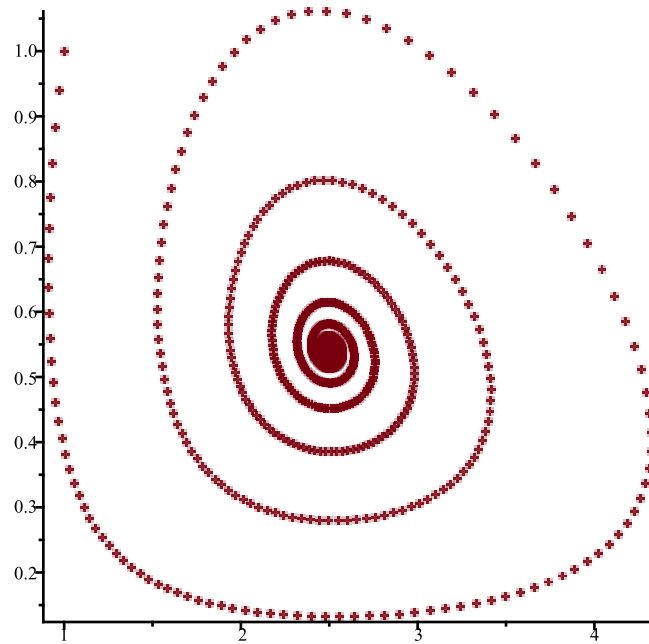
```
> TimeSeries(Fv_2, [x, y], [1, 1], 0.01, 10, 1)
```



```
> TimeSeries(Fv_2, [x, y], [1, 1], 0.01, 10, 2)
```



> *PhaseDiag*(Fv_2, [x, y], [1, 1], 0.01, 10)

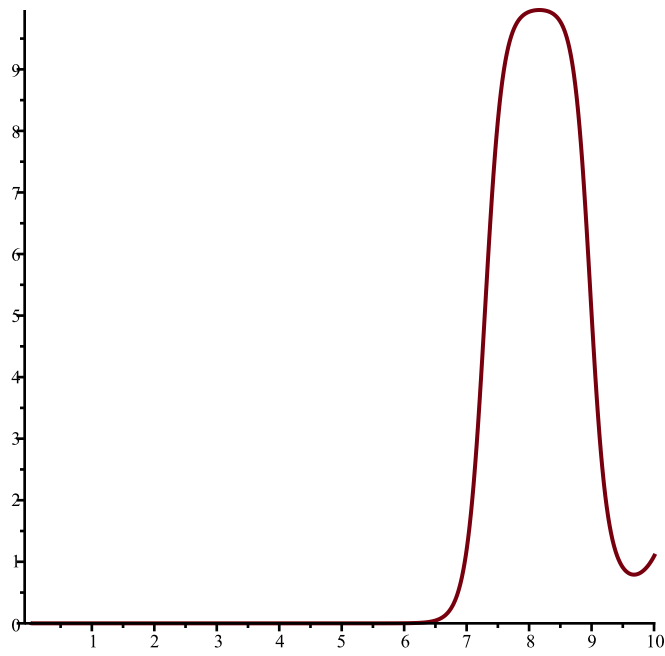


> *SEquP*(Fv_2, [x, y])

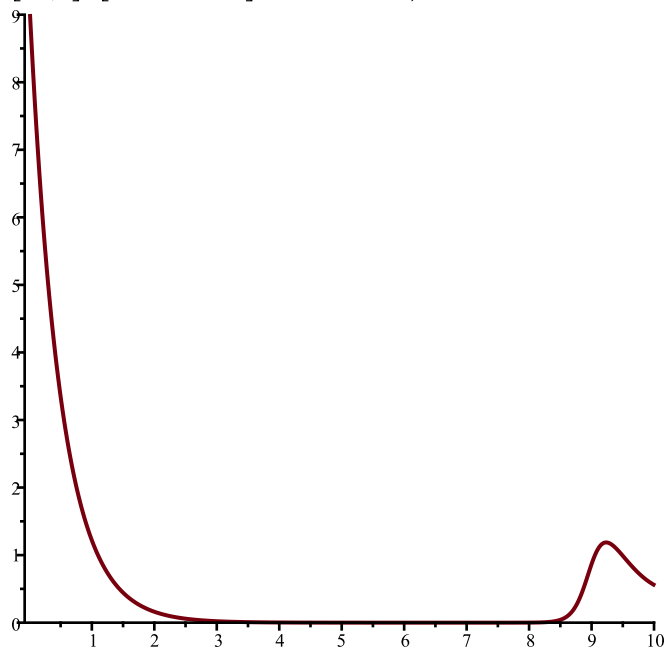
{[2.500000000, 0.5416666667]}

(85)

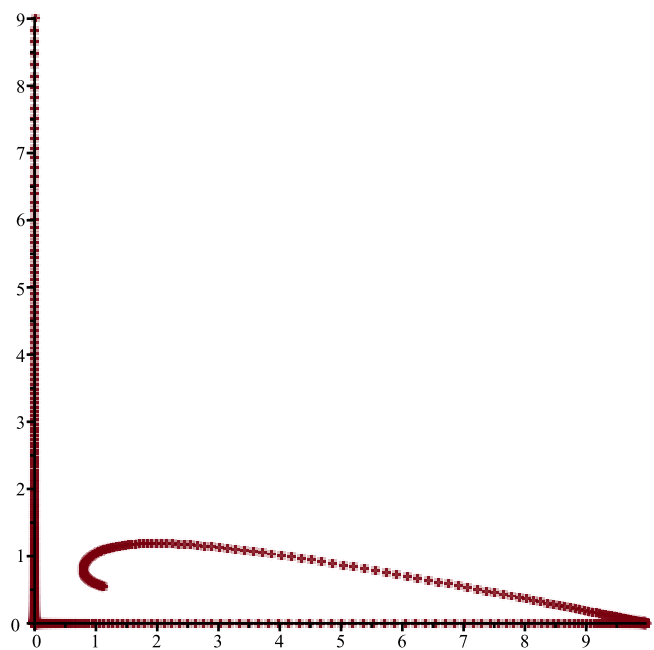
> *TimeSeries*(Fv_3, [x, y], [0.01, 9.01], 0.01, 10, 1)



> *TimeSeries*(Fv_3, [x, y], [0.01, 9.01], 0.01, 10, 2)



> *PhaseDiag*(Fv_3, [x, y], [0.01, 9.01], 0.01, 10)



```
> SEquP(Fv_3, [x, y])
```

```
{[2., 0.7000000000]}
```

(86)