```
> #John Hermitt hw21
  read "/John/Rutgers/Senior Fall/Dynamic Models/DMB.txt"
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

--- **(1)**

```
> #ChemoStat
  a1 := trunc(evalf(rand( ) * 10^(-11)));
  a2 := trunc(evalf(rand( ) * 10^(-11)));
  F := ChemoStat(N, C, a1, a2);
  SEquP(F, [N, C]);
  TimeSeries(F, [N, C], [a1, a2], 0.01, 10, 1);
  TimeSeries(F, [N, C], [a1, a2], 0.01, 10, 2);
  PhaseDiag(F, [N, C], [a1, a2], 0.01, 10);

  a1 := trunc(evalf(rand( ) * 10^(-11)));
```

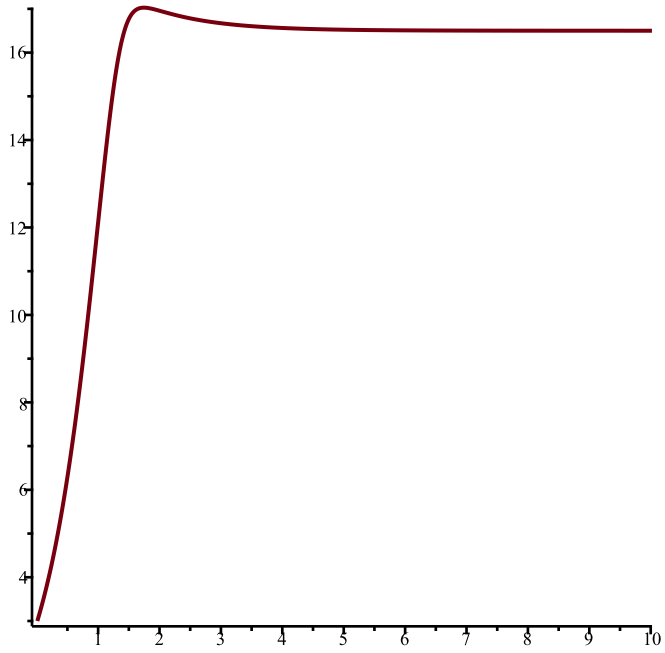
```
a2 := trunc(evalf(rand( ) * 10^(-11)));
F := ChemoStat(N, C, a1, a2);
SEquP(F, [N, C]);
TimeSeries(F, [N, C], [a1, a2], 0.01, 10, 1);
TimeSeries(F, [N, C], [a1, a2], 0.01, 10, 2);
PhaseDiag(F, [N, C], [a1, a2], 0.01, 10);

a1 := trunc(evalf(rand( ) * 10^(-11)));
a2 := trunc(evalf(rand( ) * 10^(-11)));
F := ChemoStat(N, C, a1, a2);
SEquP(F, [N, C]);
TimeSeries(F, [N, C], [a1, a2], 0.01, 10, 1);
TimeSeries(F, [N, C], [a1, a2], 0.01, 10, 2);
PhaseDiag(F, [N, C], [a1, a2], 0.01, 10);
```

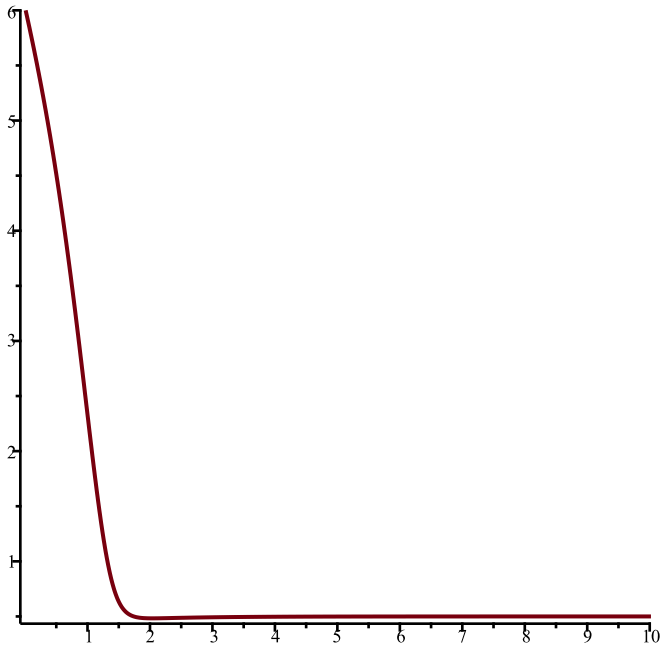
$$a1 := 3$$

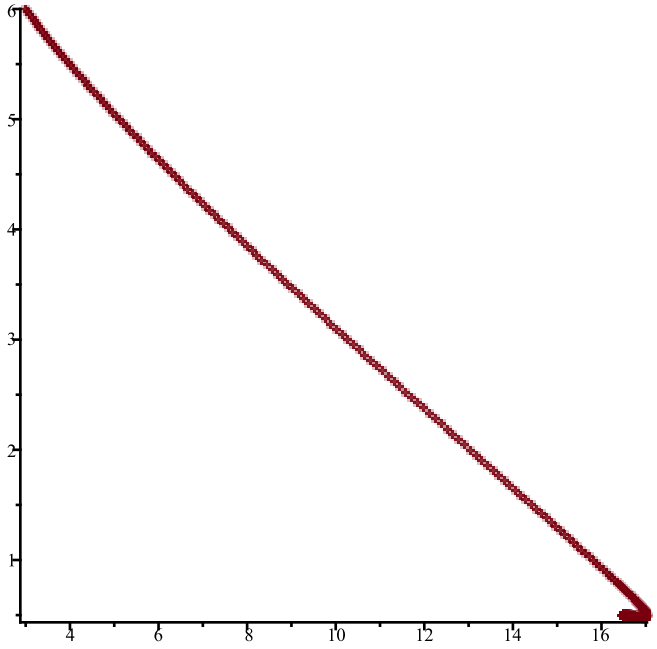
$$a2 := 6$$

$$F := \left[\frac{3\,C\,N}{C+1} - N,\ -\frac{C\,N}{C+1} - C + 6 \right]$$

$$\{[16.50000000, 0.5000000000]\}$$





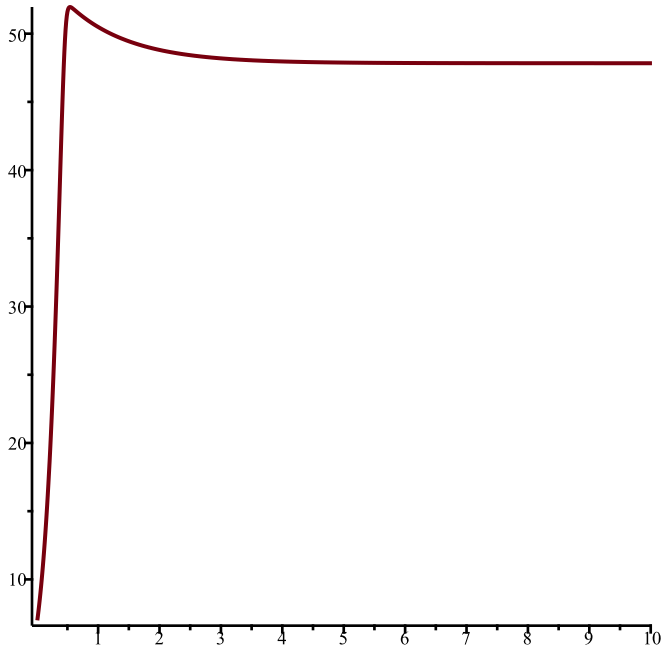


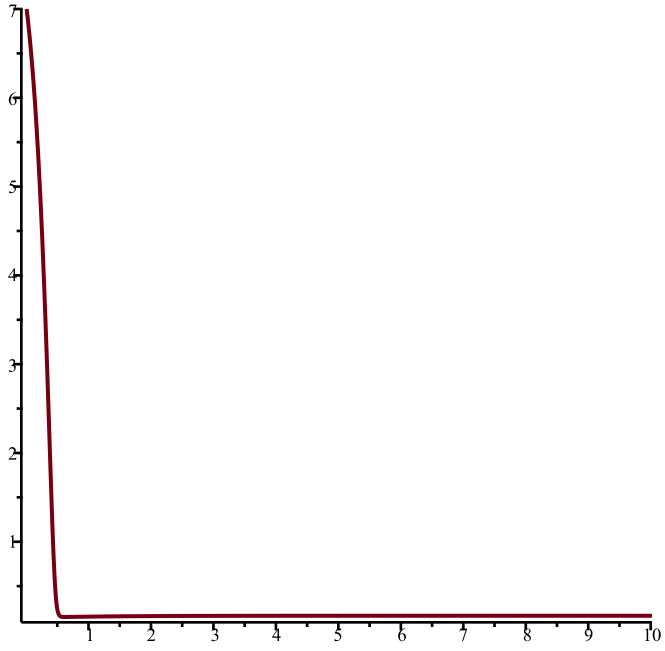
$$a1 := 7$$

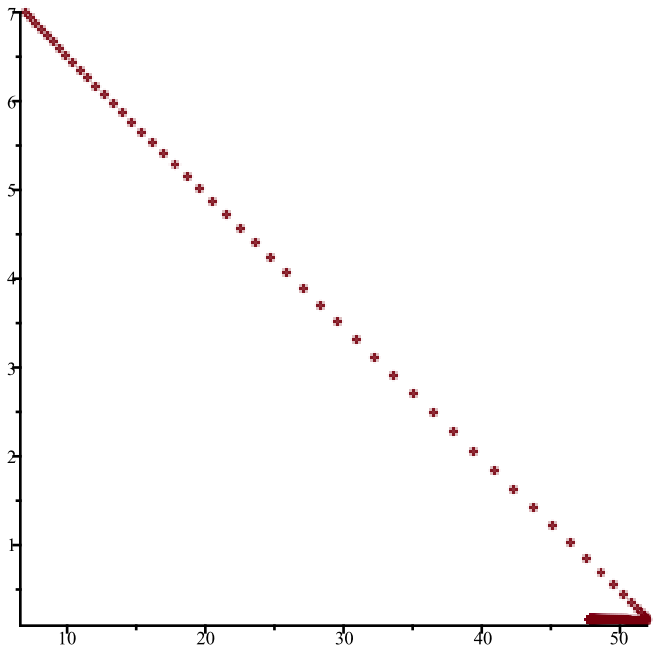
$$a2 := 7$$

$$F := \left[\frac{7\,C\,N}{C+1} - N,\ -\frac{C\,N}{C+1} - C + 7 \right]$$

$$\{[47.83333333,\ 0.1666666667]\}$$


50
40
30
20
10
1
2
3
4
5
6
7
8
9
10


7
6
5
4
3
2
1
1
2
3
4
5
6
7
8
9
10

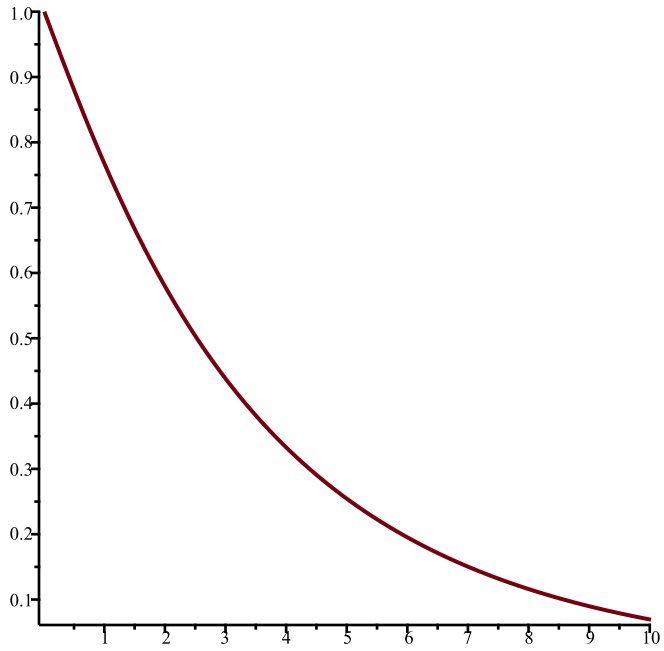


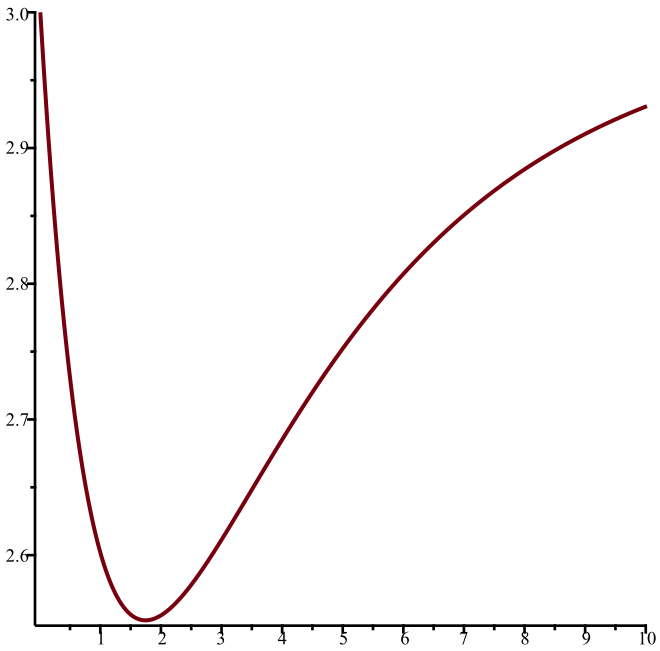
$$a1 := 1$$

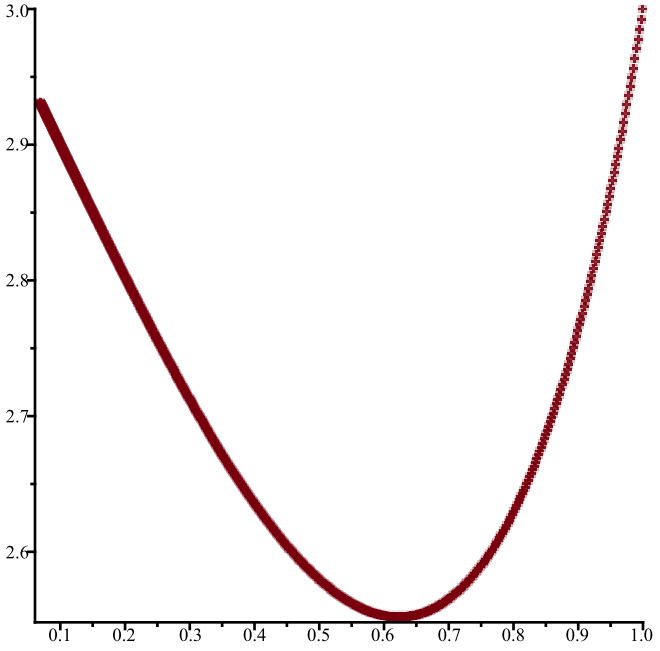
$$a2 := 3$$

$$F := \left[\frac{C\,N}{C+1} - N,\ -\frac{C\,N}{C+1} - C + 3 \right]$$

$$\{[0., 3.]\}$$








```
> #GeneNet
  a0 := trunc(evalf(rand( ) * 10^(-11)));
  a := trunc(evalf(rand( ) * 10^(-11)));
  b := trunc(evalf(rand( ) * 10^(-11)));
  n := trunc(evalf(rand( ) * 10^(-11)));
  G := GeneNet(a0, a, b, n, m01, m02, m03, p01, p02, p03);
  m1 := trunc(evalf(rand( ) * 10^(-11)));
  m2 := trunc(evalf(rand( ) * 10^(-11)));
  m3 := trunc(evalf(rand( ) * 10^(-11)));
  p1 := trunc(evalf(rand( ) * 10^(-11)));
  p2 := trunc(evalf(rand( ) * 10^(-11)));
  p3 := trunc(evalf(rand( ) * 10^(-11)));
  SEquP(G, [m01, m02, m03, p01, p02, p03]);
```

```
TimeSeries(G, [m01, m02, m03, p01, p02, p03], [m1, m2, m3, p1, p2, p3], 0.01, 10, 1);

a0 := trunc(evalf(rand( ) * 10^(-11)));
a := trunc(evalf(rand( ) * 10^(-11)));
b := trunc(evalf(rand( ) * 10^(-11)));
n := trunc(evalf(rand( ) * 10^(-11)));
G := GeneNet(a0, a, b, n, m01, m02, m03, p01, p02, p03);
m1 := trunc(evalf(rand( ) * 10^(-11)));
m2 := trunc(evalf(rand( ) * 10^(-11)));
m3 := trunc(evalf(rand( ) * 10^(-11)));
p1 := trunc(evalf(rand( ) * 10^(-11)));
p2 := trunc(evalf(rand( ) * 10^(-11)));
p3 := trunc(evalf(rand( ) * 10^(-11)));
SEquP(G, [m01, m02, m03, p01, p02, p03]);
TimeSeries(G, [m01, m02, m03, p01, p02, p03], [m1, m2, m3, p1, p2, p3], 0.01, 10, 1);

a0 := trunc(evalf(rand( ) * 10^(-11)));
a := trunc(evalf(rand( ) * 10^(-11)));
b := trunc(evalf(rand( ) * 10^(-11)));
n := trunc(evalf(rand( ) * 10^(-11)));
G := GeneNet(a0, a, b, n, m01, m02, m03, p01, p02, p03);
m1 := trunc(evalf(rand( ) * 10^(-11)));
m2 := trunc(evalf(rand( ) * 10^(-11)));
m3 := trunc(evalf(rand( ) * 10^(-11)));
p1 := trunc(evalf(rand( ) * 10^(-11)));
p2 := trunc(evalf(rand( ) * 10^(-11)));
p3 := trunc(evalf(rand( ) * 10^(-11)));
SEquP(G, [m01, m02, m03, p01, p02, p03]);
TimeSeries(G, [m01, m02, m03, p01, p02, p03], [m1, m2, m3, p1, p2, p3], 0.01, 10, 1);
```

$$a0 := 2$$

$$a := 0$$

$$b := 7$$

$$n := 5$$

$$G := [-m01 + 2, -m02 + 2, -m03 + 2, -7\,p01 + 7\,m01, -7\,p02 + 7\,m02, -7\,p03 + 7\,m03]$$

$$m1 := 7$$

$$m2 := 4$$

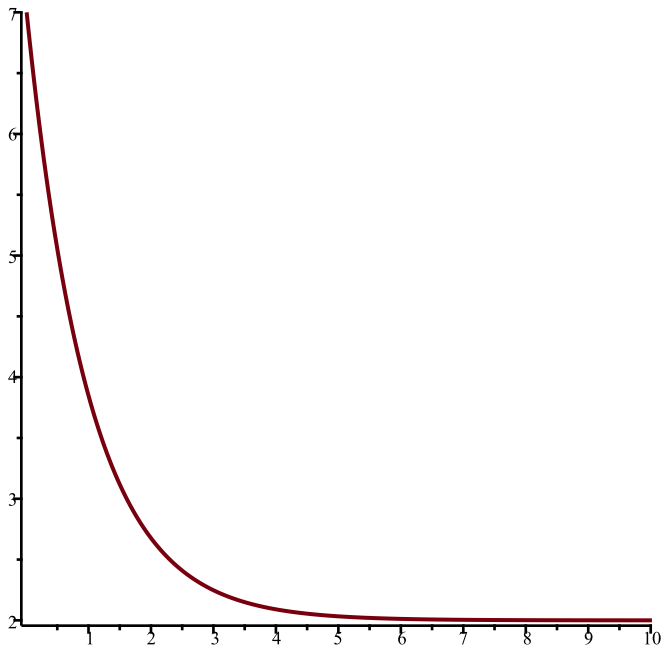
$$m3 := 8$$

$$p1 := 8$$

$$p2 := 9$$

$$p3 := 4$$

$$\{[2., 2., 2., 2., 2., 2.]\}$$



$$a0 := 5$$

$$a := 3$$

$$b := 9$$

$$n := 7$$

$$G := \left[-m01 + \frac{3}{p03^7 + 1} + 5,\ -m02 + \frac{3}{p01^7 + 1} + 5,\ -m03 + \frac{3}{p02^7 + 1} + 5,\ -9\,p01 + 9\,m01,\ -9\,p02 + 9\,m02,\ -9\,p03 + 9\,m03 \right]$$

$$m1 := 6$$

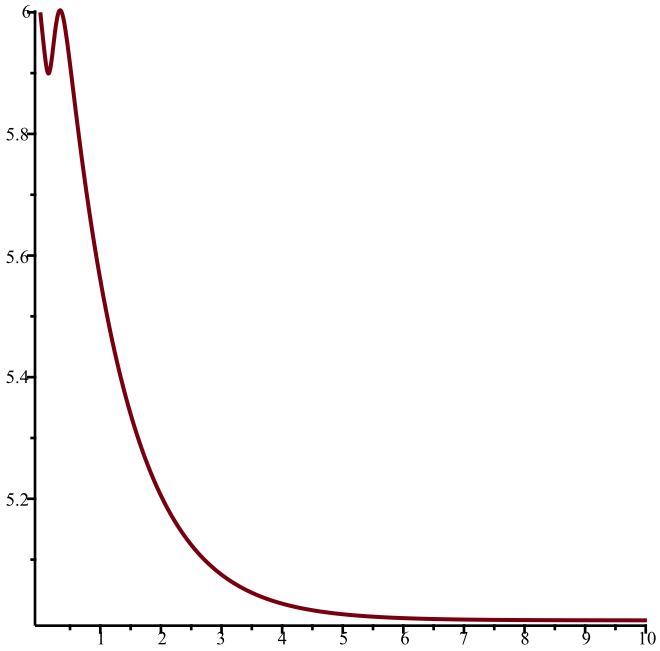
$$m2 := 6$$

$$m3 := 0$$

$$p1 := 0$$

$$p2 := 4$$

$$p3 := 3$$



$$a0 := 0$$

$$a := 7$$

$$b := 3$$

$$n := 7$$

$$G := \left[-m01 + \frac{7}{p03^7 + 1}, -m02 + \frac{7}{p01^7 + 1}, -m03 + \frac{7}{p02^7 + 1}, -3\,p01 + 3\,m01, -3\,p02 + 3\,m02, -3\,p03 + 3\,m03 \right]$$

$$m1 := 1$$

$$m2 := 4$$

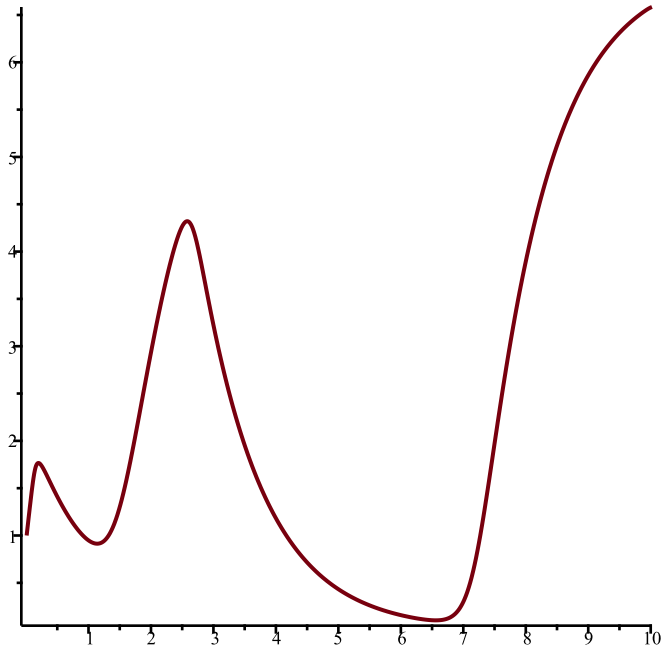
$$m3 := 3$$

$$p1 := 9$$

$$p2 := 4$$

$$p3 := 0$$

$$\varnothing$$




```
> #Lotka
 r1 := trunc(evalf(rand( ) * 10^(-11)));
r2 := trunc(evalf(rand( ) * 10^(-11)));
k1 := trunc(evalf(rand( ) * 10^(-11)));
k2 := trunc(evalf(rand( ) * 10^(-11)));
b12 := trunc(evalf(rand( ) * 10^(-11)));
b21 := trunc(evalf(rand( ) * 10^(-11)));
L := Lotka(r1, k1, r2, k2, b12, b21, N1, N2);
n1 := trunc(evalf(rand( ) * 10^(-11)));
n2 := trunc(evalf(rand( ) * 10^(-11)));
SEquP(L, [N1, N2]);
TimeSeries(L, [N1, N2], [n1, n2], 0.01, 10, 1);
TimeSeries(L, [N1, N2], [n1, n2], 0.01, 10, 2);
PhaseDiag(L, [N1, N2], [n1, n2], 0.01, 10);

r1 := trunc(evalf(rand( ) * 10^(-11)));
r2 := trunc(evalf(rand( ) * 10^(-11)));
k1 := trunc(evalf(rand( ) * 10^(-11)));
k2 := trunc(evalf(rand( ) * 10^(-11)));
b12 := trunc(evalf(rand( ) * 10^(-11)));
b21 := trunc(evalf(rand( ) * 10^(-11)));
L := Lotka(r1, k1, r2, k2, b12, b21, N1, N2);
n1 := trunc(evalf(rand( ) * 10^(-11)));
n2 := trunc(evalf(rand( ) * 10^(-11)));
SEquP(L, [N1, N2]);
TimeSeries(L, [N1, N2], [n1, n2], 0.01, 10, 1);
TimeSeries(L, [N1, N2], [n1, n2], 0.01, 10, 2);
PhaseDiag(L, [N1, N2], [n1, n2], 0.01, 10);

r1 := trunc(evalf(rand( ) * 10^(-11)));
r2 := trunc(evalf(rand( ) * 10^(-11)));
```

```
k1 := trunc(evalf(rand( ) * 10^(-11)));
k2 := trunc(evalf(rand( ) * 10^(-11)));
b12 := trunc(evalf(rand( ) * 10^(-11)));
b21 := trunc(evalf(rand( ) * 10^(-11)));
L := Lotka(r1, k1, r2, k2, b12, b21, N1, N2);
n1 := trunc(evalf(rand( ) * 10^(-11)));
n2 := trunc(evalf(rand( ) * 10^(-11)));
SEquP(L, [N1, N2]);
TimeSeries(L, [N1, N2], [n1, n2], 0.01, 10, 1);
TimeSeries(L, [N1, N2], [n1, n2], 0.01, 10, 2);
PhaseDiag(L, [N1, N2], [n1, n2], 0.01, 10);
```

$$r1 := 7$$

$$r2 := 3$$

$$k1 := 3$$

$$k2 := 5$$

$$b12 := 9$$

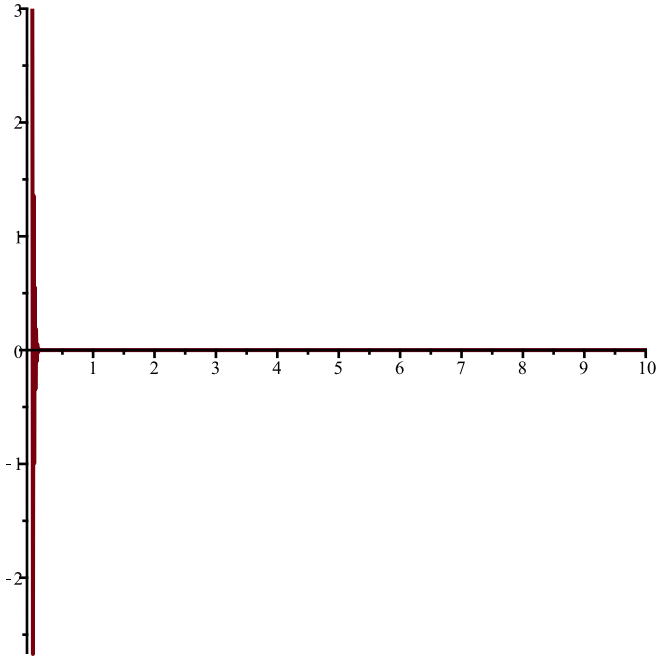
$$b21 := 6$$

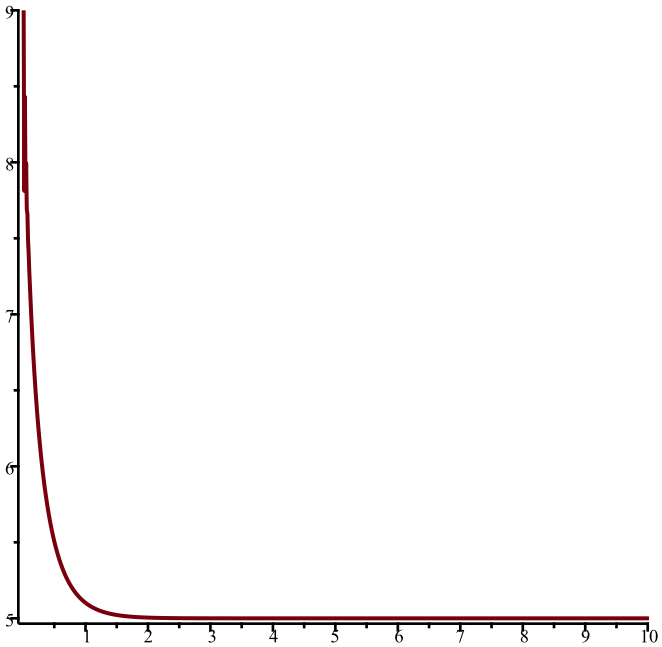
$$L := \left[\frac{7\,N1\,(3 - N1 - 9\,N2)}{3}, \frac{3\,N2\,(5 - N2 - 6\,N1)}{5} \right]$$

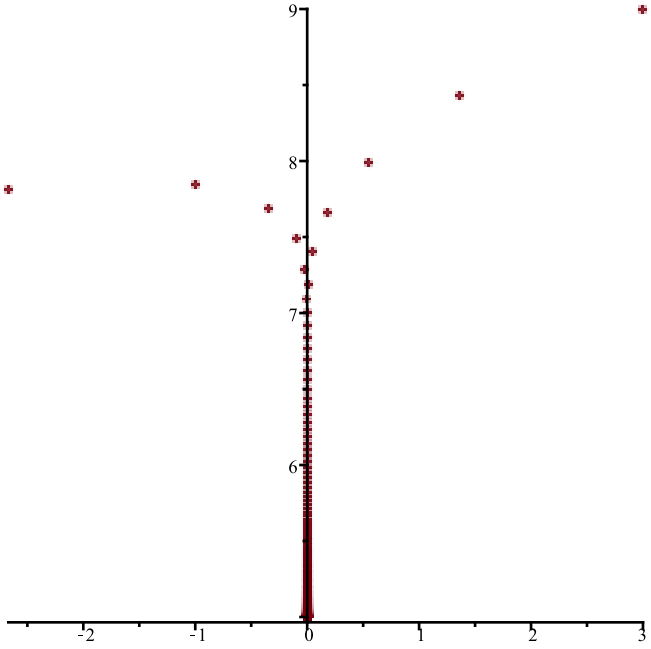
$$n1 := 3$$

$$n2 := 9$$

$$\{[0., 5.], [3., 0.]\}$$







$$r1 := 8$$

$$r2 := 5$$

$$k1 := 4$$

$$k2 := 6$$

$$b12 := 5$$

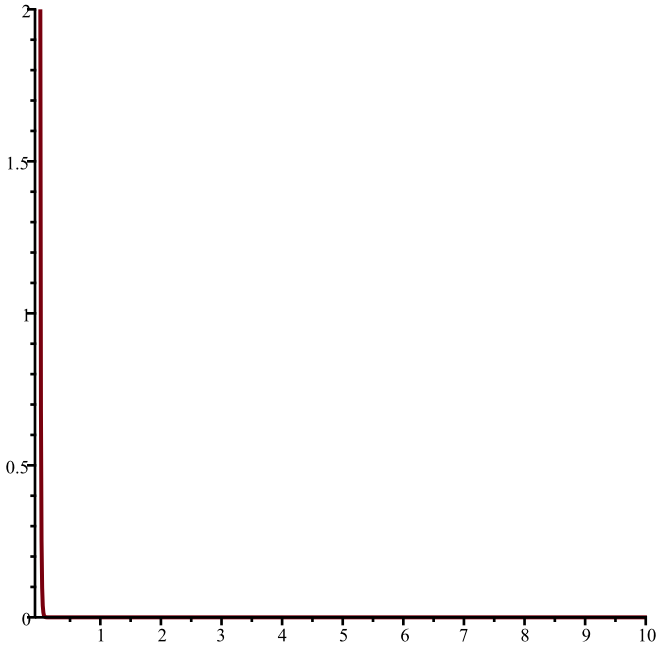
$$b21 := 0$$

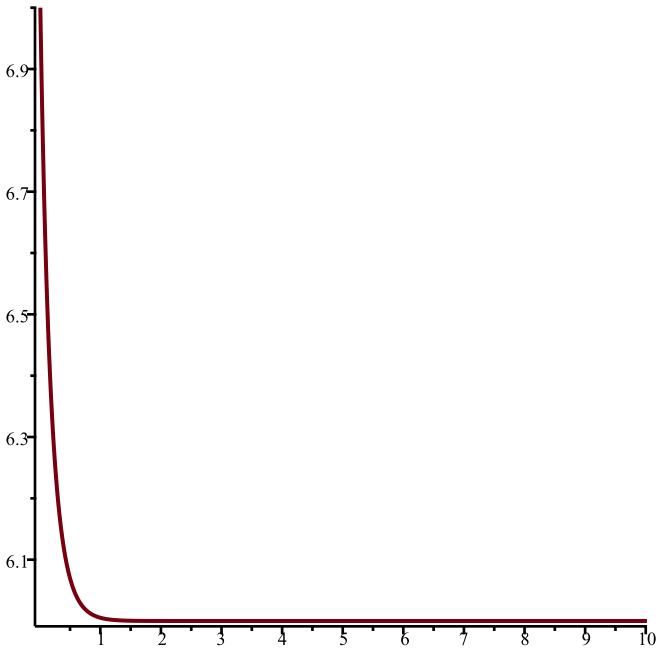
$$L := \left[2\,N1\,(4 - N1 - 5\,N2),\ \frac{5\,N2\,(6 - N2)}{6}\right]$$

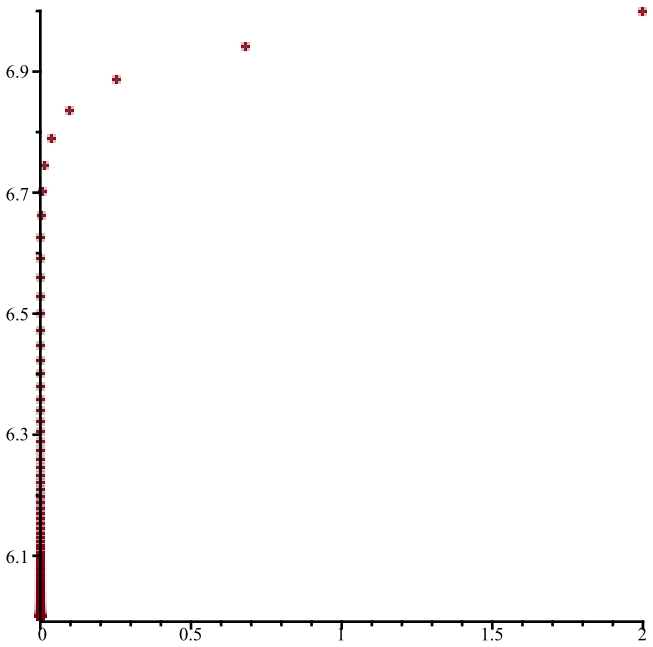
$$n1 := 2$$

$$n2 := 7$$

{[0., 6.]}







$$r1 := 8$$

$$r2 := 0$$

$$k1 := 1$$

$$k2 := 4$$

$$b12 := 9$$

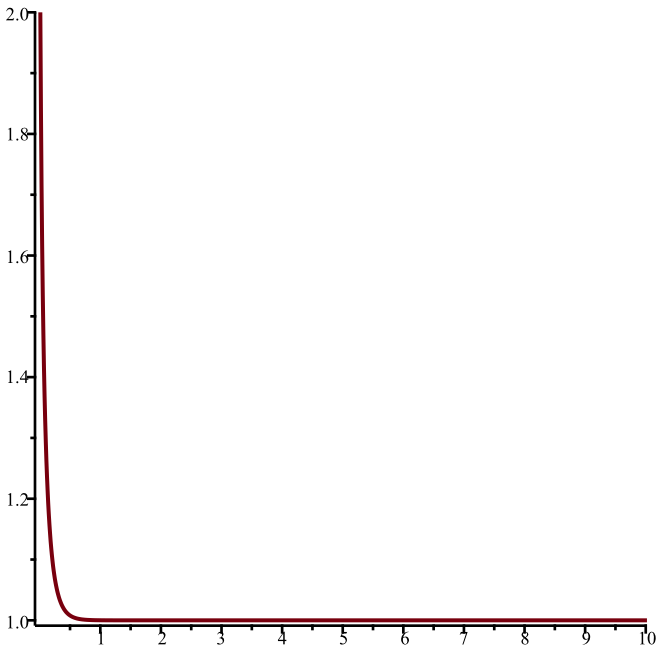
$$b21 := 6$$

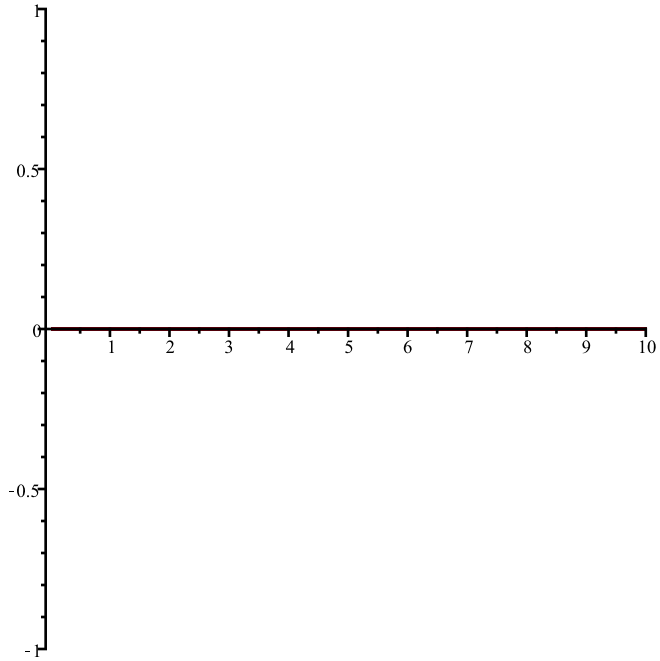
$$L := [8\,N1\,(1 - N1 - 9\,N2), 0]$$

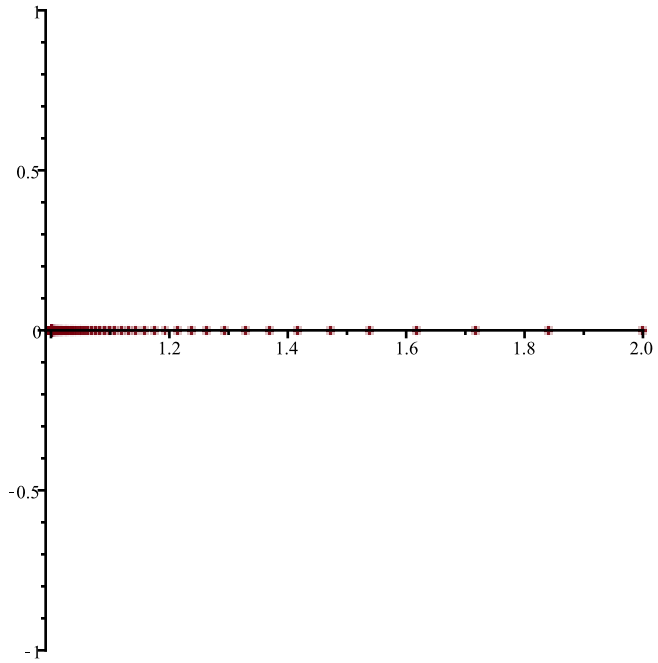
$$n1 := 2$$

$$n2 := 0$$

Error, (in SEquP) cannot determine if this expression is true or false: max(0, 8.-72.*N2) < 0 |/John/Rutgers/Senior Fall/Dynamic Models/DMB.txt:647|


2.0
1.8
1.6
1.4
1.2
1.0
1
2
3
4
5
6
7
8
9
10


1
0.5
0
-0.5
-1
1
2
3
4
5
6
7
8
9
10




```
> #Volterra
 a := trunc(evalf(rand( ) * 10^(-11)));
b := trunc(evalf(rand( ) * 10^(-11)));
c := trunc(evalf(rand( ) * 10^(-11)));
d := trunc(evalf(rand( ) * 10^(-11)));
V := Volterra(a, b, c, d, x, y);
x1 := trunc(evalf(rand( ) * 10^(-11)));
y2 := trunc(evalf(rand( ) * 10^(-11)));
SEquP(V, [x, y]);
TimeSeries(V, [x, y], [x1, y2], 0.01, 1, 1);
TimeSeries(V, [x, y], [x1, y2], 0.01, 1, 2);
PhaseDiag(V, [x, y], [x1, y2], 0.01, 10);

a := trunc(evalf(rand( ) * 10^(-11)));
b := trunc(evalf(rand( ) * 10^(-11)));
c := trunc(evalf(rand( ) * 10^(-11)));
d := trunc(evalf(rand( ) * 10^(-11)));
V := Volterra(a, b, c, d, x, y);
x1 := trunc(evalf(rand( ) * 10^(-11)));
y2 := trunc(evalf(rand( ) * 10^(-11)));
SEquP(V, [x, y]);
TimeSeries(V, [x, y], [x1, y2], 0.01, 1, 1);
TimeSeries(V, [x, y], [x1, y2], 0.01, 1, 2);
PhaseDiag(V, [x, y], [x1, y2], 0.01, 10);

a := trunc(evalf(rand( ) * 10^(-11)));
b := trunc(evalf(rand( ) * 10^(-11)));
c := trunc(evalf(rand( ) * 10^(-11)));
d := trunc(evalf(rand( ) * 10^(-11)));
V := Volterra(a, b, c, d, x, y);
x1 := trunc(evalf(rand( ) * 10^(-11)));
```

```
y2 := trunc(evalf(rand( ) * 10^( - 11) ) );
SEquP(V, [x, y]);
TimeSeries(V, [x, y], [x1, y2], 0.01, 1, 1);
TimeSeries(V, [x, y], [x1, y2], 0.01, 1, 2);
PhaseDiag(V, [x, y], [x1, y2], 0.01, 10);
```

$$a := 5$$

$$b := 6$$

$$c := 1$$

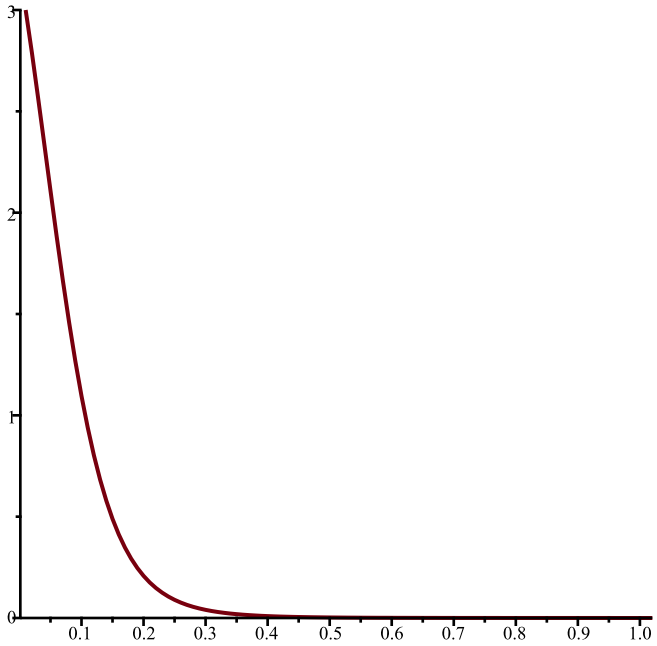
$$d := 3$$

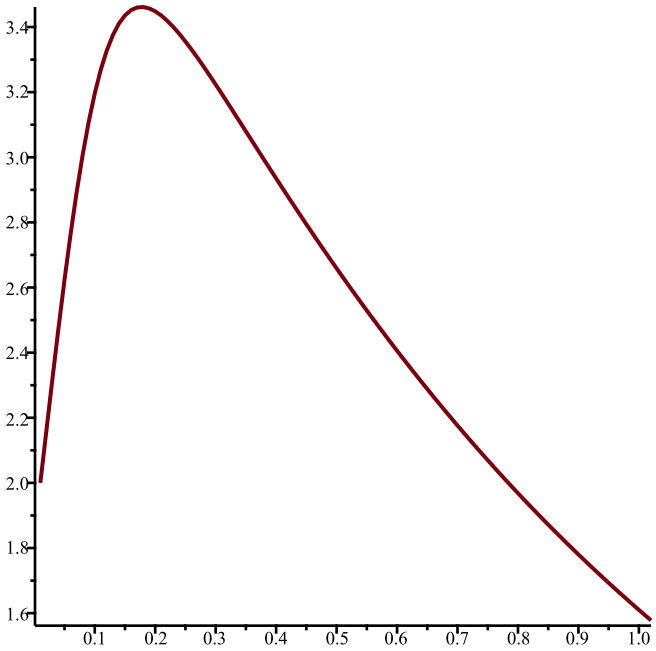
$$V := [-6\,x\,y + 5\,x,\ 3\,x\,y - y]$$

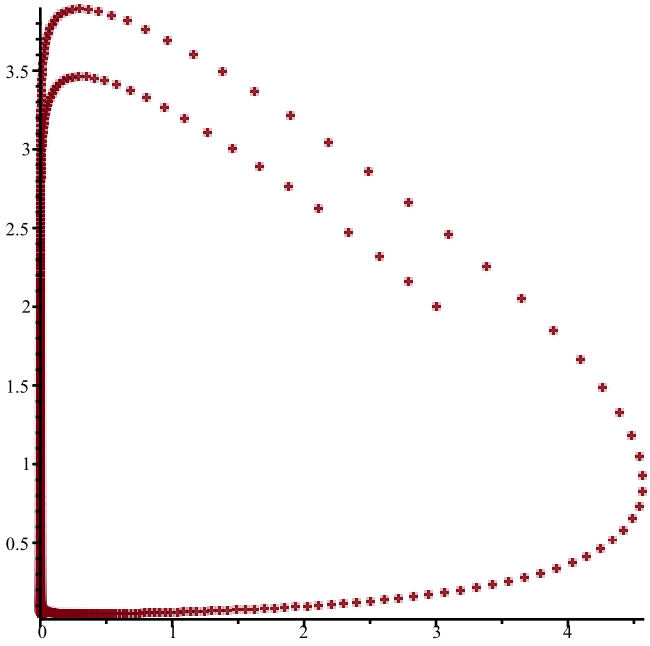
$$x1 := 3$$

$$y2 := 2$$

$$\{[0.3333333333,\ 0.8333333333]\}$$







$$a := 2$$

$$b := 7$$

$$c := 7$$

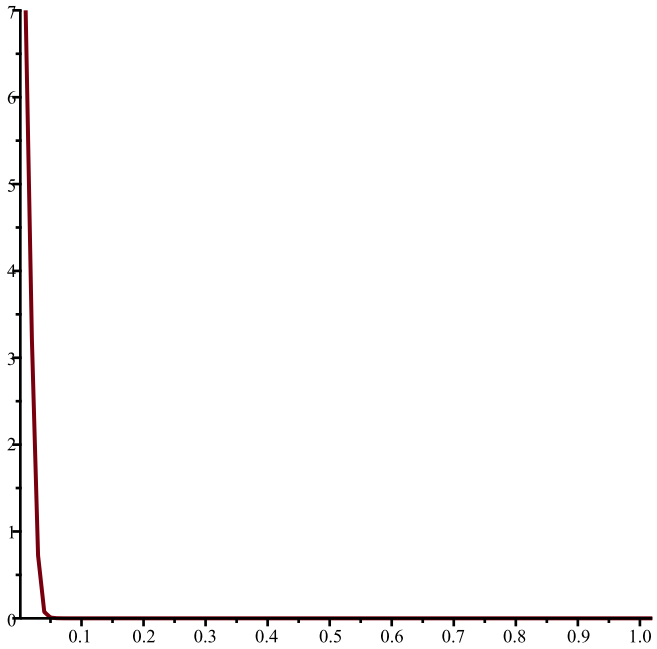
$$d := 7$$

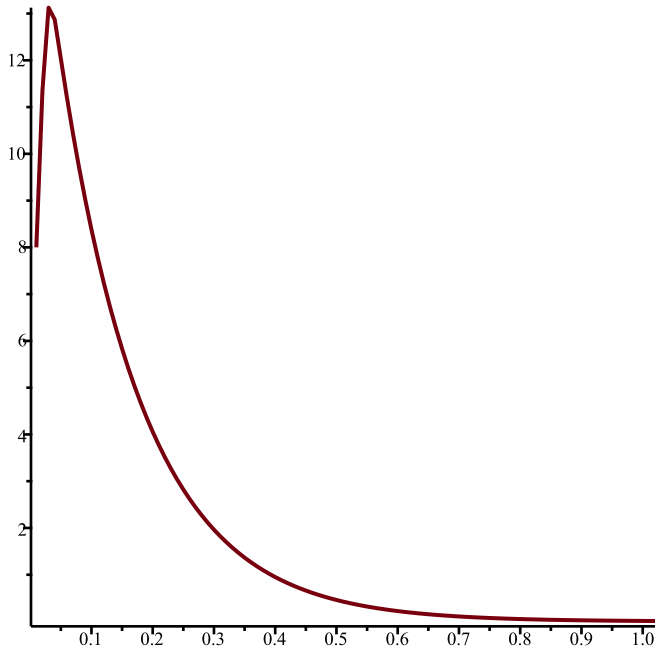
$$V := [-7\,x\,y + 2\,x,\ 7\,x\,y - 7\,y]$$

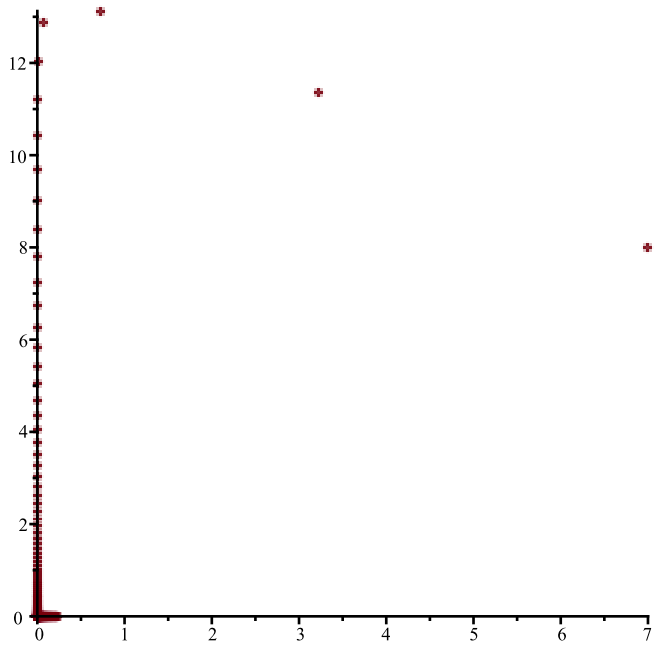
$$x1 := 7$$

$$y2 := 8$$

$$\varnothing$$


7
6
5
4
3
2
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0


12
10
8
6
4
2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0



$$a := 8$$

$$b := 2$$

$$c := 4$$

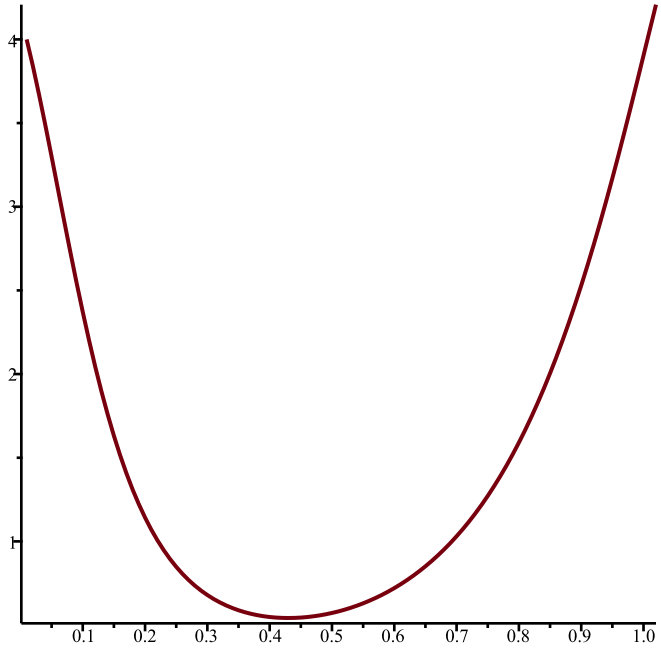
$$d := 2$$

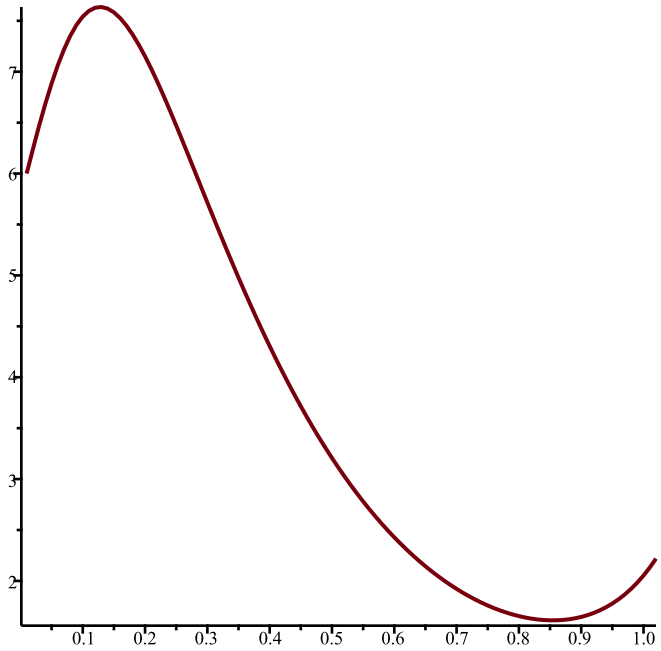
$$V := [-2\,x\,y + 8\,x,\ 2\,x\,y - 4\,y]$$

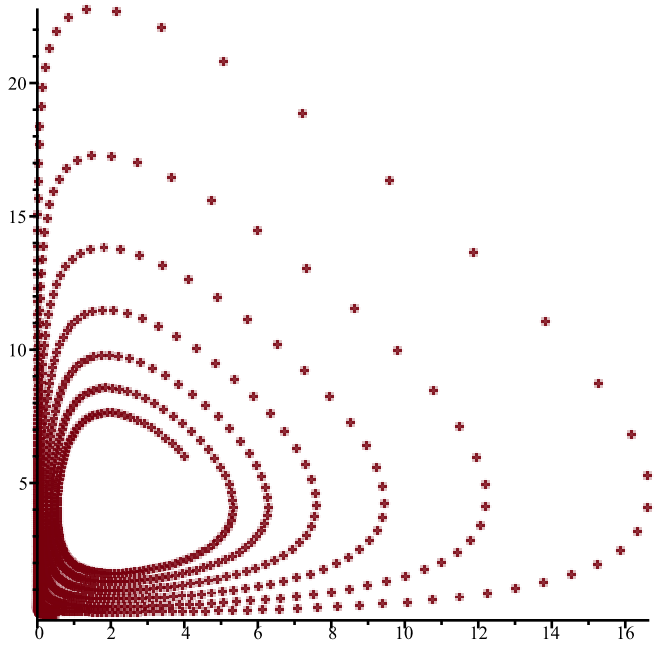
$$x1 := 4$$

$$y2 := 6$$

$$\varnothing$$








```
> #VolterraM
  a := trunc(evalf(rand( ) * 10^(-11)));
  b := trunc(evalf(rand( ) * 10^(-11)));
  c := trunc(evalf(rand( ) * 10^(-11)));
  d := trunc(evalf(rand( ) * 10^(-11)));
  K := trunc(evalf(rand( ) * 10^(-11)));
  V2 := VolterraM(a, b, c, d, K, x, y);
  x1 := trunc(evalf(rand( ) * 10^(-11)));
  y2 := trunc(evalf(rand( ) * 10^(-11)));
  SEquP(V2, [x, y]);
  TimeSeries(V2, [x, y], [x1, y2], 0.01, 10, 1);
  TimeSeries(V2, [x, y], [x1, y2], 0.01, 1, 2);
  PhaseDiag(V2, [x, y], [x1, y2], 0.01, 10);
```

```
a := trunc(evalf(rand( ) * 10^(-11)));
b := trunc(evalf(rand( ) * 10^(-11)));
c := trunc(evalf(rand( ) * 10^(-11)));
d := trunc(evalf(rand( ) * 10^(-11)));
K := trunc(evalf(rand( ) * 10^(-11)));
V2 := VolterraM(a, b, c, d, K, x, y);
x1 := trunc(evalf(rand( ) * 10^(-11)));
y2 := trunc(evalf(rand( ) * 10^(-11)));
SEquP(V2, [x, y]);
TimeSeries(V2, [x, y], [x1, y2], 0.01, 10, 1);
TimeSeries(V2, [x, y], [x1, y2], 0.01, 1, 2);
PhaseDiag(V2, [x, y], [x1, y2], 0.01, 10);

a := trunc(evalf(rand( ) * 10^(-11)));
b := trunc(evalf(rand( ) * 10^(-11)));
c := trunc(evalf(rand( ) * 10^(-11)));
d := trunc(evalf(rand( ) * 10^(-11)));
K := trunc(evalf(rand( ) * 10^(-11)));
V2 := VolterraM(a, b, c, d, K, x, y);
x1 := trunc(evalf(rand( ) * 10^(-11)));
y2 := trunc(evalf(rand( ) * 10^(-11)));
SEquP(V2, [x, y]);
TimeSeries(V2, [x, y], [x1, y2], 0.01, 10, 1);
TimeSeries(V2, [x, y], [x1, y2], 0.01, 1, 2);
 PhaseDiag(V2, [x, y], [x1, y2], 0.01, 10);
```

$$a := 4$$

$$b := 2$$

$$c := 7$$

$$d := 0$$

$$K := 4$$

Error, (in VolterraM) numeric exception: division by zero
|/John/Rutgers/Senior Fall/Dynamic Models/DMB.txt:838|

$$x1 := 3$$

$$y2 := 0$$

bad input

FAIL

bad input

FAIL

bad input

FAIL

bad input

FAIL

$$a := 5$$

$$b := 3$$

$$c := 4$$

$$d := 2$$

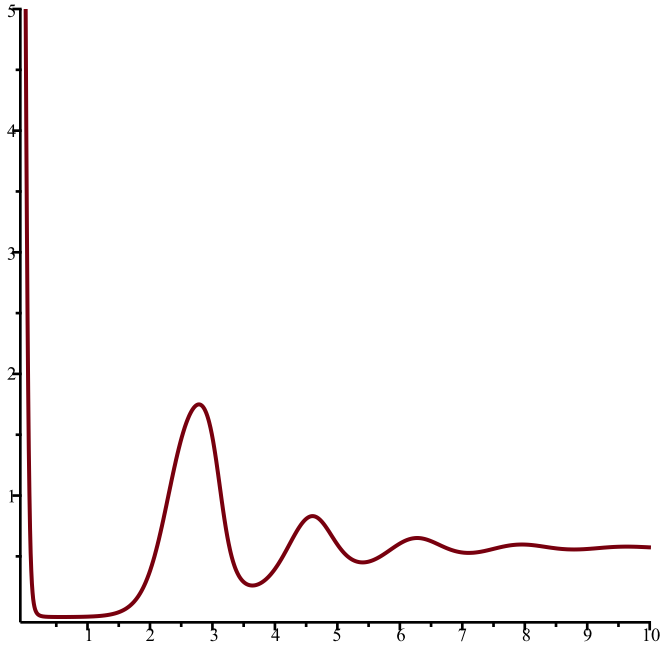
$$K := 7$$

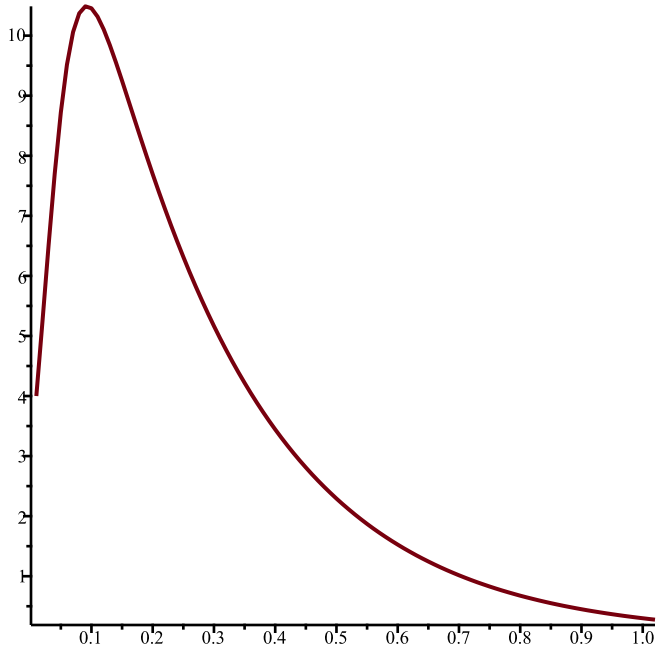
$$V2 := \left[5\,x\left(1 - \frac{x}{2}\right) - 3\,x\,y,\ 7\,x\,y - 4\,y\right]$$

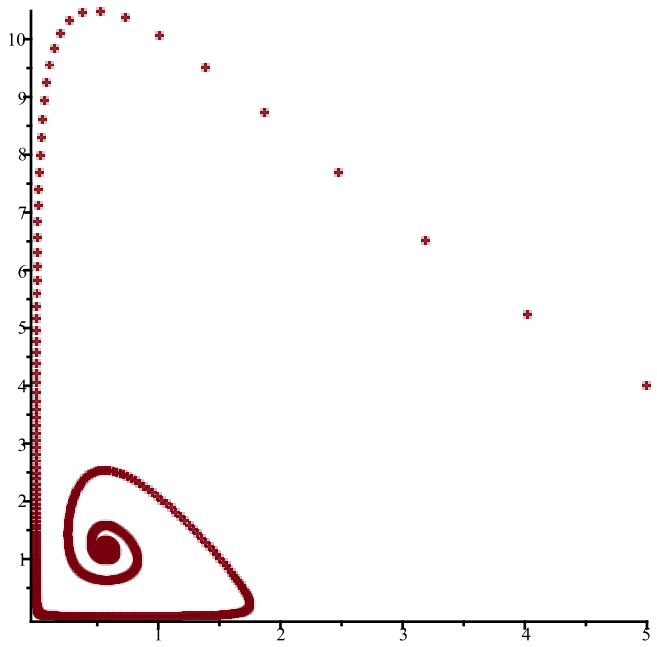
$$x1 := 5$$

$$y2 := 4$$

$$\{[0.5714285714,\ 1.190476190]\}$$







$$a := 0$$

$$b := 0$$

$$c := 1$$

$$d := 1$$

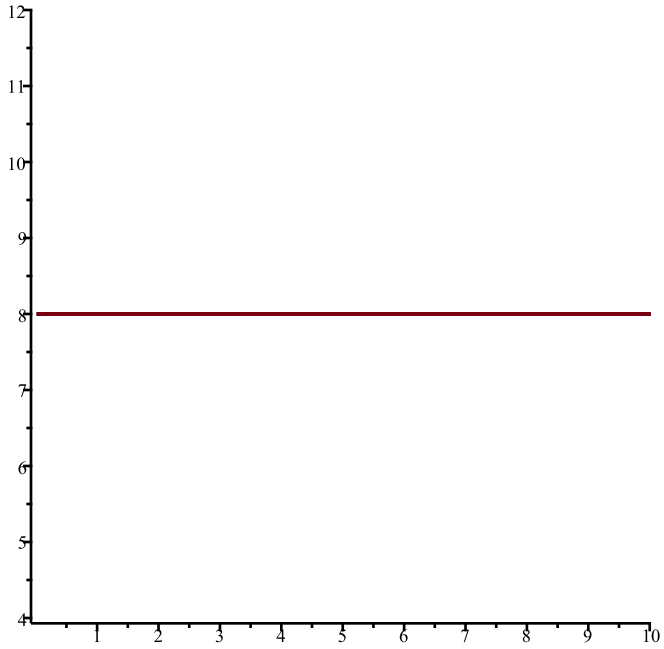
$$K := 8$$

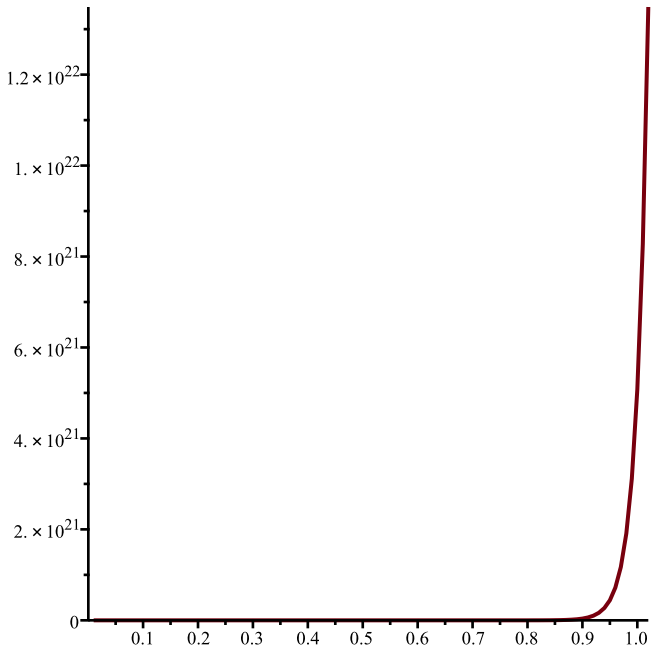
$$V2 := [0, 8\,x\,y - y]$$

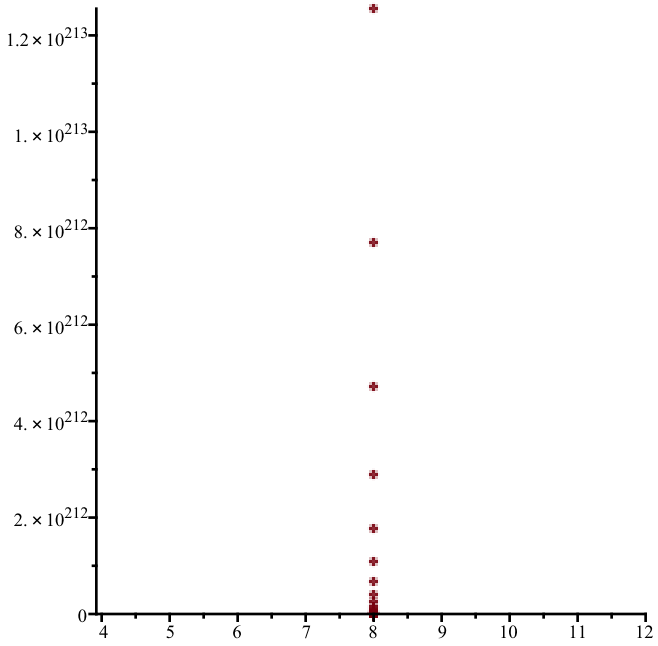
$$x1 := 8$$

$$y2 := 5$$

Error, (in SEquP) cannot determine if this expression is true or false: max(0, 8.*x-1.) < 0 |/John/Rutgers/Senior Fall/Dynamic Models/DMB.txt:647|




1.2 × 10^22
1. × 10^22
8. × 10^21
6. × 10^21
4. × 10^21
2. × 10^21
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0


1.2 × 10^213
1. × 10^213
8. × 10^212
6. × 10^212
4. × 10^212
2. × 10^212
0
4
5
6
7
8
9
10
11
12