```
> #OK to post Homework
> #Jeton Hida, Assignment 21, November 15, 2021
> read "/Users/jeton/Desktop/Math 336/DMB.txt"
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

```
> #Question 1
> Help(ChemoStat)
```

ChemoStat(N,C,a1,a2): The Chemostat continuous-time dynamical system with N=Bacterial poplulation densitty, and C=nutient Concentration in growth chamber (see Table 4.1 of Edelstein-Keshet, p. 122)
with paramerts a1, a2, Equations (19a_, (19b) in Edelestein-Keshet p. 127 (section 4.5, where they are called alpha1, alpha2). a1 and a2 can be symbolic or numeric. Try:
ChemoStat(N,C,a1,a2);

ChemoStat(N,C,2,3); **(2)**

```
> ChemoStat(N,C,a1,a2)
```

$$\left[\frac{a1\,C\,N}{C+1}-N, -\frac{C\,N}{C+1}-C+a2\right] \quad (3)$$

```
> F:=ChemoStat(N,C,2,3)
```

$$F := \left[\frac{2\,C\,N}{C+1}-N, -\frac{C\,N}{C+1}-C+3\right] \quad (4)$$

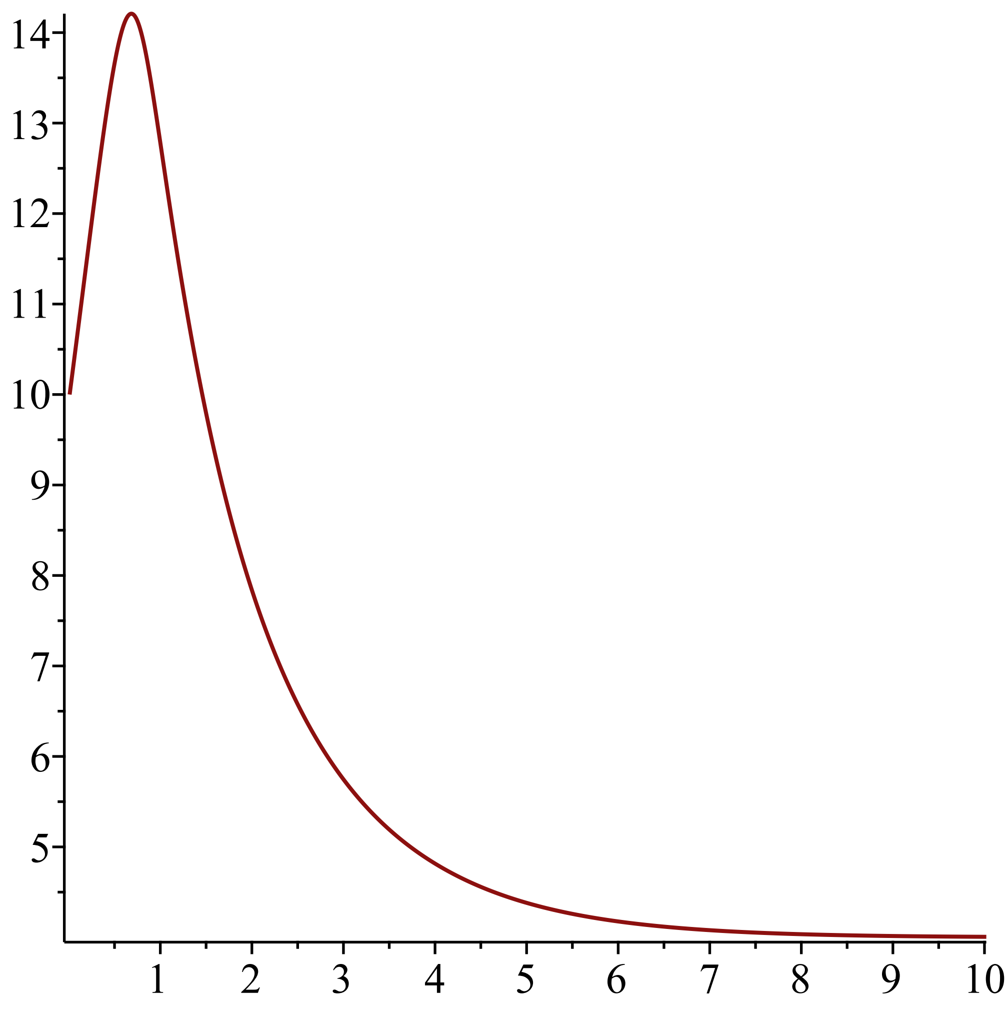
```
> EquP(F,[N,C])
```

$$\{[0, 3], [4, 1]\} \quad (5)$$

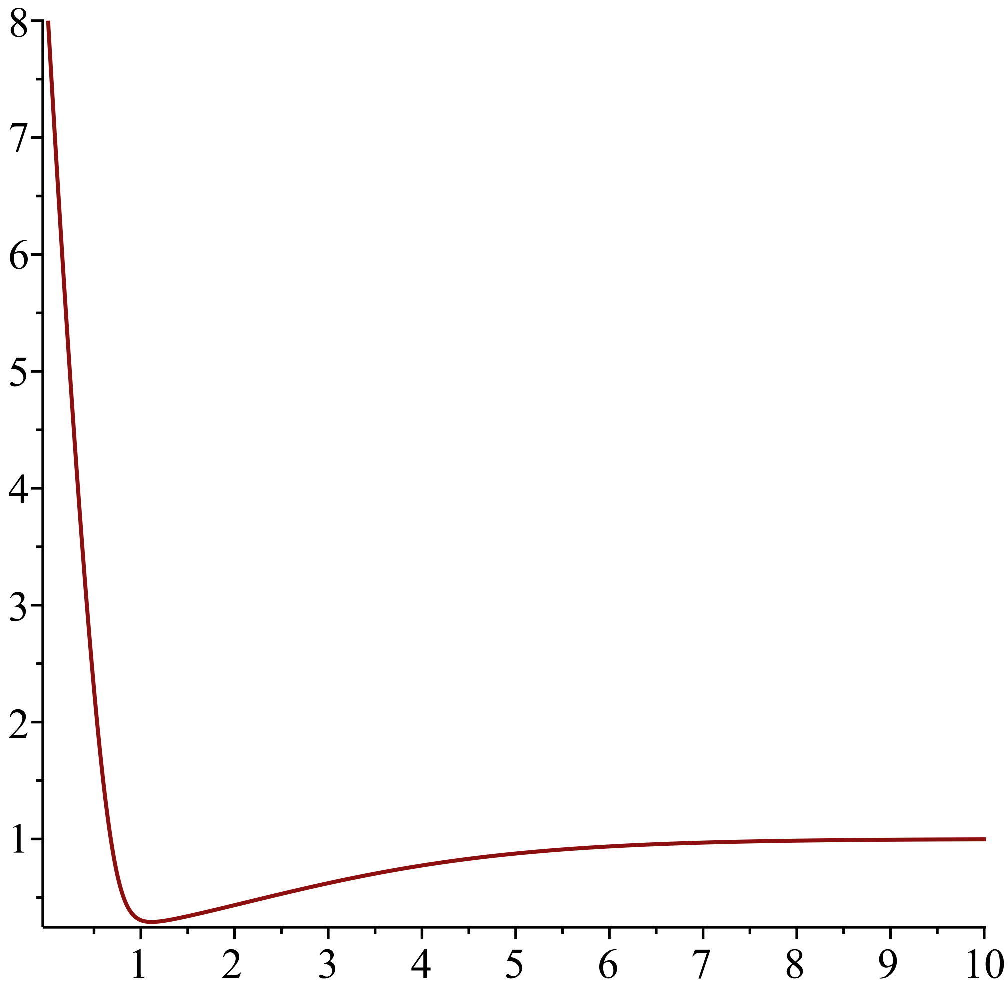
```
> SEquP(F,[N,C])
```

$$\{[4., 1.]\} \quad (6)$$

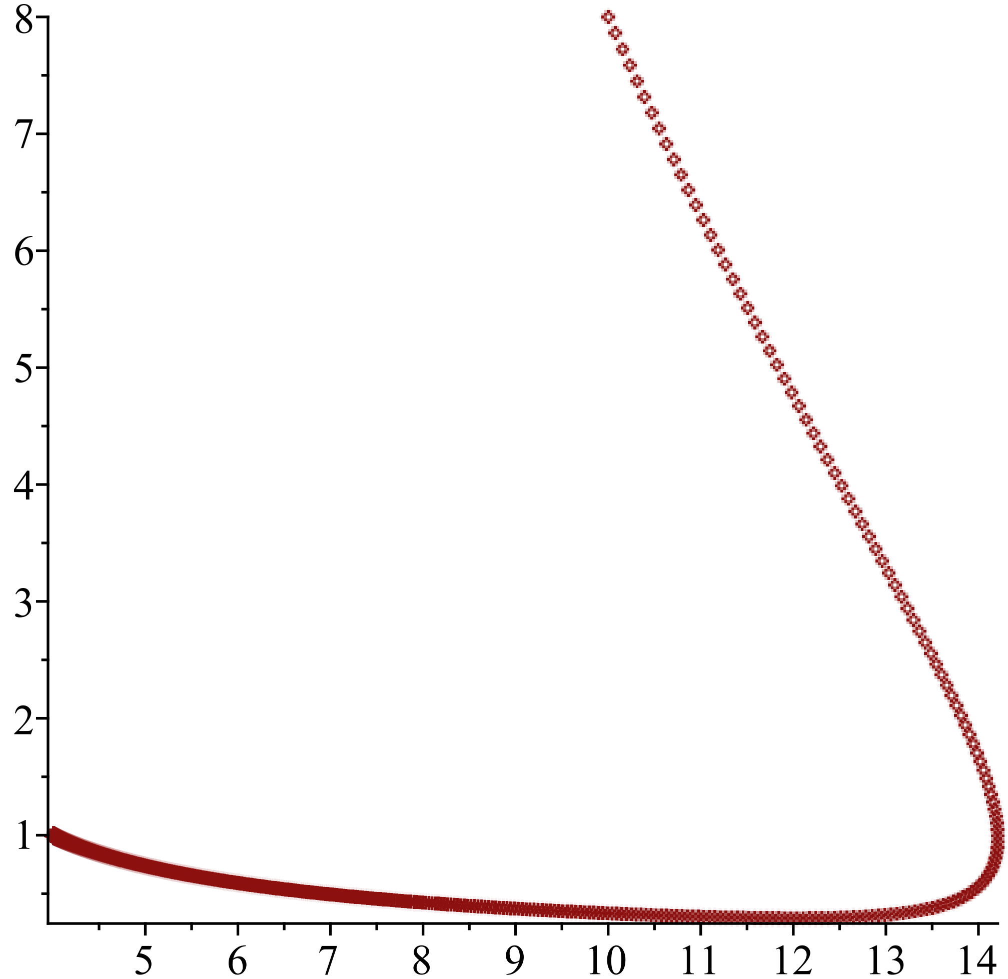
```
> TimeSeries(F,[N,C],[10,8],.01,10,1)
```




```
> TimeSeries(F,[N,C],[10,8],.01,10,2)
```




```
> PhaseDiag(F,[N,C],[10,8],.01,10)
```




```
> Help(GeneNet)
```

GeneNet(a0,a,b,n,m1,m2,m3,p1,p2,p3): The contiuous-time dynamical system, with quantities m1,m2,m3,p1,p2,p3, due to M. Elowitz and S. Leibler described in the Ellner-Guckenheimer book, Eq. (4.1) (chapter 4, p. 112) and parameers a0 (called alpha_0 there),a (called alpha there), b (called beta there) and n. Try: GeneNet(0,0.5,0.2,2,m1,m2,m3,p1,p2,p3); **(7)**

```
> GeneNet(a0,a,b,n,m1,m2,m3,p1,p2,p3)
```

$$\left[-m1+\frac{a}{1+p3^{n}}+a0,\ -m2+\frac{a}{1+p1^{n}}+a0,\ -m3+\frac{a}{1+p2^{n}}+a0,\ -b\,(p1-m1),\ -b\,(p2-m2),\ -b\,(p3-m3)\right] \tag{8}$$

```
> F:=GeneNet(0,50,.2,2,m1,m2,m3,p1,p2,p3)
```

$$F := \left[-m1+\frac{50}{p3^{2}+1},\ -m2+\frac{50}{p1^{2}+1},\ -m3+\frac{50}{p2^{2}+1},\ -0.2\,p1+0.2\,m1,\ -0.2\,p2+0.2\,m2,\ -0.2\,p3+0.2\,m3\right] \tag{9}$$

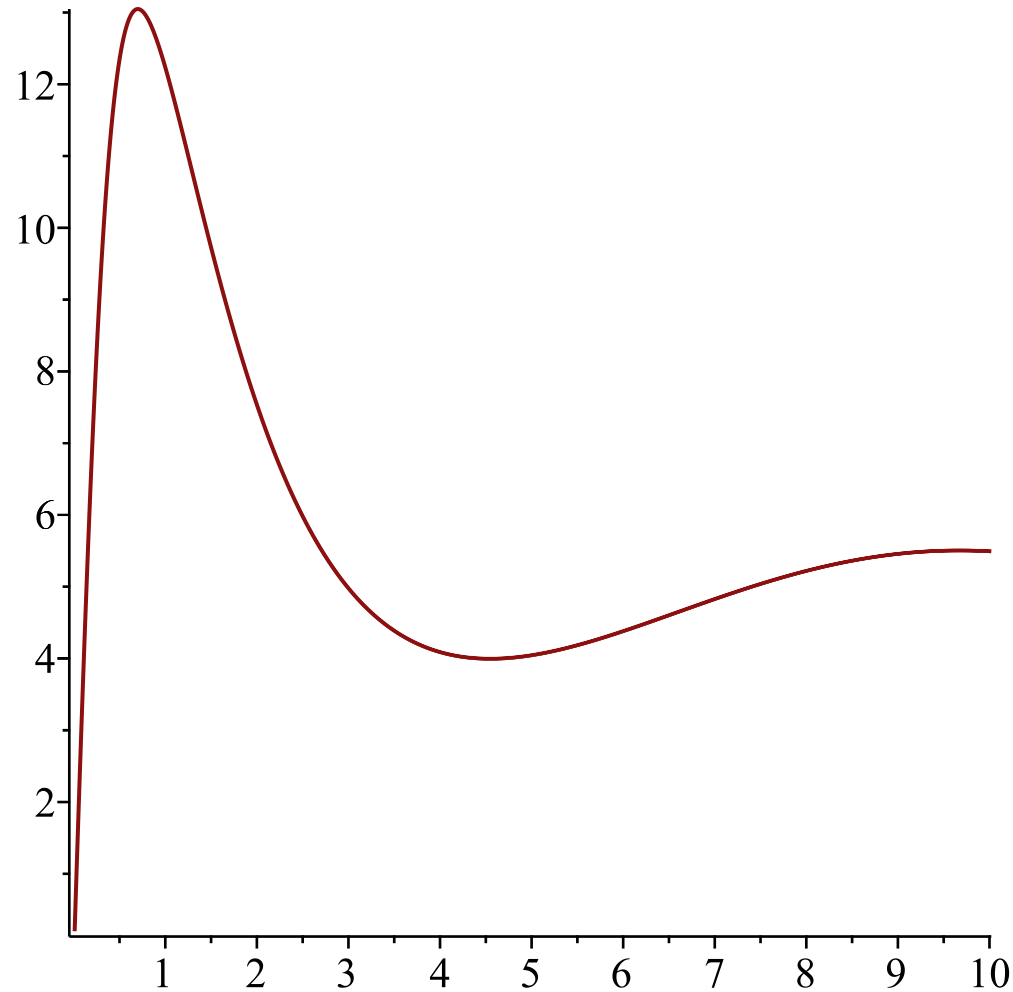
```
> EquP(F,[m1,m2,m3,p1,p2,p3])
```

{[3.593569551, 3.593569551, 3.593569551, 3.593569551, 3.593569551, 3.593569551], **(10)**
[−3.357699926 − 1.385834536 I, 2.671114375 − 2.400954221 I, 0.6965815521 + 3.769470557 I, −3.357699926 − 1.385834536 I, 2.671114375 − 2.400954221 I, 0.6965815521 + 3.769470557 I], [−3.357699926 + 1.385834536 I, 2.671114375 + 2.400954221 I, 0.6965815521 − 3.769470557 I, −3.357699926 + 1.385834536 I, 2.671114375 + 2.400954221 I, 0.6965815521 − 3.769470557 I], [−1.796784775 − 3.268838721 I, −1.796784775 − 3.268838721 I, −1.796784775 − 3.268838721 I, −1.796784775 − 3.268838721 I, −1.796784775 − 3.268838721 I, −1.796784775 − 3.268838721 I], [−1.796784775 + 3.268838721 I, −1.796784775 + 3.268838721 I, −1.796784775 + 3.268838721 I, −1.796784775 + 3.268838721 I, −1.796784775 + 3.268838721 I, −1.796784775 + 3.268838721 I], [0.6965815521 − 3.769470557 I, −3.357699926 + 1.385834536 I, 2.671114375 + 2.400954221 I, 0.6965815521 − 3.769470557 I, −3.357699926 + 1.385834536 I, 2.671114375 + 2.400954221 I], [0.6965815521 + 3.769470557 I, −3.357699926 − 1.385834536 I, 2.671114375 − 2.400954221 I, 0.6965815521 + 3.769470557 I, −3.357699926 − 1.385834536 I, 2.671114375 − 2.400954221 I], [2.671114375 − 2.400954221 I, 0.6965815521 + 3.769470557 I, −3.357699926 − 1.385834536 I, 2.671114375 − 2.400954221 I, 0.6965815521 + 3.769470557 I, −3.357699926 − 1.385834536 I], [2.671114375 + 2.400954221 I, 0.6965815521 − 3.769470557 I, −3.357699926 + 1.385834536 I, 2.671114375 + 2.400954221 I, 0.6965815521 − 3.769470557 I, −3.357699926 + 1.385834536 I]}

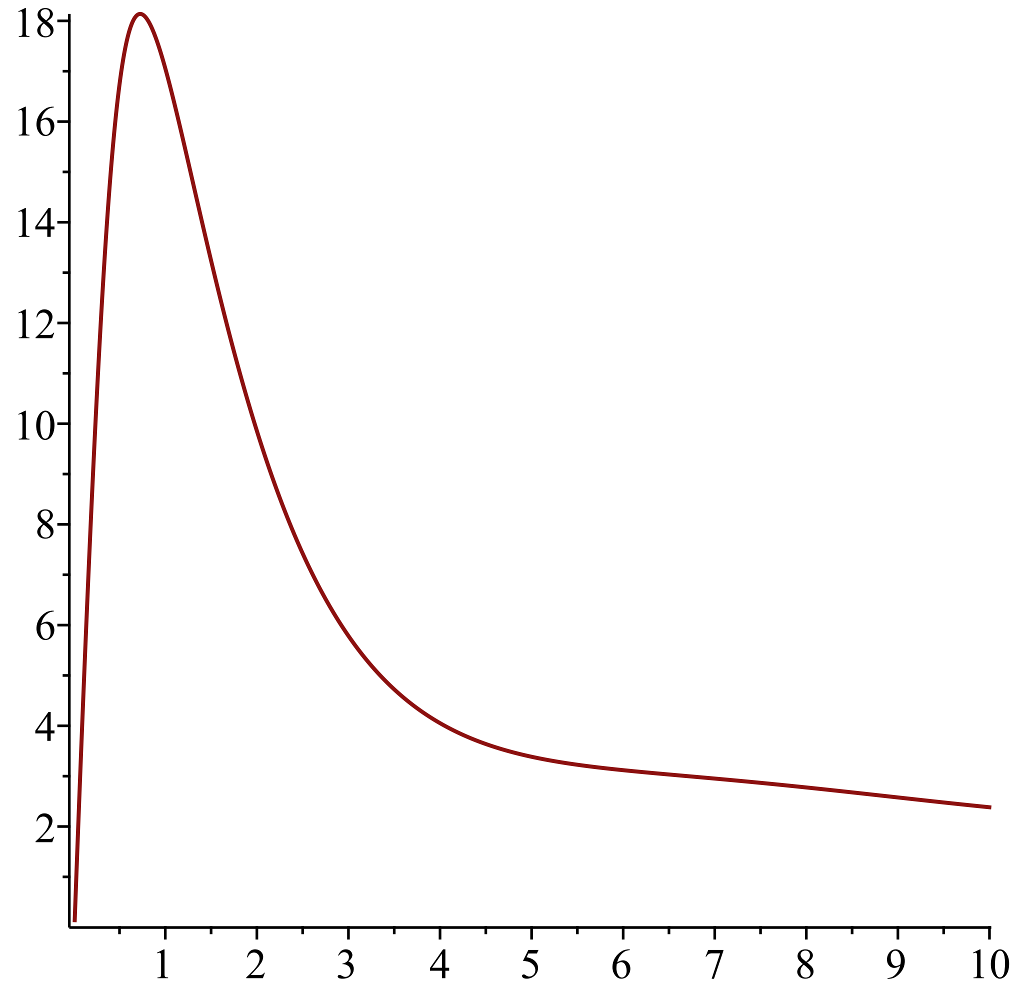
```
> SEquP(F,[m1,m2,m3,p1,p2,p3])
```

$\varnothing$ **(11)**

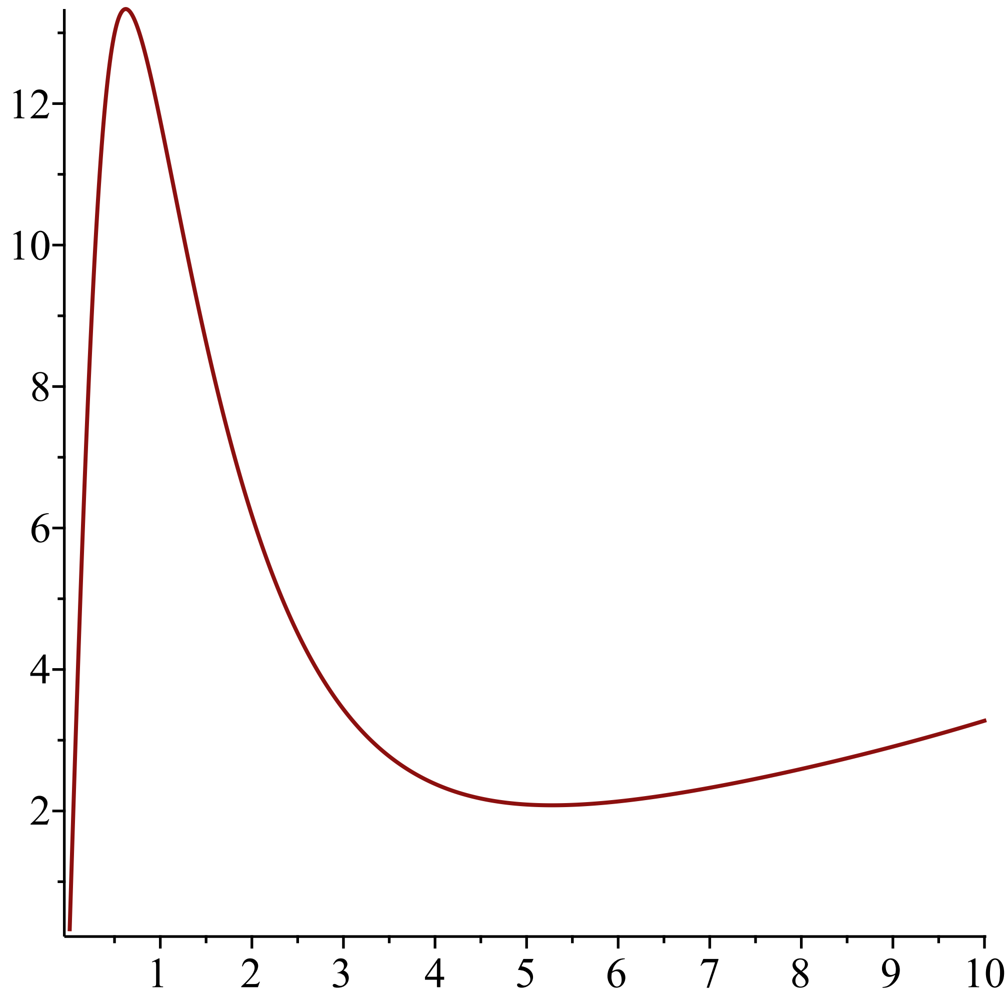
```
> TimeSeries(F,[m1,m2,m3,p1,p2,p3],[.2,.1,.3,.1,.4,.5],.01,10,1)
```



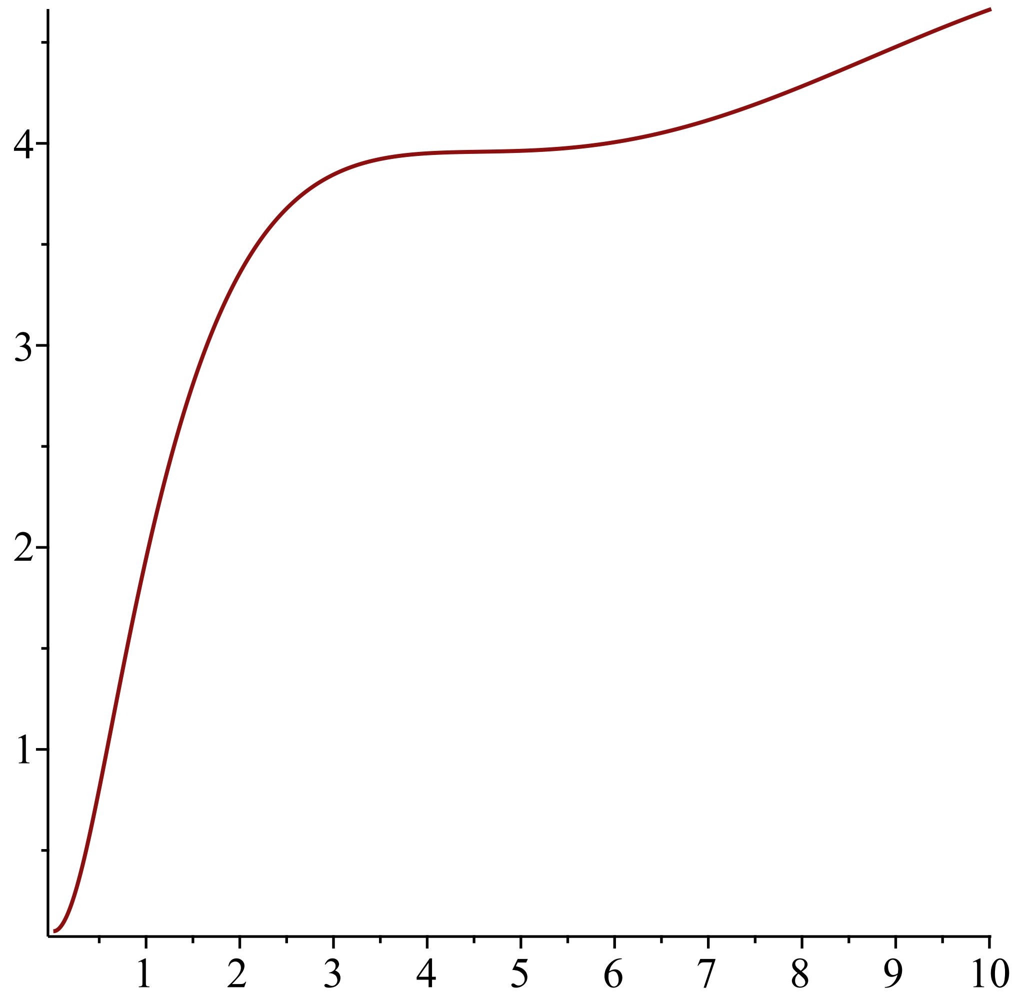

```
> TimeSeries(F,[m1,m2,m3,p1,p2,p3],[.2,.1,.3,.1,.4,.5],.01,10,2)
```



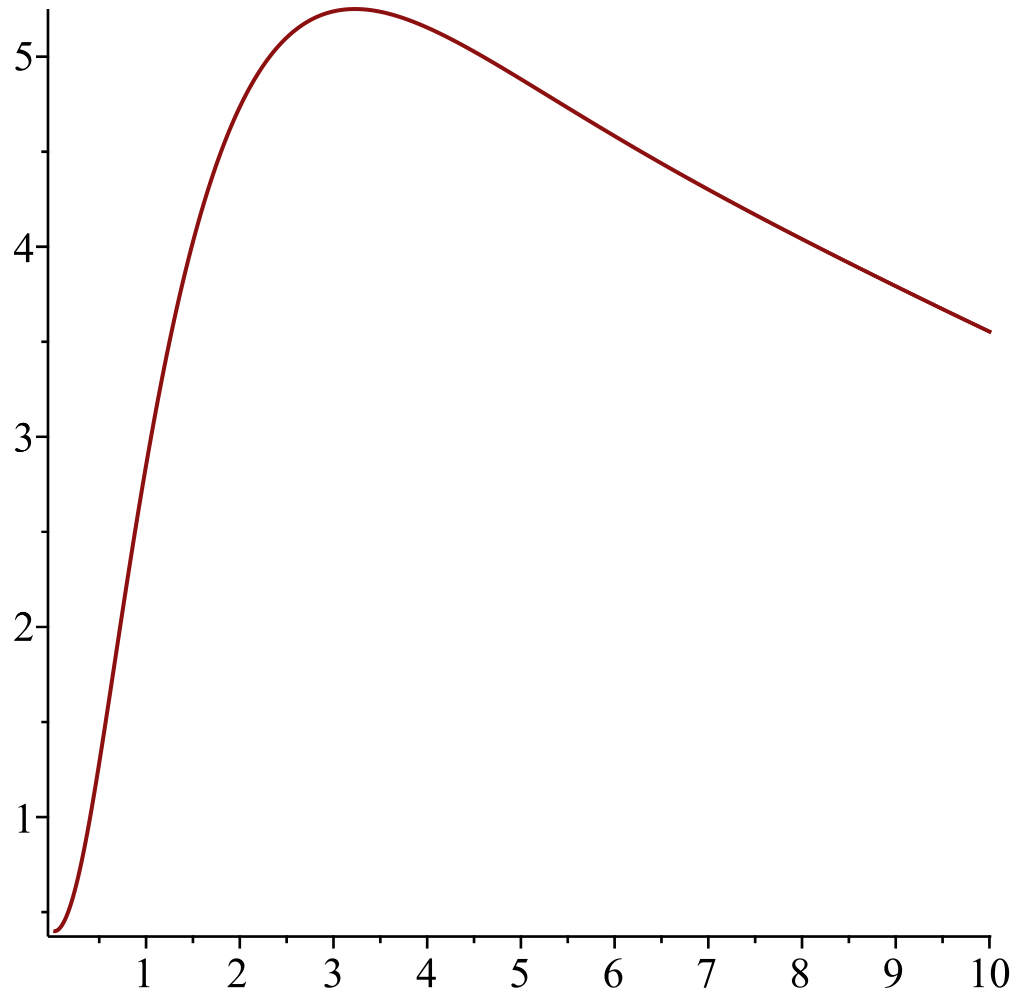

```
> TimeSeries(F,[m1,m2,m3,p1,p2,p3],[.2,.1,.3,.1,.4,.5],.01,10,3)
```



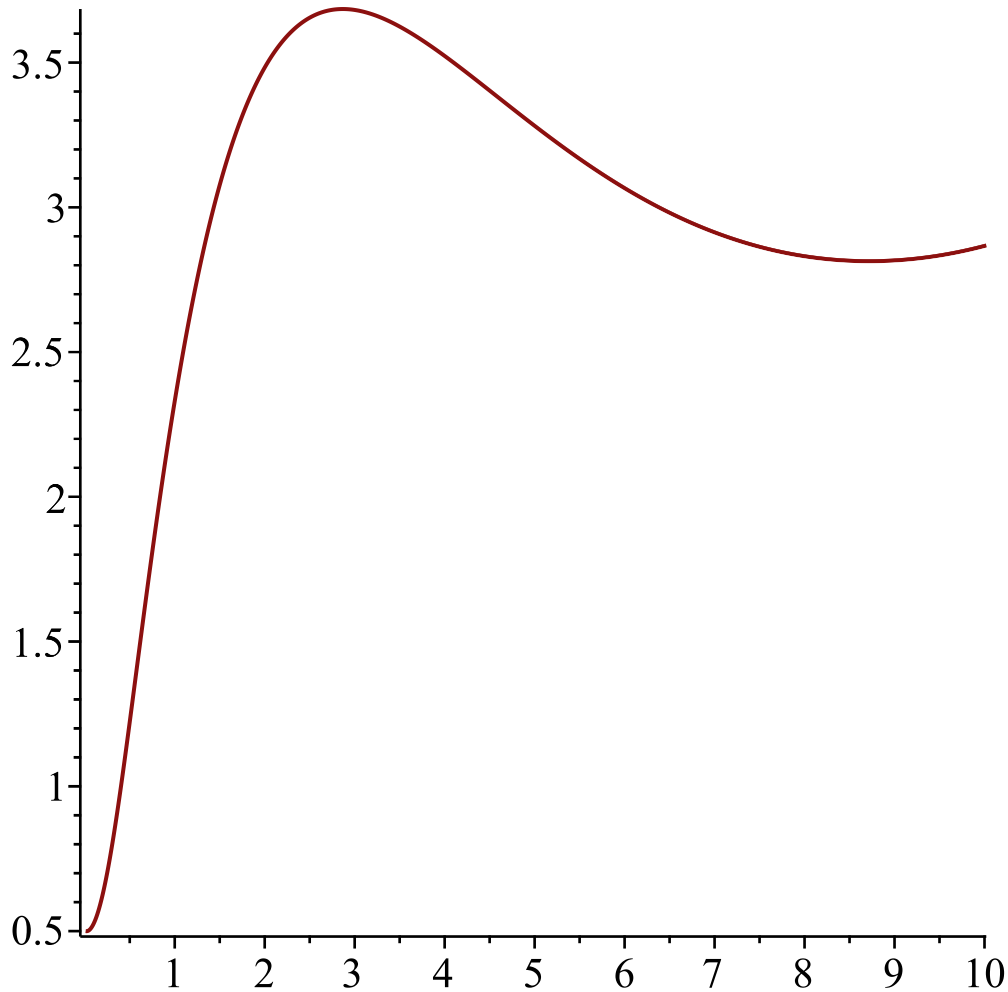

```
> TimeSeries(F,[m1,m2,m3,p1,p2,p3],[.2,.1,.3,.1,.4,.5],.01,10,4)
```




```
> TimeSeries(F,[m1,m2,m3,p1,p2,p3],[.2,.1,.3,.1,.4,.5],.01,10,5)
```




```
> TimeSeries(F,[m1,m2,m3,p1,p2,p3],[.2,.1,.3,.1,.4,.5],.01,10,6)
```




```
> Help(Lotka)
```

Lotka(r1,k1,r2,k2,b12,b21,N1,N2): The Lotka-Volterra continuous-time dynamical system, Eqs. (9a),(9b) (p. 224, section 6.3) of Edelstein-Keshet

with popoluations N1, N2, and parameters r1,r2,k1,k2, b12, b21 (called there beta_12 and beta_21)

Try:
Lotka(r1,k1,r2,k2,b12,b21,N1,N2);
Lotka(1,2,2,3,1,2,N1,N2); **(12)**

```
> Lotka(r1,k1,r2,k2,b12,b21,N1,N2)
```

$$\left[\frac{r1\,N1\,(-b12\,N2 - N1 + k1)}{k1}, \frac{r2\,N2\,(-b21\,N1 - N2 + k2)}{k2}\right] \quad \textbf{(13)}$$

```
> F:=Lotka(2,4,4,3,4,5,N1,N2)
```

$$F := \left[\frac{N1\,(4 - N1 - 4\,N2)}{2}, \frac{4\,N2\,(3 - N2 - 5\,N1)}{3}\right] \quad \textbf{(14)}$$

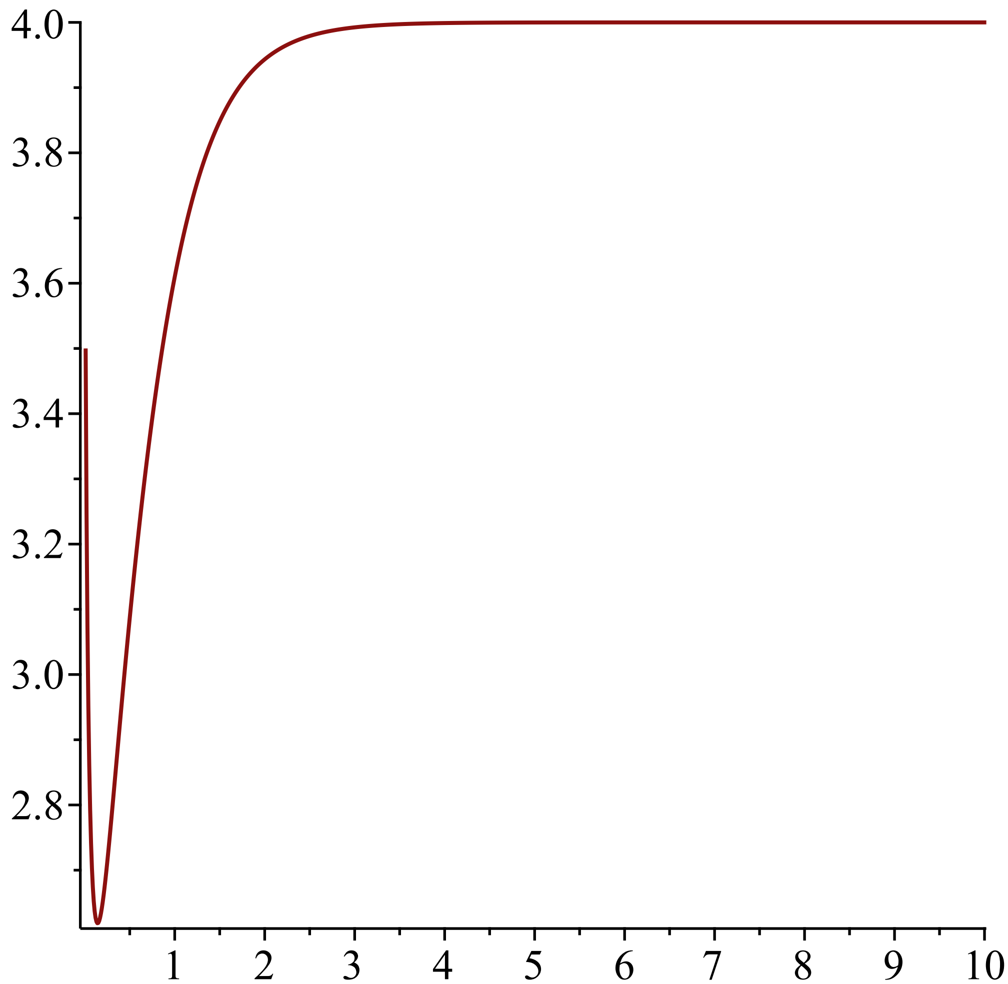
```
> EquP(F,[N1,N2])
```

$$\left\{[0, 0], [0, 3], [4, 0], \left[\frac{8}{19}, \frac{17}{19}\right]\right\} \quad \textbf{(15)}$$

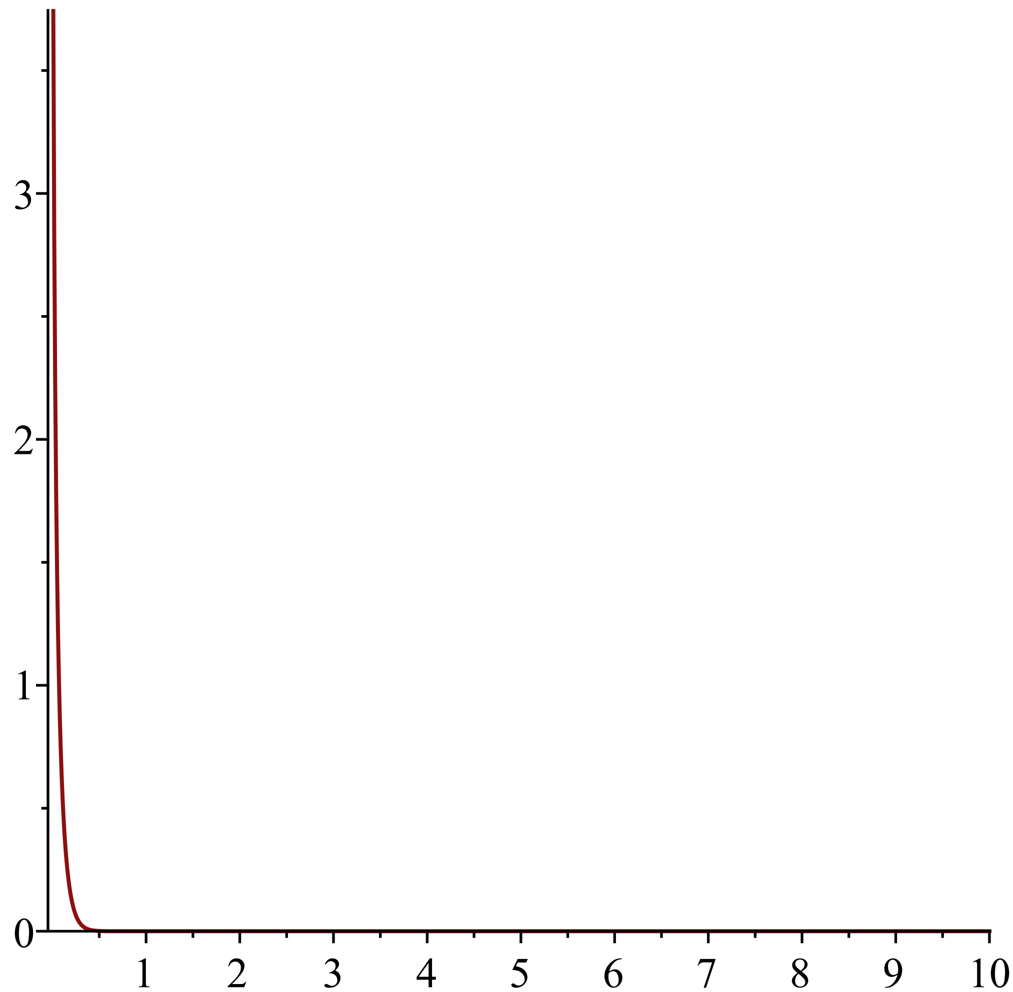
```
> SEquP(F,[N1,N2])
```

$$\{[0., 3.], [4., 0.]\} \quad (16)$$

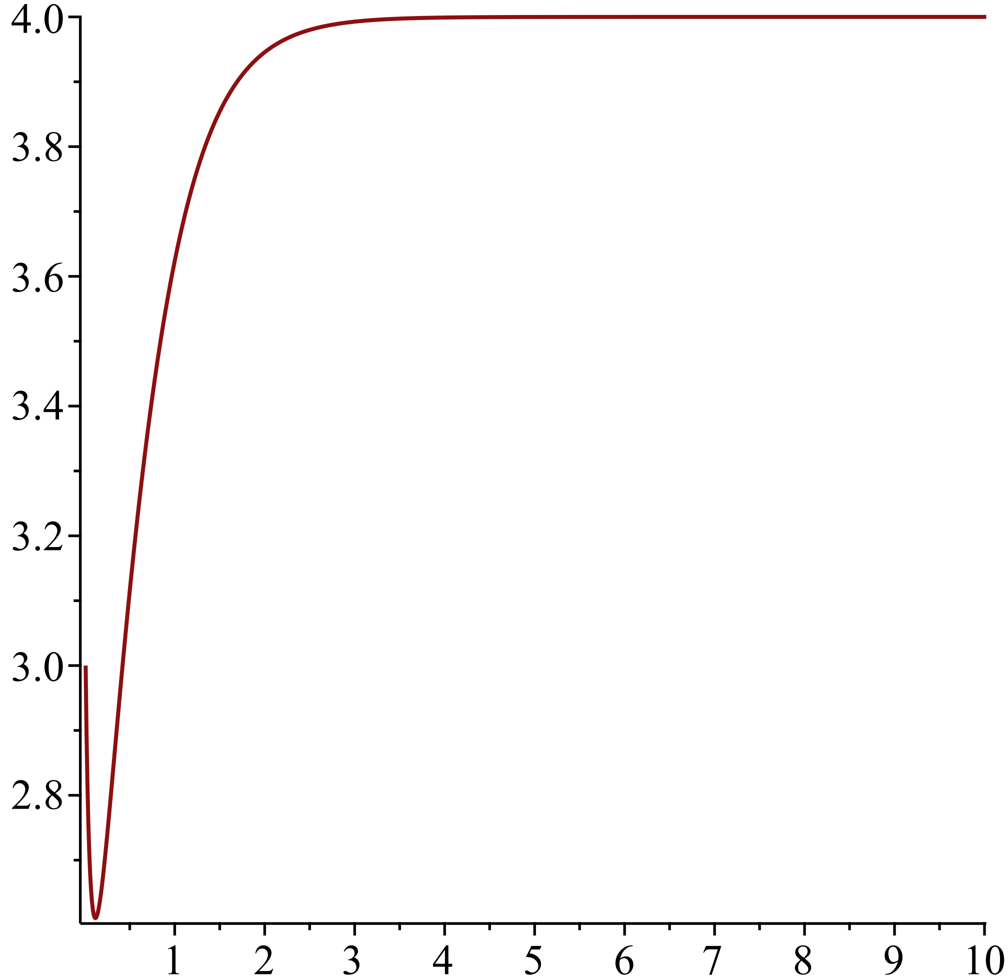
```
> TimeSeries(F,[N1,N2],[3.5,3.75],.01,10,1)
```



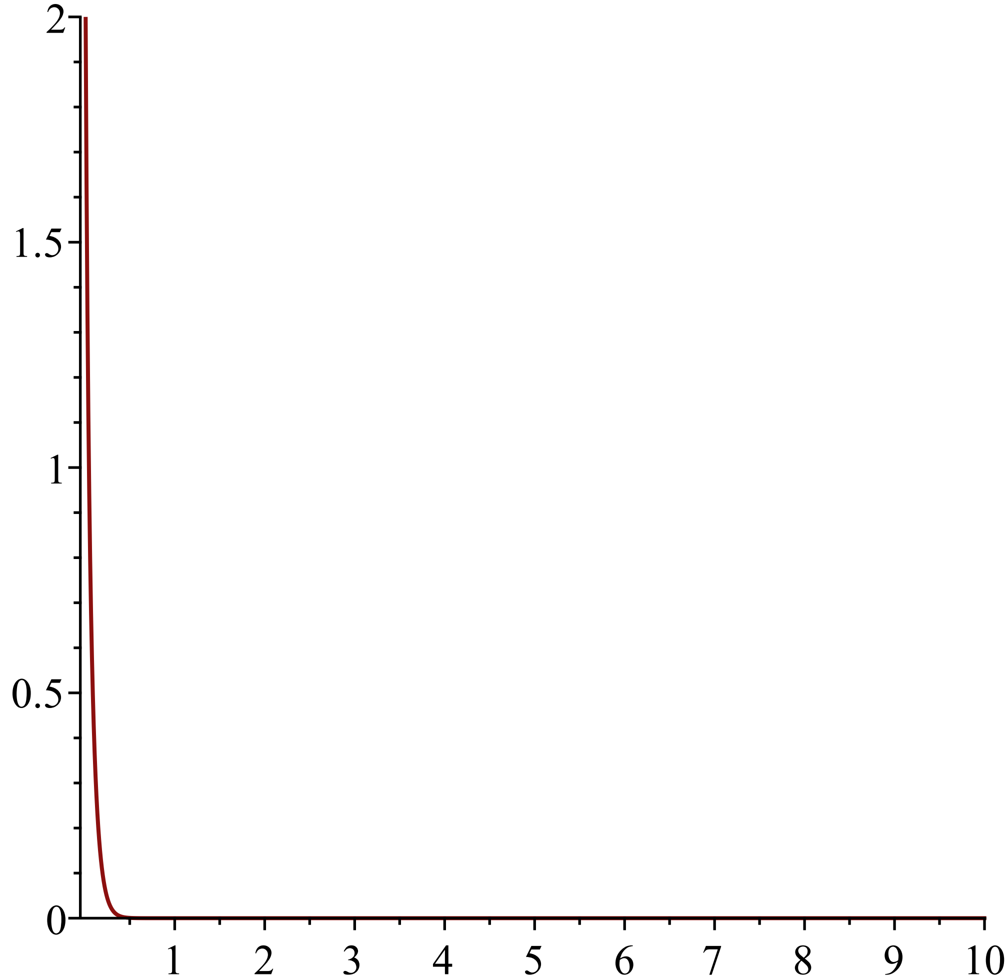

```
> TimeSeries(F,[N1,N2],[3.5,3.75],.01,10,2)
```



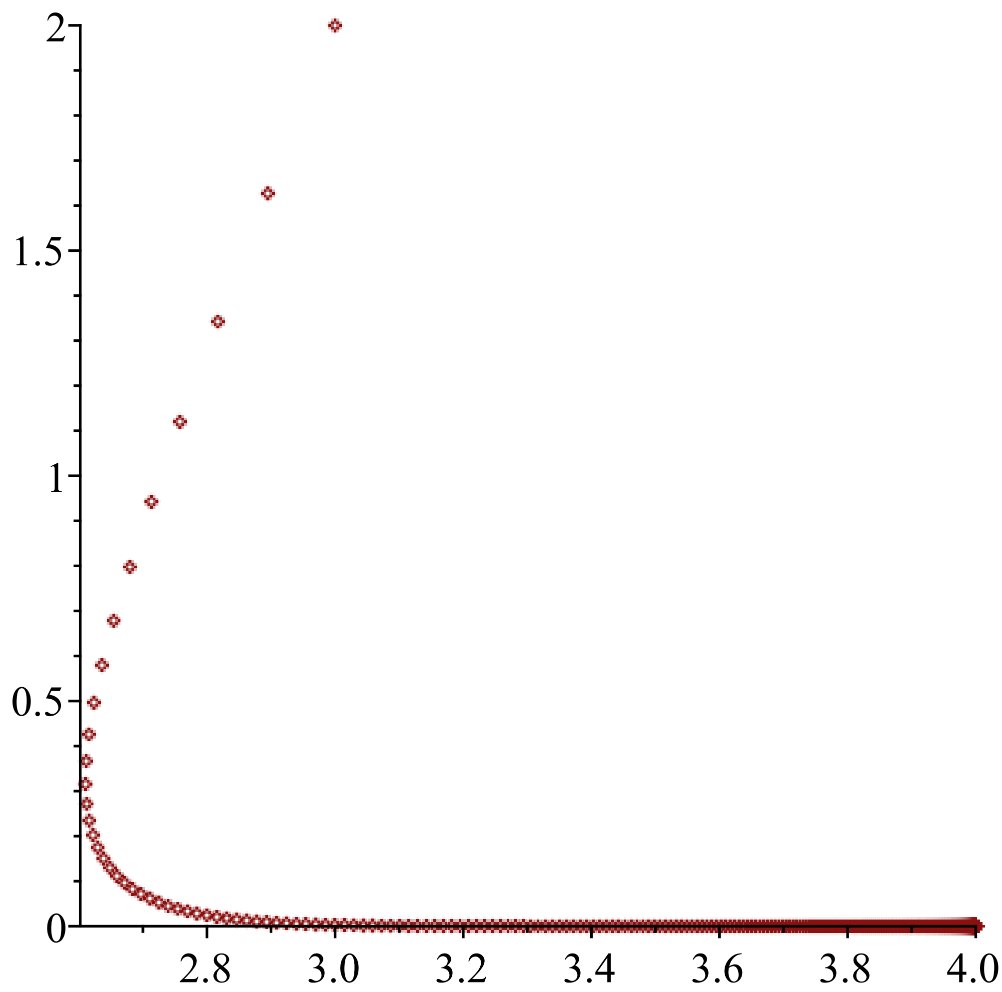

```
> TimeSeries(F,[N1,N2],[3,2],.01,10,1)
```




```
> TimeSeries(F,[N1,N2],[3,2],.01,10,2)
```




```
> PhaseDiag(F,[N1,N2],[3,2],.01,10)
```




```
> Help(Volterra)
```

Volterra(a,b,c,d,x,y): The (simple, original) Volterra predator-prey continuous-time dynamical system with parameters a,b,c,d

Given by Eqs. (7a) (7b) in Edelstein-Keshet p. 219 (section 6.2).

a,b,c,d may be symbolic or numeric

Try:

Volterra(a,b,c,d,x,y);

Volterra(1,2,3,4,x,y); **(17)**

```
> Volterra(a,b,c,d,x,y)
```

$$[-b\,x\,y + a\,x,\ d\,x\,y - c\,y] \tag{18}$$

```
> F:=Volterra(2,3,4,5,x,y)
```

$$F := [-3\,x\,y + 2\,x,\ 5\,x\,y - 4\,y] \tag{19}$$

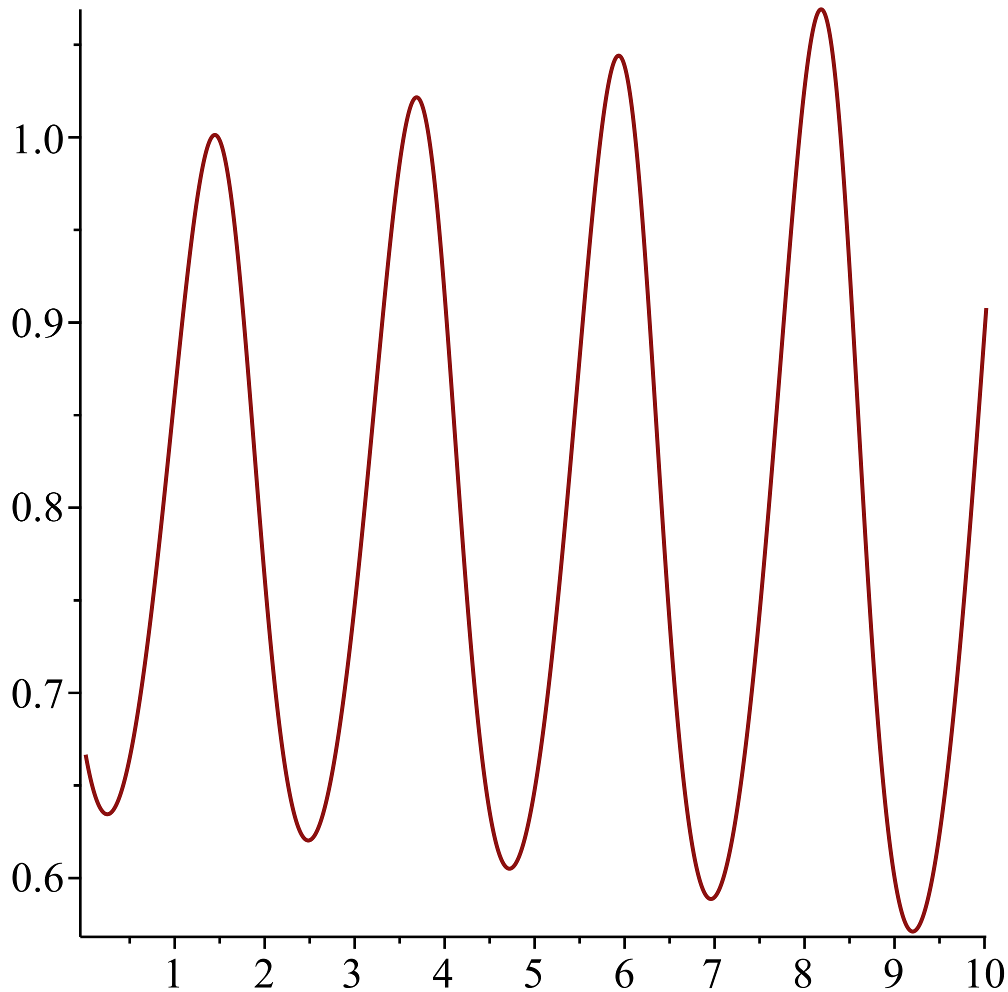
```
> EquP(F,[x,y])
```

$$\left\{[0, 0], \left[\frac{4}{5}, \frac{2}{3}\right]\right\} \tag{20}$$

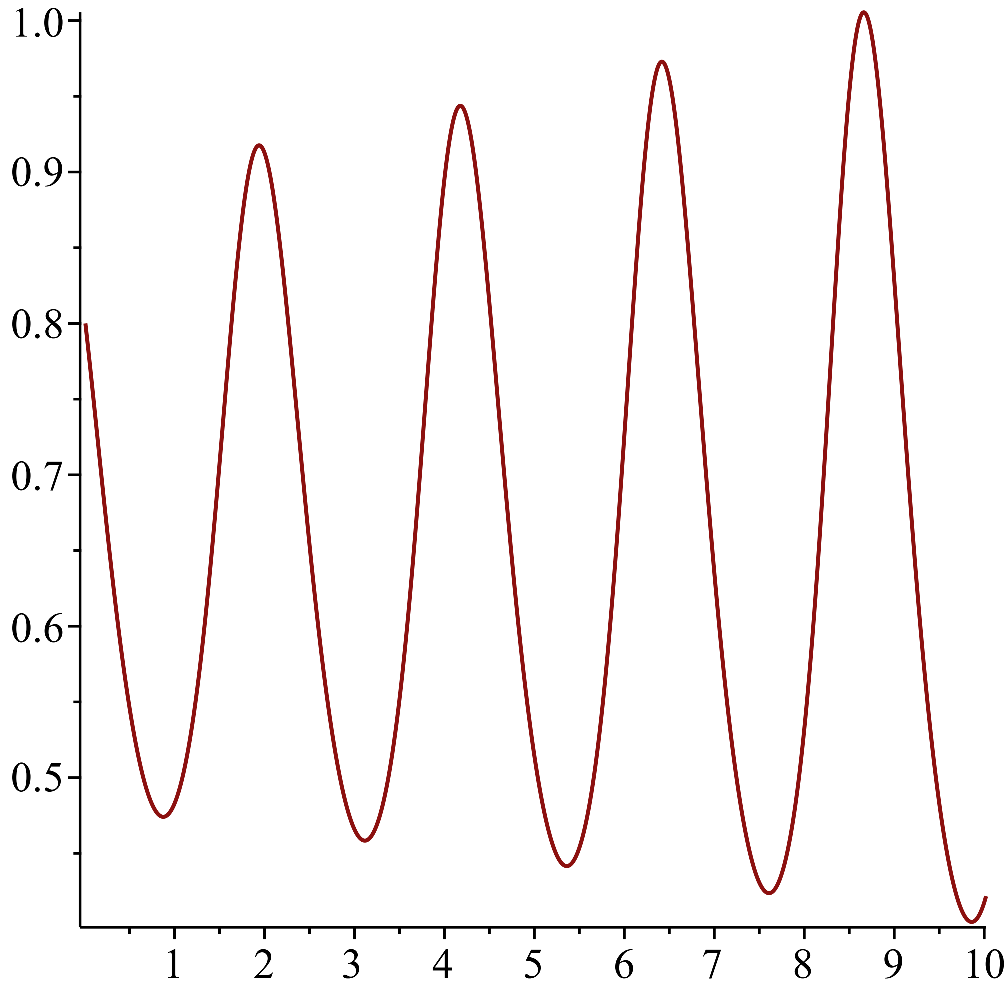
```
> SEquP(F,[x,y])
```

$$\varnothing \tag{21}$$

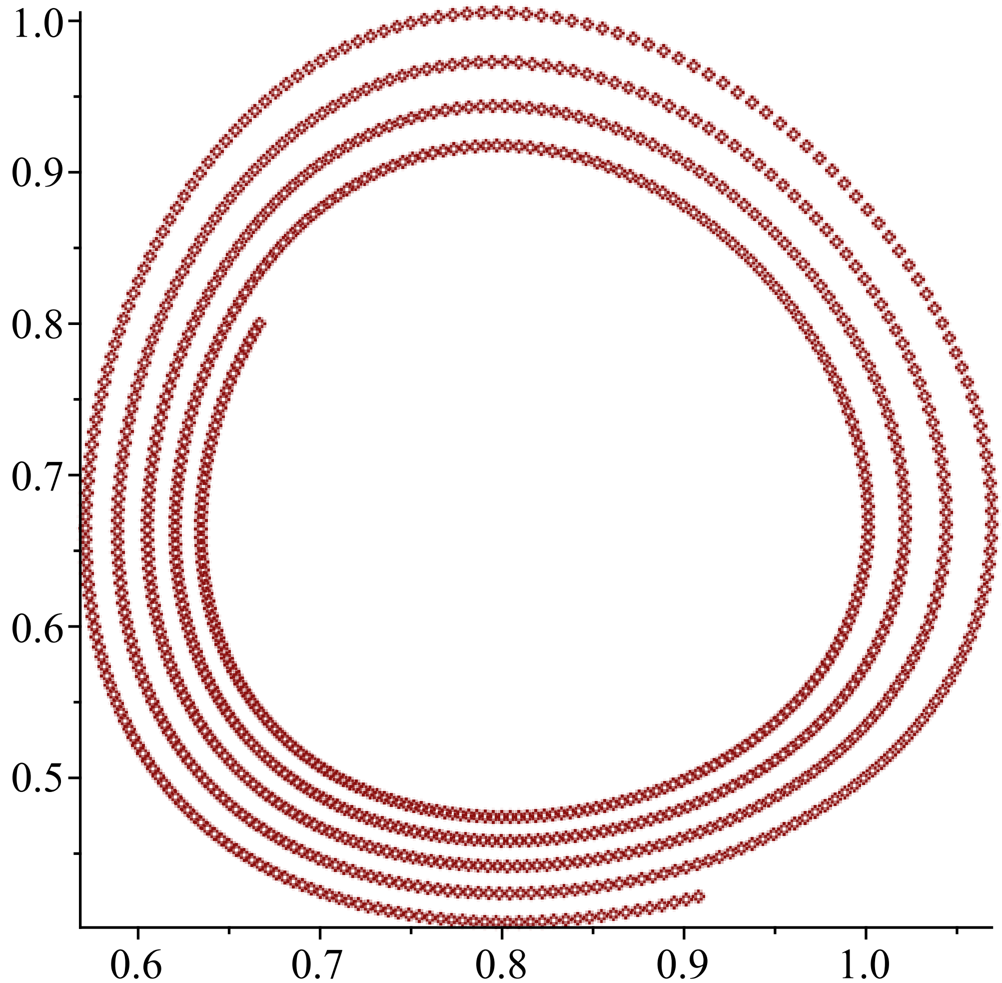
```
> TimeSeries(F,[x,y],[2/3,4/5],.01,10,1)
```




```
> TimeSeries(F,[x,y],[2/3,4/5],.01,10,2)
```




```
> PhaseDiag(F,[x,y],[2/3,4/5],.01,10)
```




```
> Help(VolterraM)
```

VolterraM(a,b,c,d,x,K,y): The MODIFIED Volterra predator-prey continuous-time dynamical system with parameters a,b,c,d,K

Given by Eqs. (8a) (8b) in Edelstein-Keshet p. 220 (section 6.2).

a,b,c,d ,Kmay be symbolic or numeric

Try:

VolterraM(a,b,c,d,K,x,y);

VolterraM(1,2,3,4,3,x,y); **(22)**

```
> VolterraM(a,b,c,d,K,x,y)
```

$$\left[a\,x\left(1-\frac{x}{d}\right)-b\,x\,y,\,K\,x\,y-c\,y\right] \quad (23)$$

```
> F:=VolterraM(3,5,6,2,3,x,y)
```

$$F:=\left[3\,x\left(1-\frac{x}{2}\right)-5\,x\,y,\,3\,x\,y-6\,y\right] \quad (24)$$

```
> EquP(F,[x,y])
```

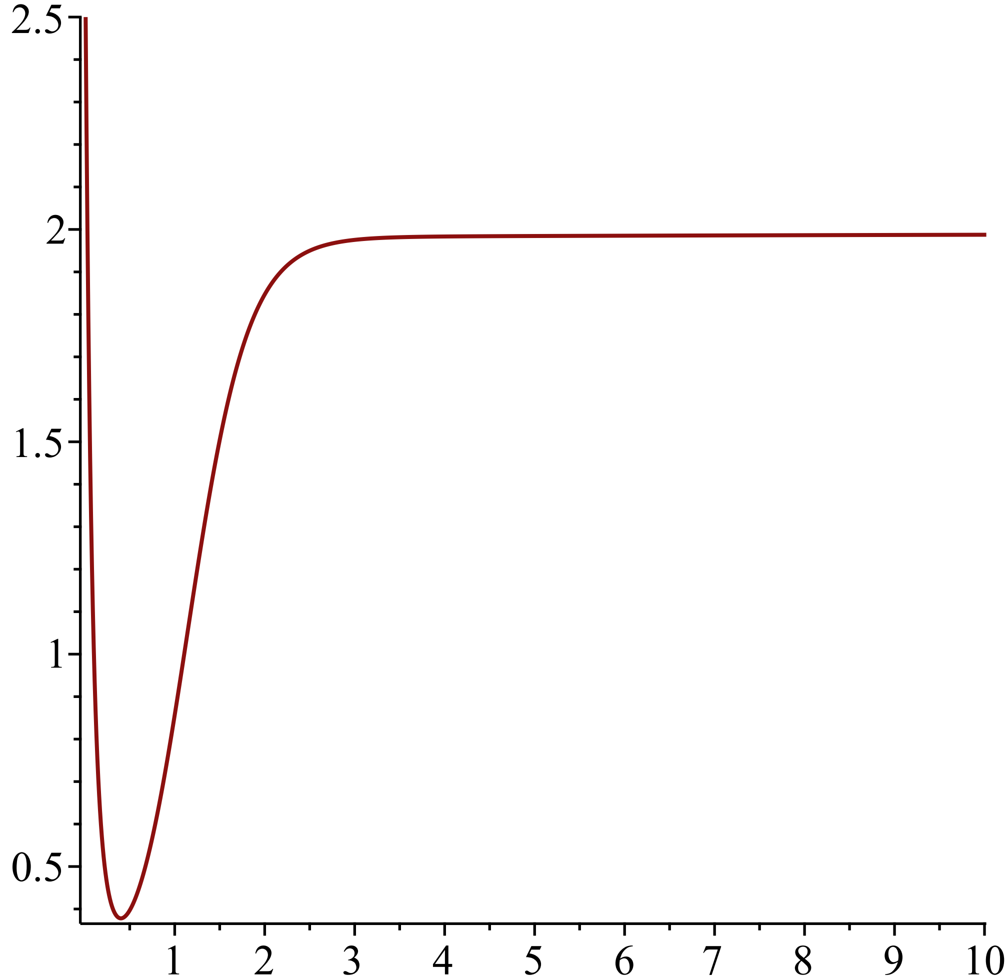
$$\{[0,0],[2,0]\} \quad (25)$$

```
> SEquP(F,[x,y])
```

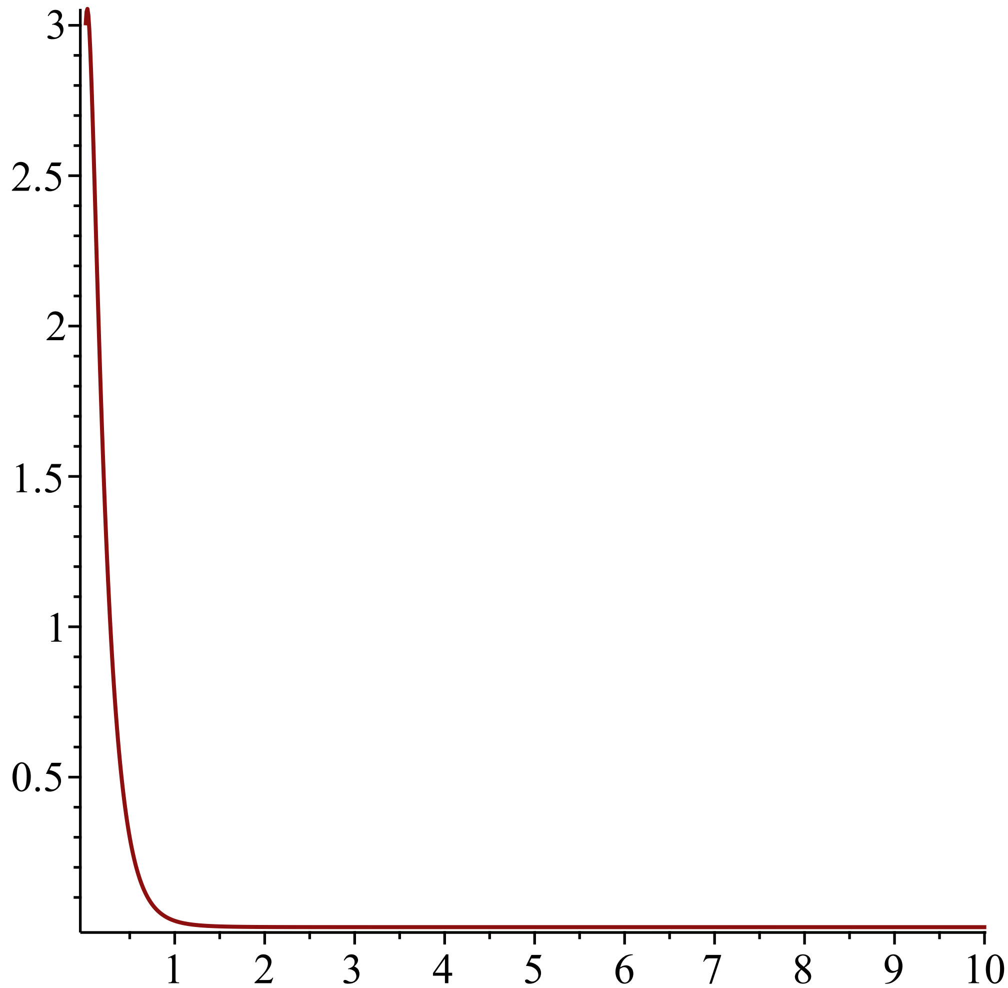
(26)

$$\varnothing \tag{26}$$

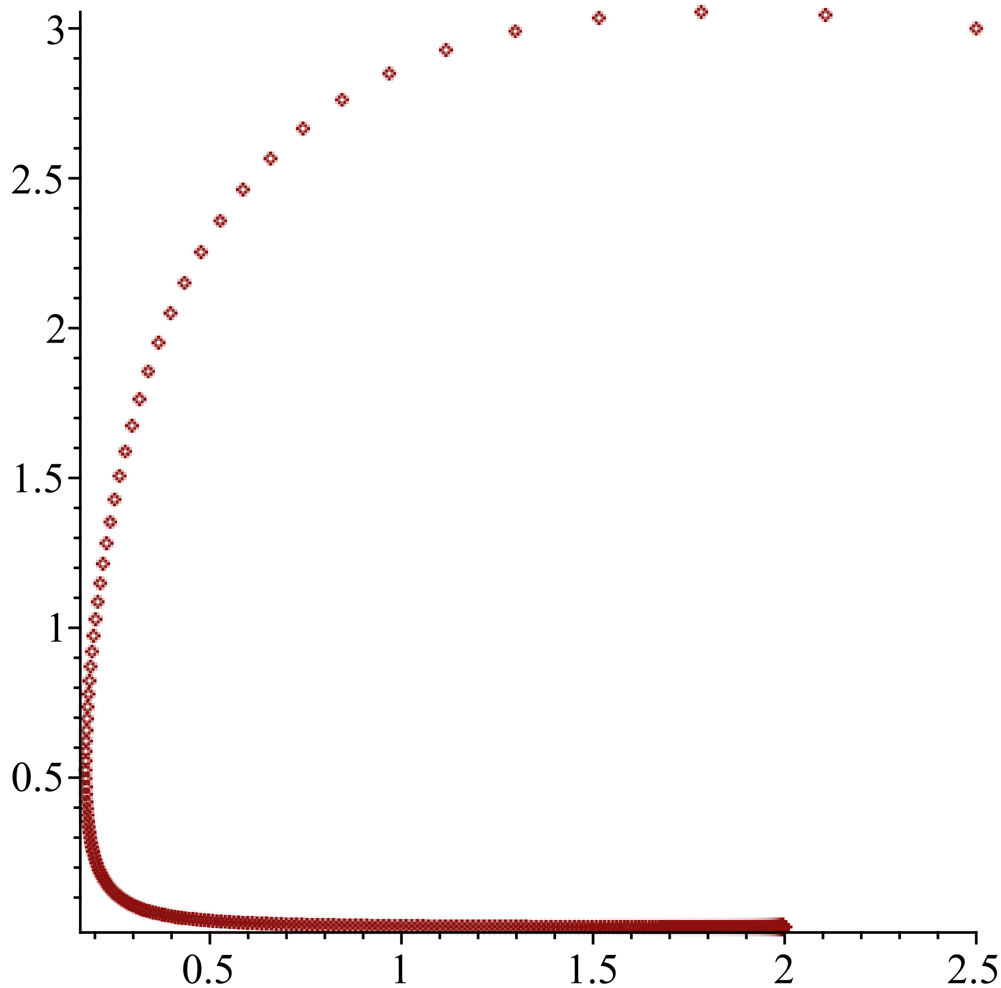
```
> TimeSeries(F,[x,y],[2.5,2],.01,10,1)
```




```
> TimeSeries(F,[x,y],[2.5,3],.01,10,2)
```




```
> PhaseDiag(F,[x,y],[2.5,3],.01,10)
```



3
2.5
2
1.5
1
0.5
0.5
1
1.5
2
2.5