> #HW 21 - Alan Ho

> #OK to post

> **read**("DMB.txt")

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

> *Help*(*ChemoStat*)

ChemoStat(N,C,a1,a2): The Chemostat continuous-time dynamical system with N=Bacterial poplulation densitty, and C=nutient Concentration in growth chamber (see Table 4.1 of Edelstein-Keshet, p. 122)
with paramerts a1, a2, Equations (19a_, (19b) in Edelestein-Keshet p. 127 (section 4.5, where they are called alpha1, alpha2). a1 and a2 can be symbolic or numeric. Try:

ChemoStat(N,C,a1,a2);

ChemoStat(N,C,2,3); **(2)**

> $F := ChemoStat(N, C, 2, 3);$

$$F := \left[\frac{2\,C\,N}{C+1} - N,\ -\frac{C\,N}{C+1} - C + 3\right] \quad \textbf{(3)}$$

> $Help(TimeSeries)$

TimeSeries(F,x,pt,h,A,i): Inputs a transformation F in the list of variables x

The time-series of x[i] vs. time of the Dynamical system approximating the the autonomous continuous dynamical process

dx/dt=F(x(t)) by a discrete time dynamical system with step-size h from t=0 to t=A

Try:

TimeSeries([x(1-y),y*(1-x)],[x,y],[0.5,0.5], 0.01, 10,1);* **(4)**

> $Help(PhaseDiag)$

PhaseDiag(F,x,pt,h,A): Inputs a transformation F in the list of variables x (of length 2), i.e. a mapping from R^2 to R^2 gives the

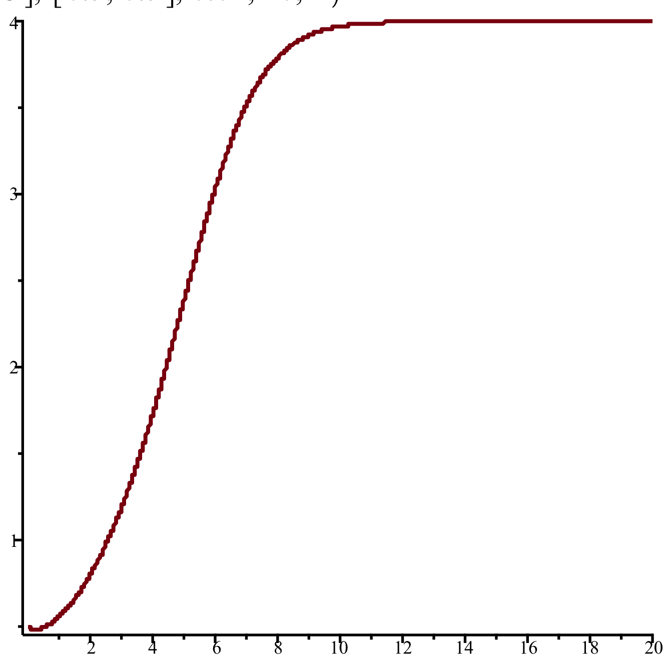
The phase diagram of the solution with initial condition x(0)=pt

dx/dt=F[1](x(t)) by a discrete time dynamical system with step-size h from t=0 to t=A

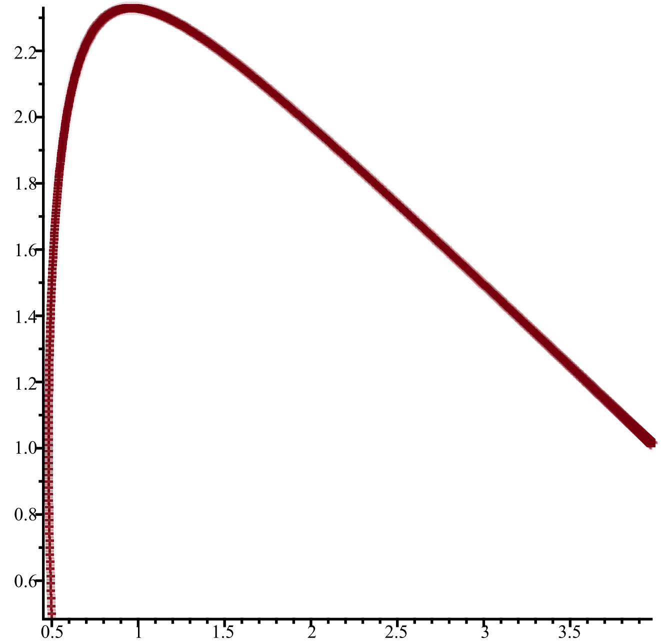
Try:

PhaseDiag([x(1-y),y*(1-x)],[x,y],[0.5,0.5], 0.01, 10);* **(5)**

> $TimeSeries(F, [N, C], [0.5, 0.5], 0.01, 20, 1)$



> $PhaseDiag(F, [N, C], [0.5, 0.5], 0.01, 10)$



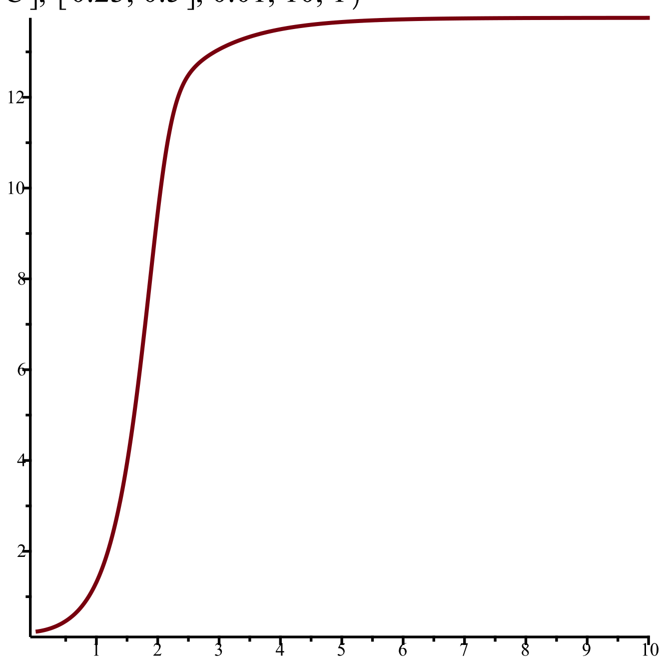
> $SEquP(F, [N, C])$

$$\{[4., 1.]\}\tag{6}$$

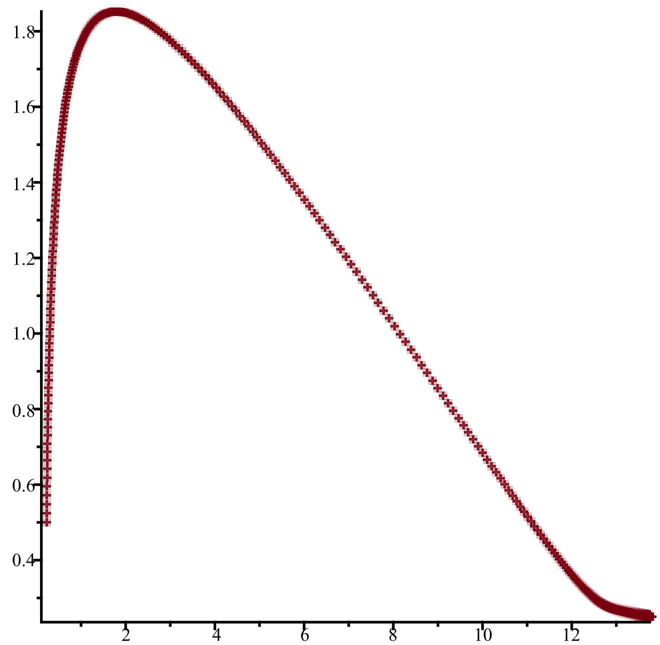
> $F := ChemoStat(N, C, 5, 3);$

$$F := \left[\frac{5\,C\,N}{C+1} - N, -\frac{C\,N}{C+1} - C + 3\right]\tag{7}$$

> $TimeSeries(F, [N, C], [0.23, 0.5], 0.01, 10, 1)$



> $PhaseDiag(F, [N, C], [0.23, 0.5], 0.01, 10)$



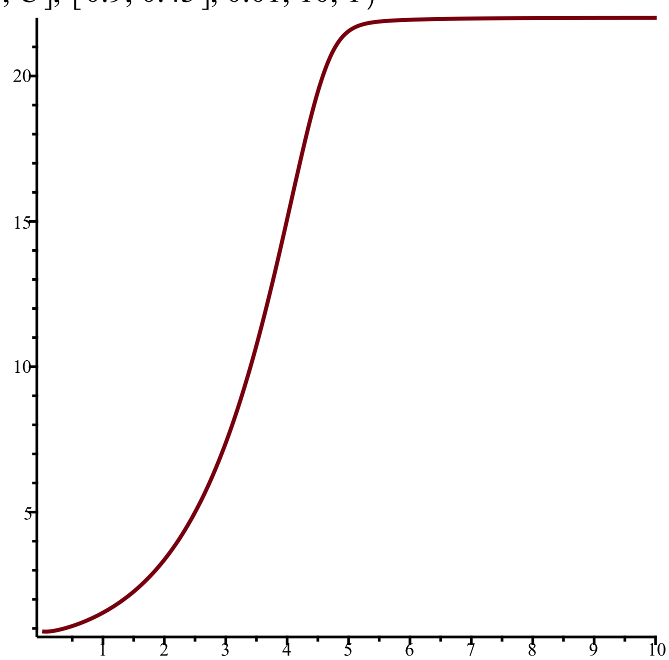
> $SEquP(F, [N, C])$

$$\{[13.75000000, 0.2500000000]\} \tag{8}$$

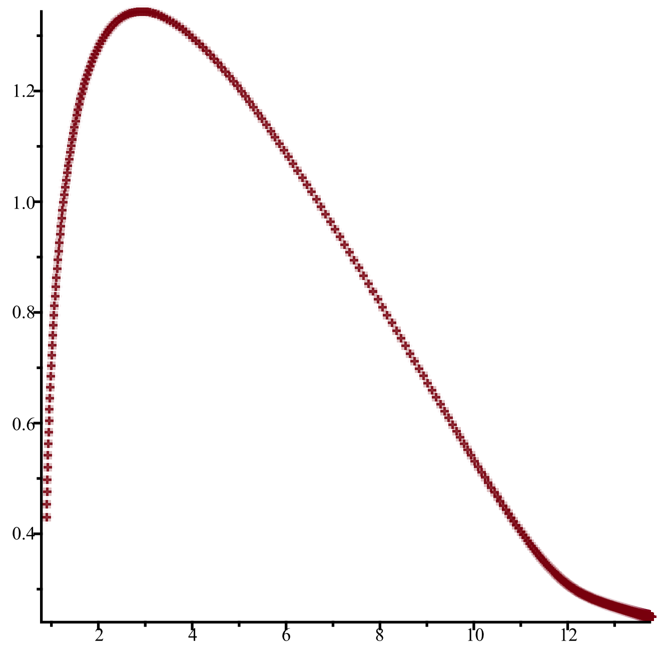
> $F := ChemoStat(N, C, 2, 12);$

$$F := \left[\frac{2\,C\,N}{C+1} - N, -\frac{C\,N}{C+1} - C + 12\right] \tag{9}$$

> $TimeSeries(F, [N, C], [0.9, 0.43], 0.01, 10, 1)$



> $PhaseDiag(F, [N, C], [0.9, 0.43], 0.01, 10)$



> $SEquP(F, [N, C])$

$$\{[22., 1.]\}\tag{10}$$

> # in all 3 cases with random parameters and random initial conditions, the horizontal asymptote obtained from TimeSeries matched with the stable equilibria predicted by SEquP

> $Help(GeneNet)$

GeneNet(a0,a,b,n,m1,m2,m3,p1,p2,p3): The contiuous-time dynamical system, with quantities m1,m2,m3,p1,p2,p3, due to M. Elowitz and S. Leibler

described in the Ellner-Guckenheimer book, Eq. (4.1) (chapter 4, p. 112)

and parameers a0 (called alpha_0 there),a (called alpha there), b (called beta there) and n.

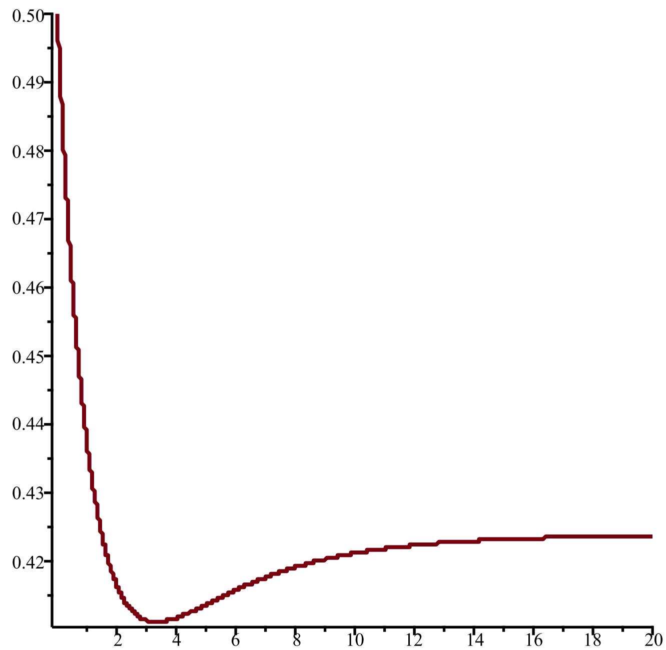
Try:

GeneNet(0,0.5,0.2,2,m1,m2,m3,p1,p2,p3); **(11)**

> $F := GeneNet(0, 0.5, 0.2, 2, m1, m2, m3, p1, p2, p3);$

$$F := \left[-m1 + \frac{0.5}{p3^2 + 1}, -m2 + \frac{0.5}{p1^2 + 1}, -m3 + \frac{0.5}{p2^2 + 1}, -0.2\, p1 + 0.2\, m1, -0.2\, p2 + 0.2\, m2, -0.2\, p3 + 0.2\, m3\right]\tag{12}$$

> $TimeSeries(F, [m1, m2, m3, p1, p2, p3], [0.5, 0.5, 0.5 , 0.5, 0.5, 0.5], 0.01, 20, 1)$



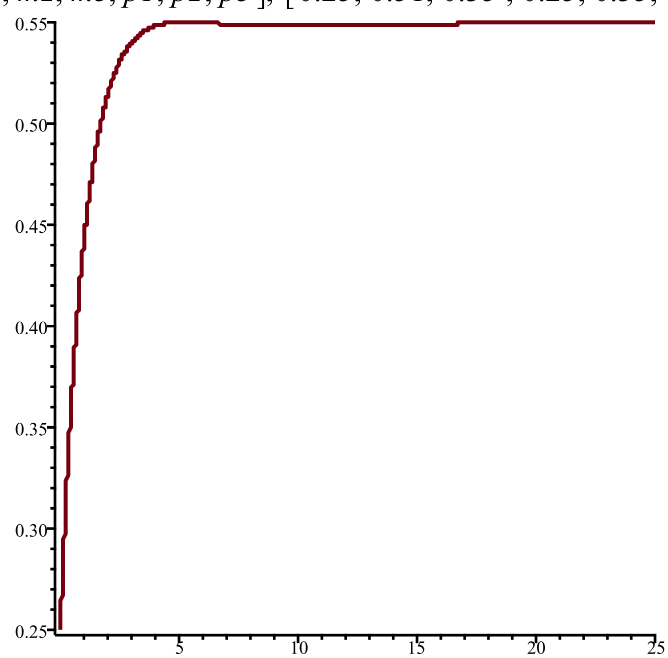
> $SEquP(F, [m1, m2, m3, p1, p2, p3])$

$$\{[0.4238537991, 0.4238537991, 0.4238537991, 0.4238537991, 0.4238537991, 0.4238537991]\} \quad (13)$$

> $F := GeneNet(0, 0.6, 0.2, 4, m1, m2, m3, p1, p2, p3);$

$$F := \left[-m1 + \frac{0.6}{p3^4 + 1}, -m2 + \frac{0.6}{p1^4 + 1}, -m3 + \frac{0.6}{p2^4 + 1}, -0.2\,p1 + 0.2\,m1, -0.2\,p2 + 0.2\,m2, -0.2\,p3 + 0.2\,m3\right] \quad (14)$$

> $TimeSeries(F, [m1, m2, m3, p1, p2, p3], [0.25, 0.51, 0.35, 0.25, 0.35, 0.55], 0.01, 25, 1)$



> $SEquP(F, [m1, m2, m3, p1, p2, p3])$

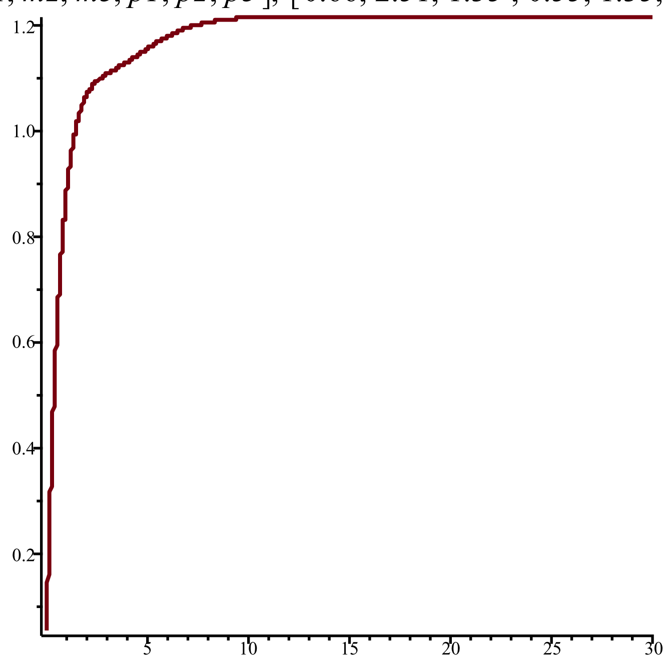
(15)

{[0.5497746038, 0.5497746038, 0.5497746038, 0.5497746038, 0.5497746038, 0.5497746038]} **(15)**

> $F := GeneNet(1, 0.6, 1.2, 3, m1, m2, m3, p1, p2, p3);$

$$F := \left[-m1 + \frac{0.6}{p3^3 + 1} + 1, -m2 + \frac{0.6}{p1^3 + 1} + 1, -m3 + \frac{0.6}{p2^3 + 1} + 1, -1.2\, p1 + 1.2\, m1, -1.2\, p2 + 1.2\, m2, -1.2\, p3 + 1.2\, m3 \right] \quad (16)$$

> TimeSeries(F, [m1, m2, m3, p1, p2, p3], [0.66, 2.51, 1.35 , 0.55, 1.35, 0.055], 0.01, 30, 6)



> SEquP(F, [m1, m2, m3, p1, p2, p3])

{[1.214832606, 1.214832606, 1.214832606, 1.214832606, 1.214832606, 1.214832606]} **(17)**

> *# in all 3 cases with random parameters and random initial conditions, the horizontal asymptote obtained from TimeSeries matched with the stable equilibria predicted by SEquP*

> *#NOTE: unable to use PhaseDiag for this system because the system is not in R2*

> Help(Lotka)

Lotka(r1,k1,r2,k2,b12,b21,N1,N2): The Lotka-Volterra continuous-time dynamical system, Eqs. (9a),(9b) (p. 224, section 6.3) of Edelstein-Keshet

with popoluations N1, N2, and parameters r1,r2,k1,k2, b12, b21 (called there beta_12 and beta_21)

Try:

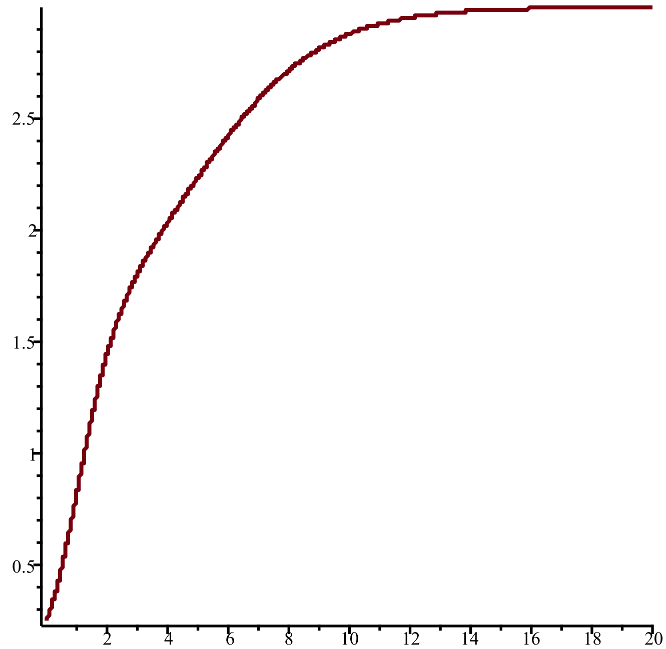
Lotka(r1,k1,r2,k2,b12,b21,N1,N2);

Lotka(1,2,2,3,1,2,N1,N2); **(18)**

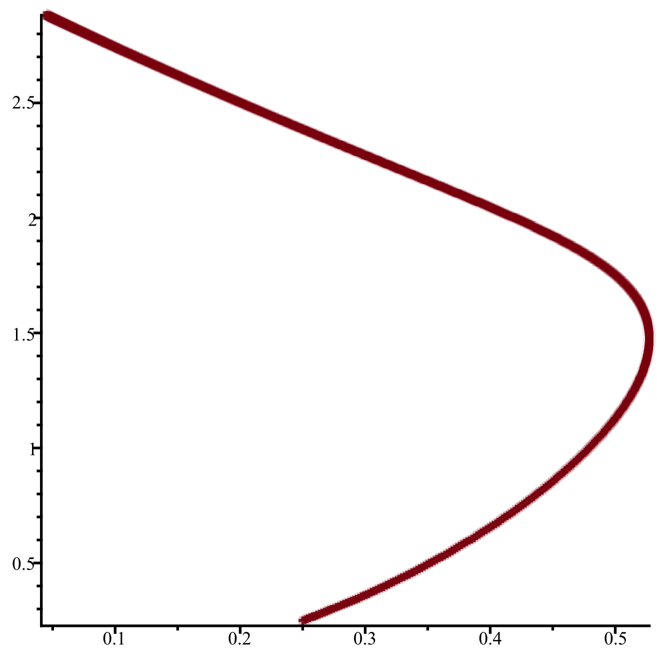
> $F := Lotka(1, 2, 2, 3, 1, 2, N1, N2);$

$$F := \left[\frac{N1\ (2 - N1 - N2)}{2}, \frac{2\, N2\ (3 - N2 - 2\, N1)}{3} \right] \quad (19)$$

> *TimeSeries*(*F*, [*N1*, *N2*], [.25, .25], 0.01, 20, 2)



> *PhaseDiag*(*F*, [*N1*, *N2*], [0.25, 0.25], 0.01, 10)



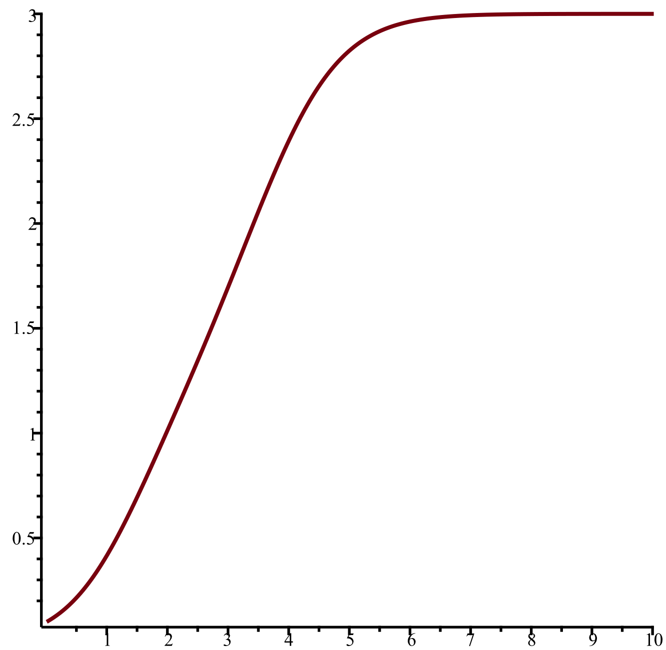
> *SEquP*(*F*, [*N1*, *N2*])

$$\{[0., 3.], [2., 0.]\} \qquad \textbf{(20)}$$

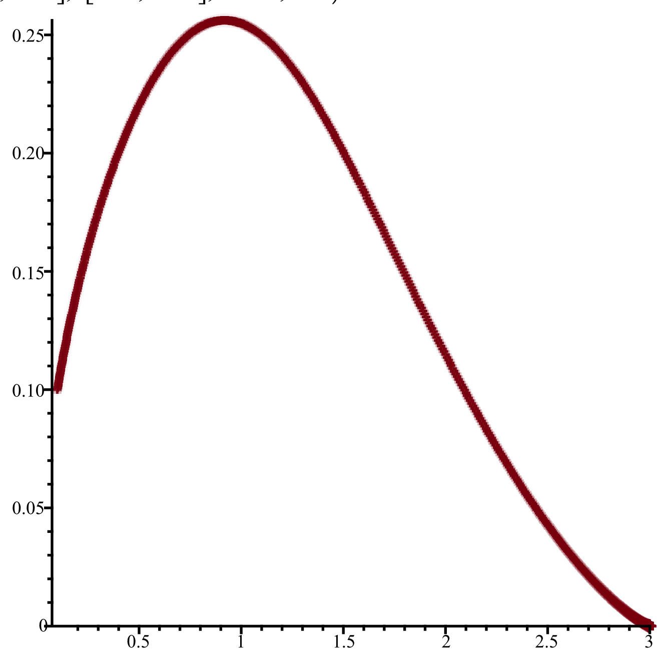
> *F* := *Lotka*(2, 3, 1, 3, 4, 3, *N1*, *N2*);

$$F := \left[\frac{2\,N1\,(3 - N1 - 4\,N2)}{3}, \frac{N2\,(3 - N2 - 3\,N1)}{3} \right] \qquad \textbf{(21)}$$

> *TimeSeries*(*F*, [*N1*, *N2*], [0.1, 0.1], 0.01, 10, 1)



> *PhaseDiag*(*F*, [*N1*, *N2*], [0.1, 0.1], 0.01, 10)



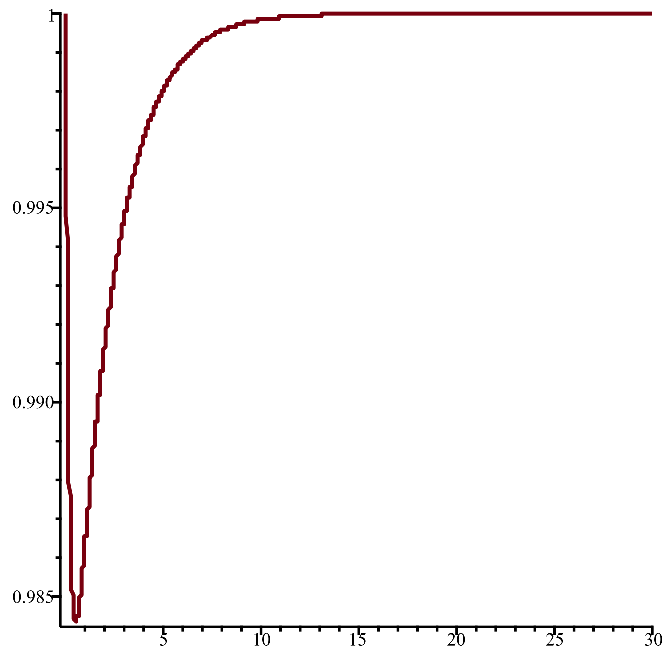
> *SEquP*(*F*, [*N1*, *N2*])

$$\{[0., 3.], [3., 0.]\} \tag{22}$$

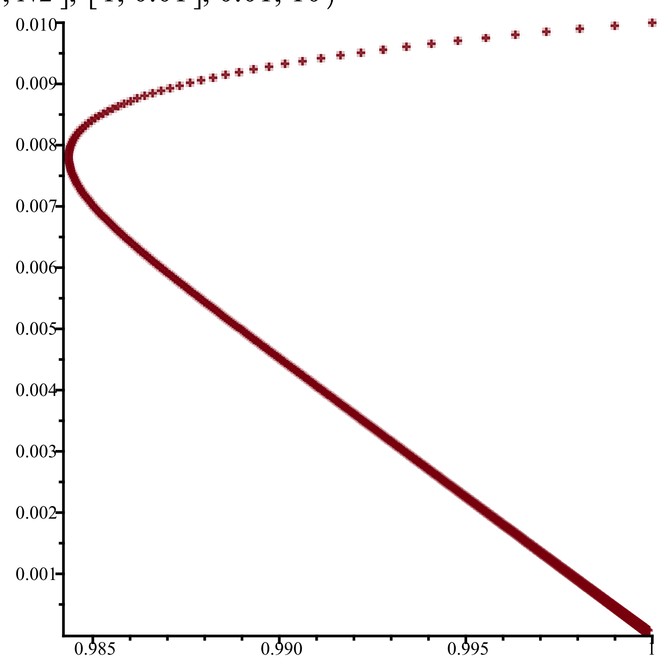
> *F* ≔ *Lotka*(5, 1, 2, 2, 2, 2.5, *N1*, *N2*);

$$F := [5\,N1\,(1 - N1 - 2\,N2), N2\,(2 - N2 - 2.5\,N1)] \tag{23}$$

> *TimeSeries*(*F*, [*N1*, *N2*], [1, 0.01], 0.01, 30, 1)



> $PhaseDiag(F, [N1, N2], [1, 0.01], 0.01, 10)$



> $SEquP(F, [N1, N2])$

$$\{[0., 2.], [1., 0.]\} \quad (24)$$

>

 # in all 3 cases with random parameters and random initial conditions, the horizontal asymptote obtained from TimeSeries matched with the stable equilibria predicted by SEquP

> $Help(Volterra)$

Volterra(a,b,c,d,x,y): The (simple, original) Volterra predator-prey continuous-time dynamical system with parameters a,b,c,d

Given by Eqs. (7a) (7b) in Edelstein-Keshet p. 219 (section 6.2).

a,b,c,d may be symbolic or numeric

Try:

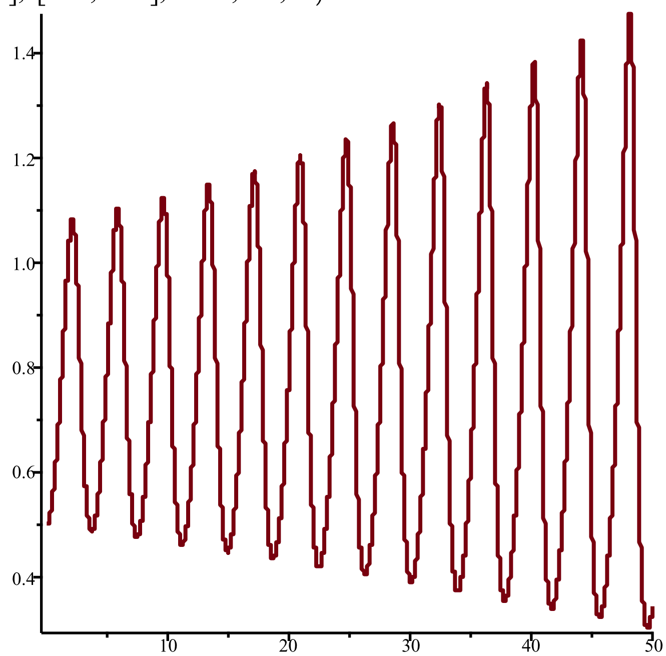
Volterra(a,b,c,d,x,y);

Volterra(1,2,3,4,x,y); **(25)**

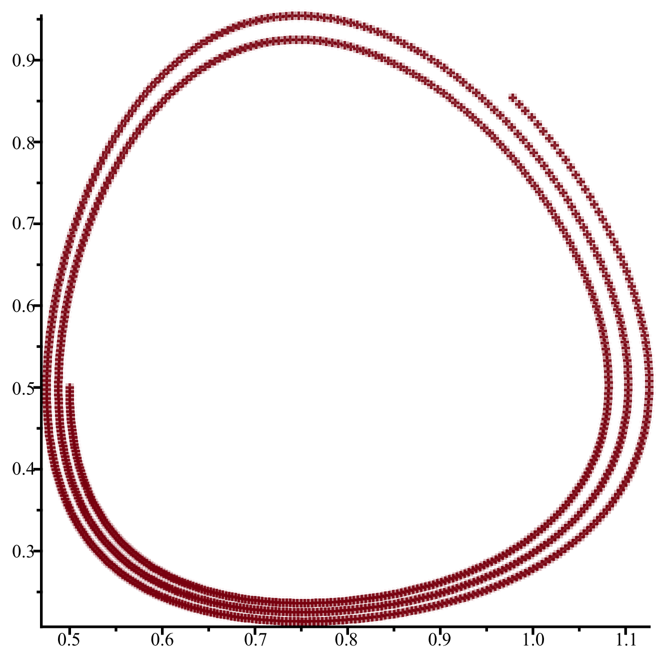
> $F := Volterra(1, 2, 3, 4, x, y);$

$$F := [-2\,x\,y + x,\ 4\,x\,y - 3\,y] \quad \textbf{(26)}$$

> $TimeSeries(F, [x, y], [0.5, 0.5], 0.01, 50, 1)$



> $PhaseDiag(F, [x, y], [0.5, 0.5], 0.01, 10)$



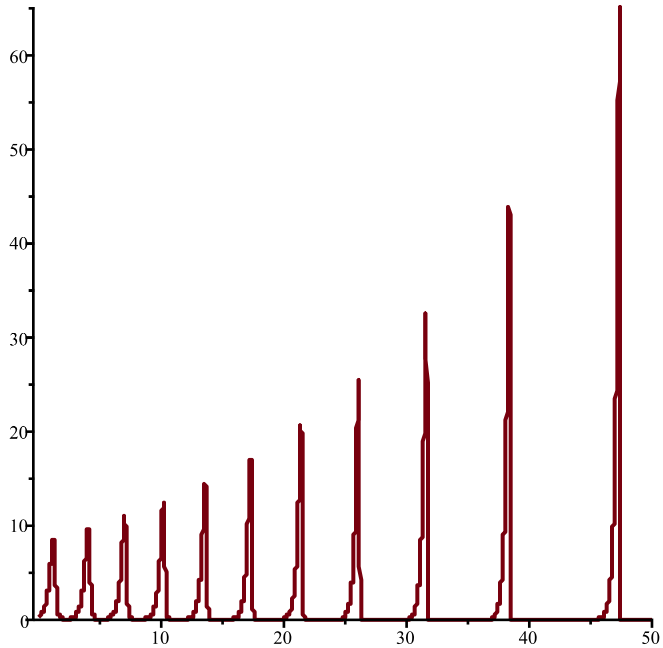
> $SEquP(F, [x, y])$

$$\varnothing \quad \textbf{(27)}$$

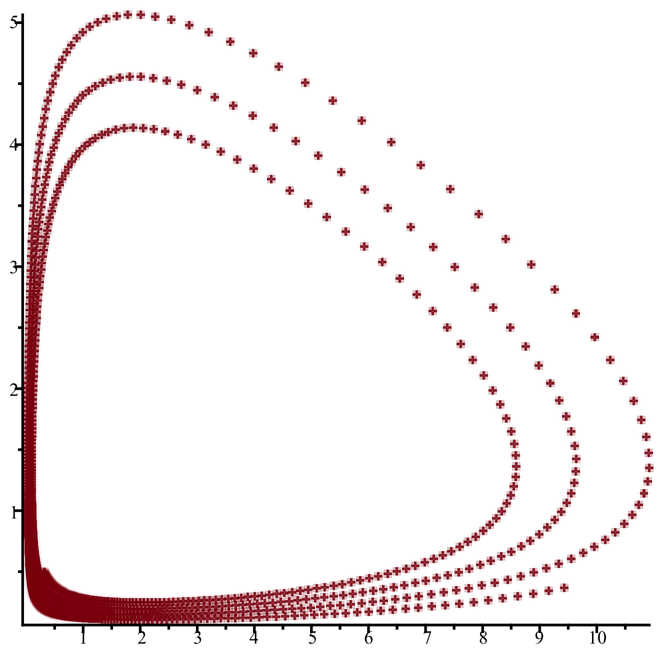
> $F := Volterra(4, 3, 2, 1, x, y);$

$$F := [-3\,x\,y + 4\,x, x\,y - 2\,y] \quad (28)$$

> $TimeSeries(F, [x, y], [0.32, 0.5], 0.01, 50, 1)$



> $PhaseDiag(F, [x, y], [0.32, 0.5], 0.01, 10)$



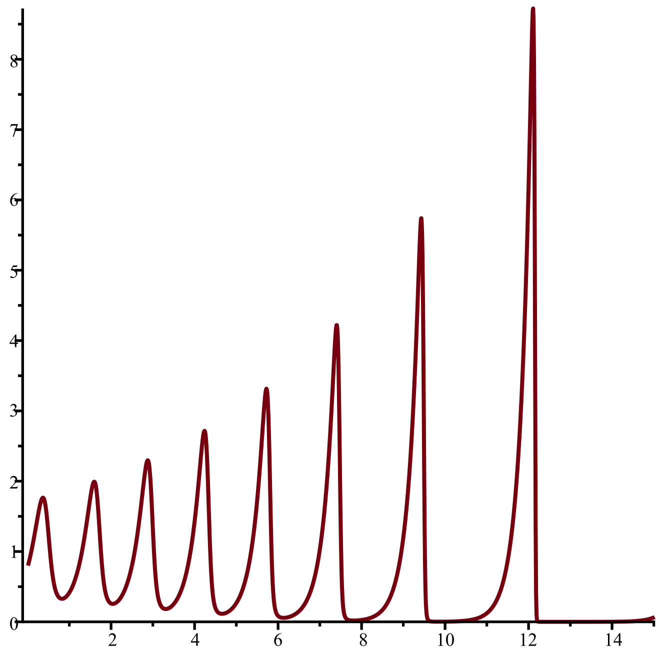
> $SEquP(F, [x, y])$

$$\varnothing \quad (29)$$

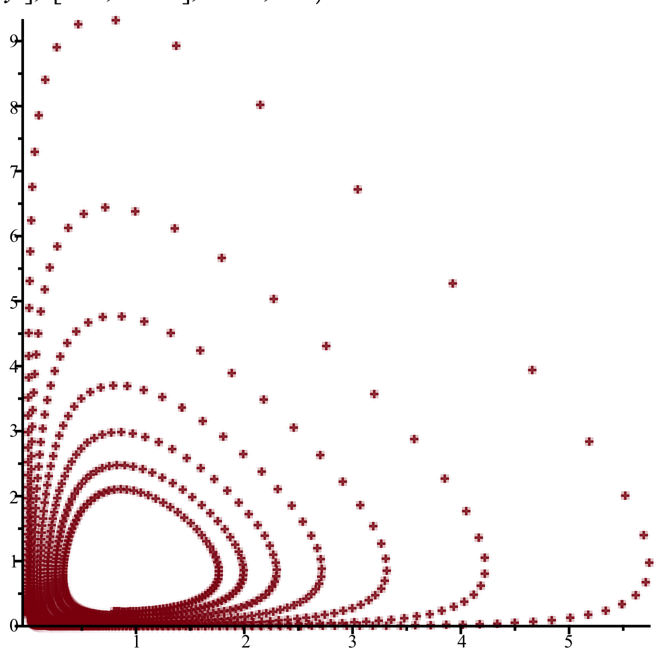
> $F := Volterra(4, 5, 8, 9, x, y);$

$$F := [-5\,x\,y + 4\,x, 9\,x\,y - 8\,y] \quad (30)$$

> $TimeSeries(F, [x, y], [0.8, 0.23], 0.01, 15, 1)$



> $PhaseDiag(F, [x, y], [0.8, 0.23], 0.01, 10)$



> $SEquP(F, [x, y])$

$$\varnothing \quad (31)$$

> #As seen by the TimeSeries and Phase Diagrams, the Volterra dynamical system will have no stable equilibria no matter the parameter or initial point

> $Help(VolterraM)$

VolterraM(a,b,c,d,x,K,y): The MODIFIED Volterra predator-prey continuous-time dynamical system with parameters a,b,c,d,K

Given by Eqs. (8a) (8b) in Edelstein-Keshet p. 220 (section 6.2).

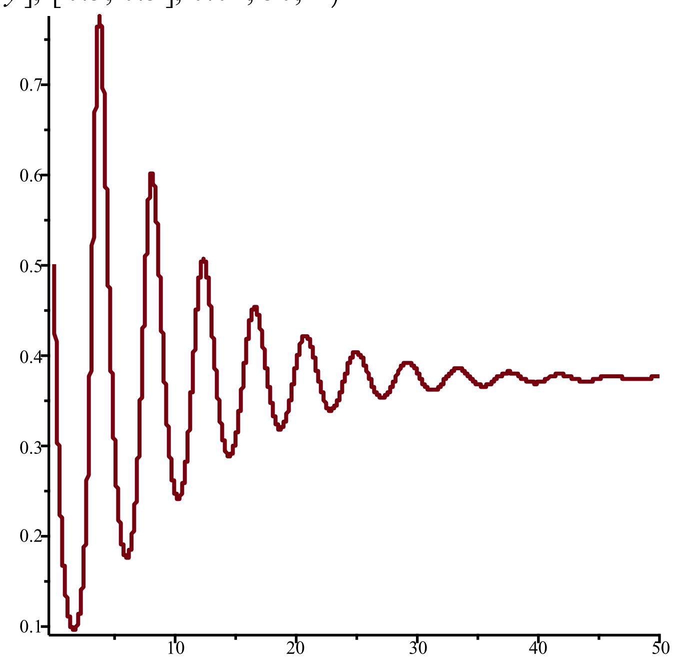
a,b,c,d ,Kmay be symbolic or numeric

Try:
VolterraM(a,b,c,d,K,x,y);
VolterraM(1,2,3,4,3,x,y); **(32)**

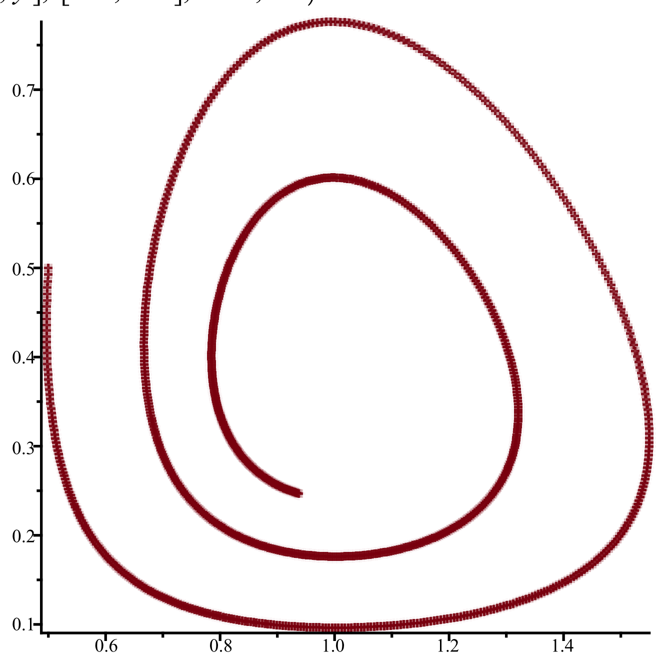
> $F := VolterraM(1, 2, 3, 4, 3, x, y);$

$$F := \left[x\left(1-\frac{x}{4}\right)-2\,x\,y,\ 3\,x\,y-3\,y\right] \quad \textbf{(33)}$$

> $TimeSeries(F, [x, y], [0.5, 0.5], 0.01, 50, 2)$



> $PhaseDiag(F, [x, y], [0.5, 0.5], 0.01, 10)$



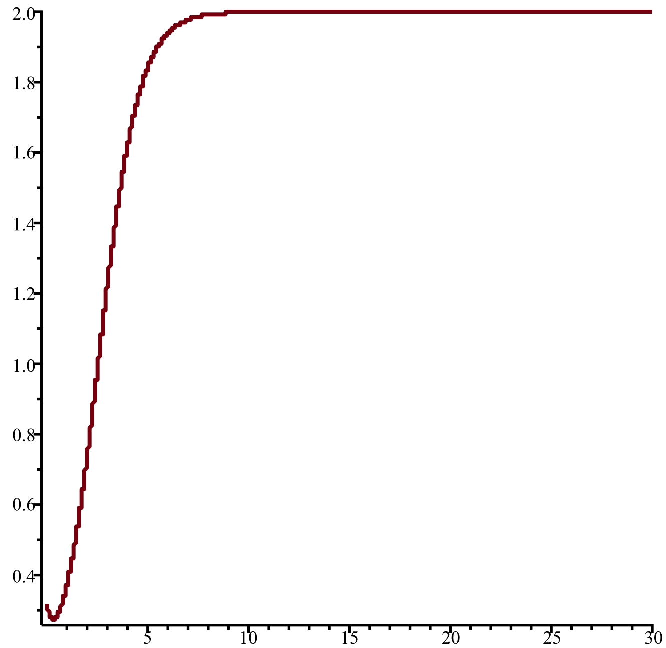
> $SEquP(F, [x, y])$

$$\{[1., 0.3750000000]\} \quad \textbf{(34)}$$

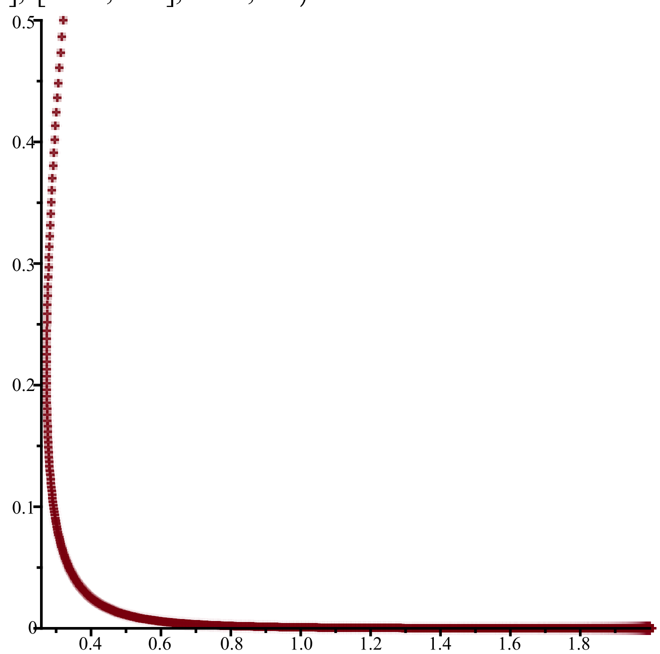
> $F := VolterraM(1, 4, 3, 2, 1, x, y);$

$$F := \left[x\left(1 - \frac{x}{2}\right) - 4\,x\,y, x\,y - 3\,y\right] \tag{35}$$

> $TimeSeries(F, [x, y], [0.32, 0.5], 0.01, 30, 1)$



> $PhaseDiag(F, [x, y], [0.32, 0.5], 0.01, 10)$



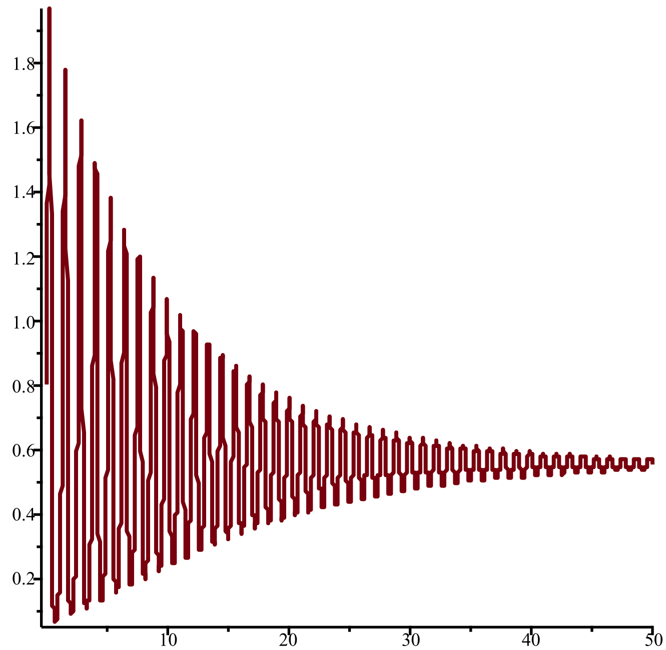
> $SEquP(F, [x, y])$

$$\{[2., 0.]\} \tag{36}$$

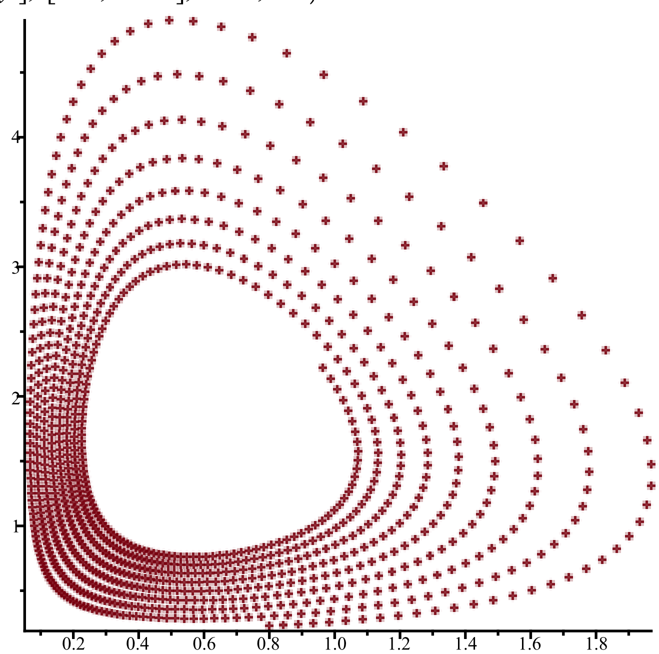
> $F := VolterraM(7, 4, 5, 8, 9, x, y);$

$$F := \left[7\,x\left(1 - \frac{x}{8}\right) - 4\,x\,y, 9\,x\,y - 5\,y\right] \tag{37}$$

> $TimeSeries(F, [x, y], [0.8, 0.23], 0.01, 50, 1)$



> *PhaseDiag*(*F*, [*x*, *y*], [0.8, 0.23], 0.01, 10)



> *SEquP*(*F*, [*x*, *y*])

{[0.5555555556, 1.628472222]} **(38)**

> *#Unlike Voterra, this modified system had stable equilibrium points that were proved graphically and confirmed numerically.*