

> read "/Users/jjj104/Documents/DMB.txt" :

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)

accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

*The most current version is available on WWW at:
<http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt> .
Please report all bugs to: DoronZeil at gmail dot com .*

*For general help, and a list of the MAIN functions,
type "Help():". For specific help type "Help(procedure_name);"*

*For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);*

*For a list of the functions that give examples of Discrete-time dynamical systems (some famous),
type: HelpDDM());*

For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM());

For help with any of them type: Help(ProcedureName);

(1)

> #ChemoStat

*a1 := trunc(evalf(rand() * 10^(-11)));*

*a2 := trunc(evalf(rand() * 10^(-11)));*

F := ChemoStat(N, C, a1, a2);

SEquP(F, [N, C]);

TimeSeries(F, [N, C], [a1, a2], 0.01, 10, 1);

TimeSeries(F, [N, C], [a1, a2], 0.01, 10, 2);

PhaseDiag(F, [N, C], [a1, a2], 0.01, 10);

```
a1 := trunc(evalf(rand() * 10^(-11)));
a2 := trunc(evalf(rand() * 10^(-11)));
```

```
F := ChemoStat(N, C, a1, a2);
SEquP(F, [N, C]);
TimeSeries(F, [N, C], [a1, a2], 0.01, 10, 1);
TimeSeries(F, [N, C], [a1, a2], 0.01, 10, 2);
PhaseDiag(F, [N, C], [a1, a2], 0.01, 10);
a1 := trunc(evalf(rand() * 10^(-11)));
a2 := trunc(evalf(rand() * 10^(-11)));
```

```
F := ChemoStat(N, C, a1, a2);
SEquP(F, [N, C]);
TimeSeries(F, [N, C], [a1, a2], 0.01, 10, 1);
TimeSeries(F, [N, C], [a1, a2], 0.01, 10, 2);
PhaseDiag(F, [N, C], [a1, a2], 0.01, 10);
```

a1 := 3

a2 := 1

F := ChemoStat(N, C, 3, 1)

SEquP(ChemoStat(N, C, 3, 1), [N, C])

TimeSeries(ChemoStat(N, C, 3, 1), [N, C], [3, 1], 0.01, 10, 1)

TimeSeries(ChemoStat(N, C, 3, 1), [N, C], [3, 1], 0.01, 10, 2)

PhaseDiag(ChemoStat(N, C, 3, 1), [N, C], [3, 1], 0.01, 10)

a1 := 0

a2 := 8

F := ChemoStat(N, C, 0, 8)

SEquP(ChemoStat(N, C, 0, 8), [N, C])

TimeSeries(ChemoStat(N, C, 0, 8), [N, C], [0, 8], 0.01, 10, 1)

TimeSeries(ChemoStat(N, C, 0, 8), [N, C], [0, 8], 0.01, 10, 2)

PhaseDiag(ChemoStat(N, C, 0, 8), [N, C], [0, 8], 0.01, 10)

a1 := 4

a2 := 8

F := ChemoStat(N, C, 4, 8)

SEquP(ChemoStat(N, C, 4, 8), [N, C])

TimeSeries(ChemoStat(N, C, 4, 8), [N, C], [4, 8], 0.01, 10, 1)

TimeSeries(ChemoStat(N, C, 4, 8), [N, C], [4, 8], 0.01, 10, 2)

PhaseDiag(ChemoStat(N, C, 4, 8), [N, C], [4, 8], 0.01, 10)

(2)

> #GeneNet

```
a0 := trunc(evalf(rand() * 10^(-11)));
a := trunc(evalf(rand() * 10^(-11)));
b := trunc(evalf(rand() * 10^(-11)));
n := trunc(evalf(rand() * 10^(-11)));
```

```

G := GeneNet(a0, a, b, n, m01, m02, m03, p01, p02, p03);
m1 := trunc(evalf(rand() * 10^(-11)));
m2 := trunc(evalf(rand() * 10^(-11)));
m3 := trunc(evalf(rand() * 10^(-11)));
p1 := trunc(evalf(rand() * 10^(-11)));
p2 := trunc(evalf(rand() * 10^(-11)));
p3 := trunc(evalf(rand() * 10^(-11)));
SEquP(G, [m01, m02, m03, p01, p02, p03]);
TimeSeries(G, [m01, m02, m03, p01, p02, p03], [m1, m2, m3, p1, p2, p3], 0.01, 10, 1);
a0 := trunc(evalf(rand() * 10^(-11)));
a := trunc(evalf(rand() * 10^(-11)));
b := trunc(evalf(rand() * 10^(-11)));
n := trunc(evalf(rand() * 10^(-11)));

```

```

G := GeneNet(a0, a, b, n, m01, m02, m03, p01, p02, p03);
m1 := trunc(evalf(rand() * 10^(-11)));
m2 := trunc(evalf(rand() * 10^(-11)));
m3 := trunc(evalf(rand() * 10^(-11)));
p1 := trunc(evalf(rand() * 10^(-11)));
p2 := trunc(evalf(rand() * 10^(-11)));
p3 := trunc(evalf(rand() * 10^(-11)));
SEquP(G, [m01, m02, m03, p01, p02, p03]);
TimeSeries(G, [m01, m02, m03, p01, p02, p03], [m1, m2, m3, p1, p2, p3], 0.01,
10, 1);
a0 := trunc(evalf(rand() * 10^(-11)));
a := trunc(evalf(rand() * 10^(-11)));
b := trunc(evalf(rand() * 10^(-11)));
n := trunc(evalf(rand() * 10^(-11)));

```

```

G := GeneNet(a0, a, b, n, m01, m02, m03, p01, p02, p03);
m1 := trunc(evalf(rand() * 10^(-11)));
m2 := trunc(evalf(rand() * 10^(-11)));
m3 := trunc(evalf(rand() * 10^(-11)));
p1 := trunc(evalf(rand() * 10^(-11)));
p2 := trunc(evalf(rand() * 10^(-11)));
p3 := trunc(evalf(rand() * 10^(-11)));
SEquP(G, [m01, m02, m03, p01, p02, p03]);
TimeSeries(G, [m01, m02, m03, p01, p02, p03], [m1, m2, m3, p1, p2, p3], 0.01, 10, 1);

```

$a0 := 3$

$a := 6$

$b := 7$

$n := 7$

$$G := \left[-m01 + \frac{6}{p03^7 + 1} + 3, -m02 + \frac{6}{p01^7 + 1} + 3, -m03 + \frac{6}{p02^7 + 1} + 3, -7 p01 \right. \\
\left. + 7 m01, -7 p02 + 7 m02, -7 p03 + 7 m03 \right]$$

m1 := 1

m2 := 3

m3 := 9

p1 := 2

p2 := 7

p3 := 4

(3)

```
> #Lotka
Help(Lotka);
r1 := trunc(evalf(rand() * 10^(-11)));
r2 := trunc(evalf(rand() * 10^(-11)));
k1 := trunc(evalf(rand() * 10^(-11)));
k2 := trunc(evalf(rand() * 10^(-11)));
b12 := trunc(evalf(rand() * 10^(-11)));
b21 := trunc(evalf(rand() * 10^(-11)));
L := Lotka(r1, k1, r2, k2, b12, b21, N1, N2);
n1 := trunc(evalf(rand() * 10^(-11)));
n2 := trunc(evalf(rand() * 10^(-11)));

SEquP(L, [N1, N2]);
TimeSeries(L, [N1, N2], [n1, n2], 0.01, 10, 1);
TimeSeries(L, [N1, N2], [n1, n2], 0.01, 10, 2);
PhaseDiag(L, [N1, N2], [n1, n2], 0.01, 10);
r1 := trunc(evalf(rand() * 10^(-11)));
r2 := trunc(evalf(rand() * 10^(-11)));
k1 := trunc(evalf(rand() * 10^(-11)));
k2 := trunc(evalf(rand() * 10^(-11)));
b12 := trunc(evalf(rand() * 10^(-11)));
b21 := trunc(evalf(rand() * 10^(-11)));
L := Lotka(r1, k1, r2, k2, b12, b21, N1, N2);
n1 := trunc(evalf(rand() * 10^(-11)));
n2 := trunc(evalf(rand() * 10^(-11)));

SEquP(L, [N1, N2]);
TimeSeries(L, [N1, N2], [n1, n2], 0.01, 10, 1);
TimeSeries(L, [N1, N2], [n1, n2], 0.01, 10, 2);
PhaseDiag(L, [N1, N2], [n1, n2], 0.01, 10);
r1 := trunc(evalf(rand() * 10^(-11)));
r2 := trunc(evalf(rand() * 10^(-11)));
k1 := trunc(evalf(rand() * 10^(-11)));
k2 := trunc(evalf(rand() * 10^(-11)));
b12 := trunc(evalf(rand() * 10^(-11)));
b21 := trunc(evalf(rand() * 10^(-11)));
L := Lotka(r1, k1, r2, k2, b12, b21, N1, N2);
n1 := trunc(evalf(rand() * 10^(-11)));
n2 := trunc(evalf(rand() * 10^(-11)));

SEquP(L, [N1, N2]);
```

TimeSeries(*L*, [*N1*, *N2*], [*n1*, *n2*], 0.01, 10, 1);
TimeSeries(*L*, [*N1*, *N2*], [*n1*, *n2*], 0.01, 10, 2);
PhaseDiag(*L*, [*N1*, *N2*], [*n1*, *n2*], 0.01, 10);

Help(*Lotka*)

r1 := 4

r2 := 9

k1 := 3

k2 := 6

b12 := 7

b21 := 7

L := *Lotka*(4, 3, 9, 6, 7, 7, *N1*, *N2*)

n1 := 1

n2 := 3

SEquP(*Lotka*(4, 3, 9, 6, 7, 7, *N1*, *N2*), [*N1*, *N2*])

TimeSeries(*Lotka*(4, 3, 9, 6, 7, 7, *N1*, *N2*), [*N1*, *N2*], [1, 3], 0.01, 10, 1)

TimeSeries(*Lotka*(4, 3, 9, 6, 7, 7, *N1*, *N2*), [*N1*, *N2*], [1, 3], 0.01, 10, 2)

PhaseDiag(*Lotka*(4, 3, 9, 6, 7, 7, *N1*, *N2*), [*N1*, *N2*], [1, 3], 0.01, 10)

r1 := 9

r2 := 2

k1 := 7

k2 := 4

b12 := 7

b21 := 3

L := *Lotka*(9, 7, 2, 4, 7, 3, *N1*, *N2*)

n1 := 6

n2 := 8

SEquP(*Lotka*(9, 7, 2, 4, 7, 3, *N1*, *N2*), [*N1*, *N2*])

TimeSeries(*Lotka*(9, 7, 2, 4, 7, 3, *N1*, *N2*), [*N1*, *N2*], [6, 8], 0.01, 10, 1)

TimeSeries(*Lotka*(9, 7, 2, 4, 7, 3, *N1*, *N2*), [*N1*, *N2*], [6, 8], 0.01, 10, 2)

PhaseDiag(*Lotka*(9, 7, 2, 4, 7, 3, *N1*, *N2*), [*N1*, *N2*], [6, 8], 0.01, 10)

r1 := 4

r2 := 3

k1 := 8

k2 := 4

b12 := 2

b21 := 0

L := *Lotka*(4, 8, 3, 4, 2, 0, *N1*, *N2*)

n1 := 7

$n2 := 5$

SEquP(Lotka(4, 8, 3, 4, 2, 0, $N1$, $N2$), [$N1$, $N2$])

TimeSeries(Lotka(4, 8, 3, 4, 2, 0, $N1$, $N2$), [$N1$, $N2$], [7, 5], 0.01, 10, 1)

TimeSeries(Lotka(4, 8, 3, 4, 2, 0, $N1$, $N2$), [$N1$, $N2$], [7, 5], 0.01, 10, 2)

PhaseDiag(Lotka(4, 8, 3, 4, 2, 0, $N1$, $N2$), [$N1$, $N2$], [7, 5], 0.01, 10)

(4)

> #Volterra

```
a := trunc(evalf(rand() * 10^(-11)));
b := trunc(evalf(rand() * 10^(-11)));
c := trunc(evalf(rand() * 10^(-11)));
d := trunc(evalf(rand() * 10^(-11)));
V := Volterra(a, b, c, d, x, y);
x1 := trunc(evalf(rand() * 10^(-11)));
y2 := trunc(evalf(rand() * 10^(-11)));
```

```
SEquP(V, [x, y]);
TimeSeries(V, [x, y], [x1, y2], 0.01, 1, 1);
TimeSeries(V, [x, y], [x1, y2], 0.01, 1, 2);
PhaseDiag(V, [x, y], [x1, y2], 0.01, 10);
a := trunc(evalf(rand() * 10^(-11)));
b := trunc(evalf(rand() * 10^(-11)));
c := trunc(evalf(rand() * 10^(-11)));
d := trunc(evalf(rand() * 10^(-11)));
V := Volterra(a, b, c, d, x, y);
x1 := trunc(evalf(rand() * 10^(-11)));
y2 := trunc(evalf(rand() * 10^(-11)));
```

```
SEquP(V, [x, y]);
TimeSeries(V, [x, y], [x1, y2], 0.01, 1, 1);
TimeSeries(V, [x, y], [x1, y2], 0.01, 1, 2);
PhaseDiag(V, [x, y], [x1, y2], 0.01, 10);
a := trunc(evalf(rand() * 10^(-11)));
b := trunc(evalf(rand() * 10^(-11)));
c := trunc(evalf(rand() * 10^(-11)));
d := trunc(evalf(rand() * 10^(-11)));
V := Volterra(a, b, c, d, x, y);
x1 := trunc(evalf(rand() * 10^(-11)));
y2 := trunc(evalf(rand() * 10^(-11)));
```

```
SEquP(V, [x, y]);
TimeSeries(V, [x, y], [x1, y2], 0.01, 1, 1);
TimeSeries(V, [x, y], [x1, y2], 0.01, 1, 2);
PhaseDiag(V, [x, y], [x1, y2], 0.01, 10);
```

$a := 7$

$b := 4$

$c := 8$

$d := 8$

```

V := Volterra(7, 4, 8, 8, x, y)
x1 := 9
y2 := 4
SEquP(Volterra(7, 4, 8, 8, x, y), [x, y])
TimeSeries(Volterra(7, 4, 8, 8, x, y), [x, y], [9, 4], 0.01, 1, 1)
TimeSeries(Volterra(7, 4, 8, 8, x, y), [x, y], [9, 4], 0.01, 1, 2)
PhaseDiag(Volterra(7, 4, 8, 8, x, y), [x, y], [9, 4], 0.01, 10)
a := 5
b := 3
c := 9
d := 7
V := Volterra(5, 3, 9, 7, x, y)
x1 := 6
y2 := 6
SEquP(Volterra(5, 3, 9, 7, x, y), [x, y])
TimeSeries(Volterra(5, 3, 9, 7, x, y), [x, y], [6, 6], 0.01, 1, 1)
TimeSeries(Volterra(5, 3, 9, 7, x, y), [x, y], [6, 6], 0.01, 1, 2)
PhaseDiag(Volterra(5, 3, 9, 7, x, y), [x, y], [6, 6], 0.01, 10)
a := 0
b := 0
c := 4
d := 3
V := Volterra(0, 0, 4, 3, x, y)
x1 := 7
y2 := 7
SEquP(Volterra(0, 0, 4, 3, x, y), [x, y])
TimeSeries(Volterra(0, 0, 4, 3, x, y), [x, y], [7, 7], 0.01, 1, 1)
TimeSeries(Volterra(0, 0, 4, 3, x, y), [x, y], [7, 7], 0.01, 1, 2)
PhaseDiag(Volterra(0, 0, 4, 3, x, y), [x, y], [7, 7], 0.01, 10)

```

(5)

```

> #VolterraM
a := trunc(evalf(rand() * 10^(-11)));
b := trunc(evalf(rand() * 10^(-11)));
c := trunc(evalf(rand() * 10^(-11)));
d := trunc(evalf(rand() * 10^(-11)));
K := trunc(evalf(rand() * 10^(-11)));
V2 := VolterraM(a, b, c, d, K, x, y);
x1 := trunc(evalf(rand() * 10^(-11)));
y2 := trunc(evalf(rand() * 10^(-11)));

SEquP(V2, [x, y]);

```

```

TimeSeries(V2, [x, y], [x1, y2], 0.01, 10, 1);
TimeSeries(V2, [x, y], [x1, y2], 0.01, 1, 2);
PhaseDiag(V2, [x, y], [x1, y2], 0.01, 10);
a := trunc(evalf(rand() * 10^(-11)));
b := trunc(evalf(rand() * 10^(-11)));
c := trunc(evalf(rand() * 10^(-11)));
d := trunc(evalf(rand() * 10^(-11)));
K := trunc(evalf(rand() * 10^(-11)));
V2 := VolterraM(a, b, c, d, K, x, y);
x1 := trunc(evalf(rand() * 10^(-11)));
y2 := trunc(evalf(rand() * 10^(-11)));

```

```

SEquP(V2, [x, y]);
TimeSeries(V2, [x, y], [x1, y2], 0.01, 10, 1);
TimeSeries(V2, [x, y], [x1, y2], 0.01, 1, 2);
PhaseDiag(V2, [x, y], [x1, y2], 0.01, 10);
a := trunc(evalf(rand() * 10^(-11)));
b := trunc(evalf(rand() * 10^(-11)));
c := trunc(evalf(rand() * 10^(-11)));
d := trunc(evalf(rand() * 10^(-11)));
K := trunc(evalf(rand() * 10^(-11)));
V2 := VolterraM(a, b, c, d, K, x, y);
x1 := trunc(evalf(rand() * 10^(-11)));
y2 := trunc(evalf(rand() * 10^(-11)));

```

```

SEquP(V2, [x, y]);
TimeSeries(V2, [x, y], [x1, y2], 0.01, 10, 1);
TimeSeries(V2, [x, y], [x1, y2], 0.01, 1, 2);
PhaseDiag(V2, [x, y], [x1, y2], 0.01, 10);

```

a := 1

b := 8

c := 3

d := 6

K := 9

V2 := VolterraM(1, 8, 3, 6, 9, *x*, *y*)

x1 := 4

y2 := 3

SEquP(VolterraM(1, 8, 3, 6, 9, *x*, *y*), [*x*, *y*])

TimeSeries(VolterraM(1, 8, 3, 6, 9, *x*, *y*), [*x*, *y*], [4, 3], 0.01, 10, 1)

TimeSeries(VolterraM(1, 8, 3, 6, 9, *x*, *y*), [*x*, *y*], [4, 3], 0.01, 1, 2)

PhaseDiag(VolterraM(1, 8, 3, 6, 9, *x*, *y*), [*x*, *y*], [4, 3], 0.01, 10)

a := 8

b := 8

c := 7


```

d := 1
K := 3
V2 := VolterraM(8, 8, 7, 1, 3, x, y)
x1 := 4
y2 := 2
SEquP(VolterraM(8, 8, 7, 1, 3, x, y), [x, y])
TimeSeries(VolterraM(8, 8, 7, 1, 3, x, y), [x, y], [4, 2], 0.01, 10, 1)
TimeSeries(VolterraM(8, 8, 7, 1, 3, x, y), [x, y], [4, 2], 0.01, 1, 2)
PhaseDiag(VolterraM(8, 8, 7, 1, 3, x, y), [x, y], [4, 2], 0.01, 10)
a := 2
b := 8
c := 5
d := 9
K := 0
V2 := VolterraM(2, 8, 5, 9, 0, x, y)
x1 := 4
y2 := 0
SEquP(VolterraM(2, 8, 5, 9, 0, x, y), [x, y])
TimeSeries(VolterraM(2, 8, 5, 9, 0, x, y), [x, y], [4, 0], 0.01, 10, 1)
TimeSeries(VolterraM(2, 8, 5, 9, 0, x, y), [x, y], [4, 0], 0.01, 1, 2)
PhaseDiag(VolterraM(2, 8, 5, 9, 0, x, y), [x, y], [4, 0], 0.01, 10)

```

(6)