

```
> #OK to post homework
#Shreya Ghosh, 11-15-2021, Assignment 20
> read "/Users/shreyaghosh/Documents/DMB.txt"
First Written: Nov. 2021
```

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous) accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilberger)

*The most current version is available on WWW at:
<http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt> .
Please report all bugs to: DoronZeil at gmail dot com .*

*For general help, and a list of the MAIN functions,
type "Help():". For specific help type "Help(procedure_name);"*

*For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);*

*For a list of the functions that give examples of Discrete-time dynamical systems (some famous),
type: HelpDDM());*

For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM());

For help with any of them type: Help(ProcedureName);

(1)

```
> #1 i (beta = 0.3  $\frac{\text{nu}}$ 
1000 ).
F := SIRS(s, i, 0.0006, 5, 2, 1000)
F := [-0.0006 s i + 5000 - 5 s - 5 i, 0.0006 s i - 2 i]
```

(2)

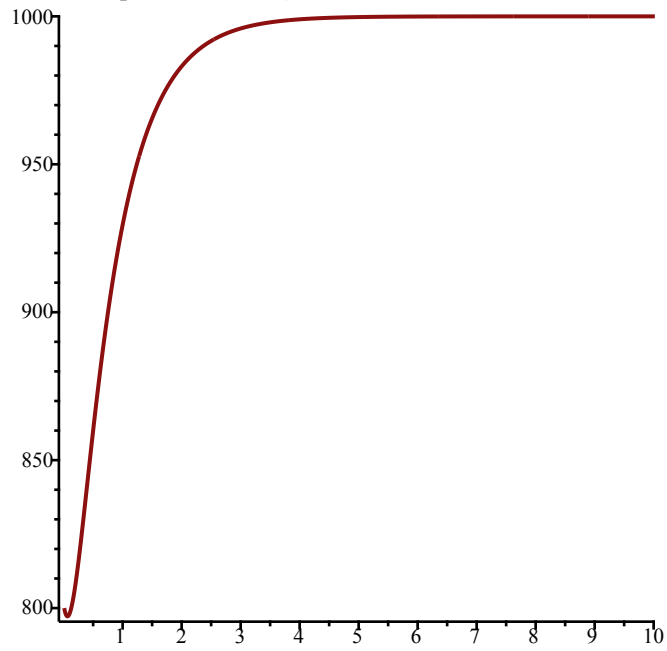
```
> EquP(F, [s, i])
{[1000., 0.], [3333.333333, -1666.666667]}
```

(3)

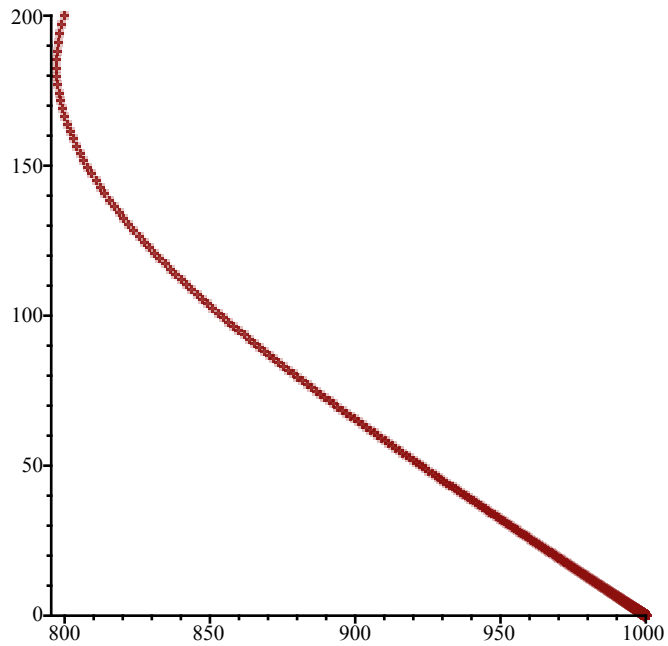
```
> SEquP(F, [s, i])
{[1000., 0.]}
```

(4)

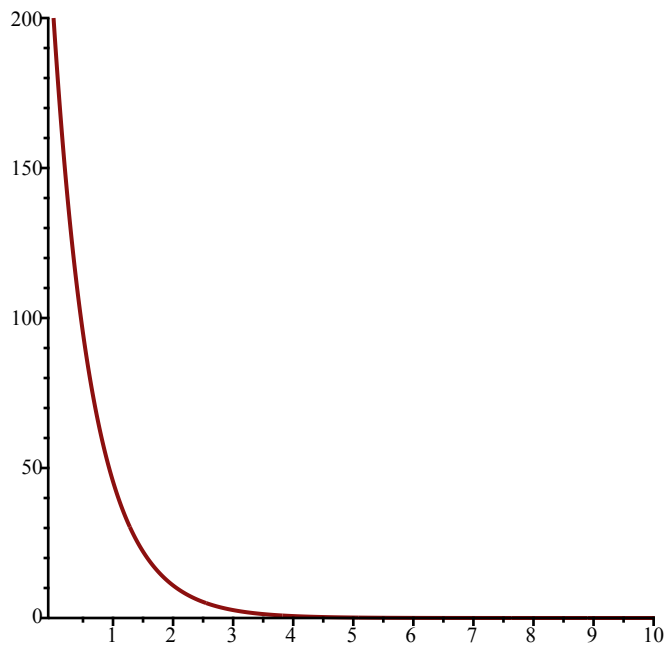
> *TimeSeries*(F , [s , i], [800, 200], 0.01, 10, 1)



> *PhaseDiag*(F , [s , i], [800, 200], 0.01, 10)



> *TimeSeries*($[0.0006 s i - 2 i, -0.0006 s i + 5000 - 5 s - 5 i]$, [i , s], [200, 800], 0.01, 10, 1)



```
>
> #1 i (beta=0.9  $\frac{nu}{1000}$  ).
> F := SIRS(s, i, 0.0018, 5, 2, 1000)
      F := [-0.0018 s i + 5000 - 5 s - 5 i, 0.0018 s i - 2 i]
```

(5)

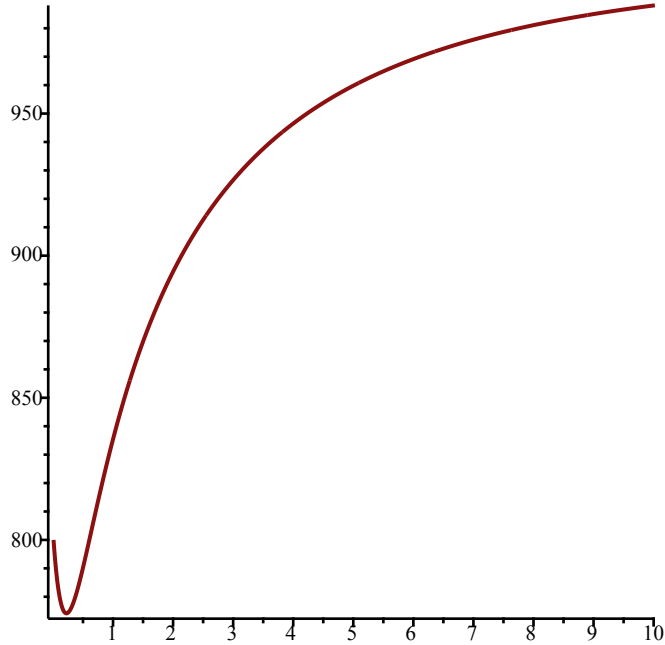
```
> EquP(F, [s, i])
      {[1000., 0.], [1111.111111, -79.36507937]}
```

(6)

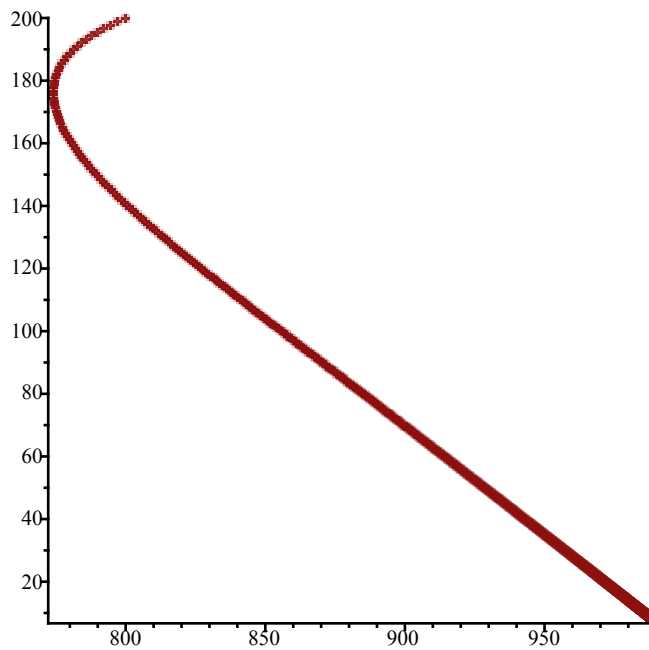
```
> SEquP(F, [s, i])
      {[1000., 0.]}
```

(7)

```
> TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1)
```



```
> PhaseDiag(F, [s, i], [800, 200], 0.01, 10)
```



> #1 i (beta=0.9 $\frac{nu}{1000}$).

> F := SIRS(s, i, 0.0018, 5, 2, 1000)

$$F := [-0.0018 s i + 5000 - 5 s - 5 i, 0.0018 s i - 2 i]$$

(8)

> EquP(F, [s, i])

$$\{[1000., 0.], [1111.111111, -79.36507937]\}$$

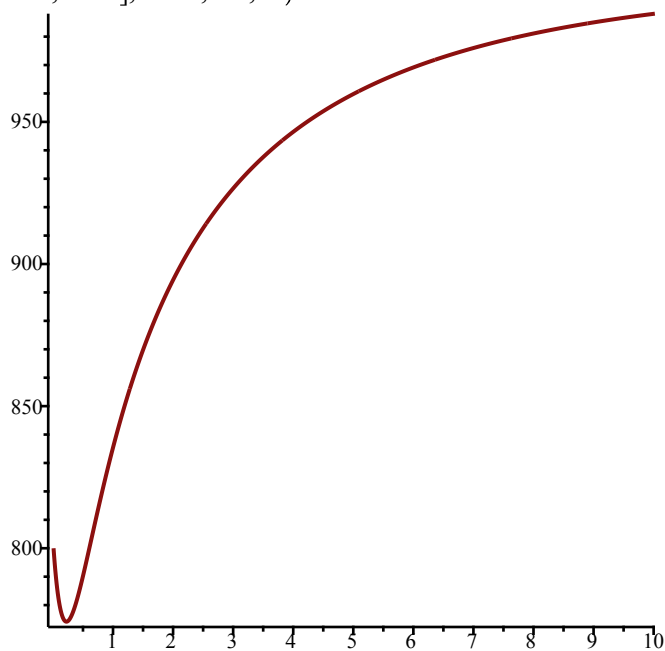
(9)

> SEquP(F, [s, i])

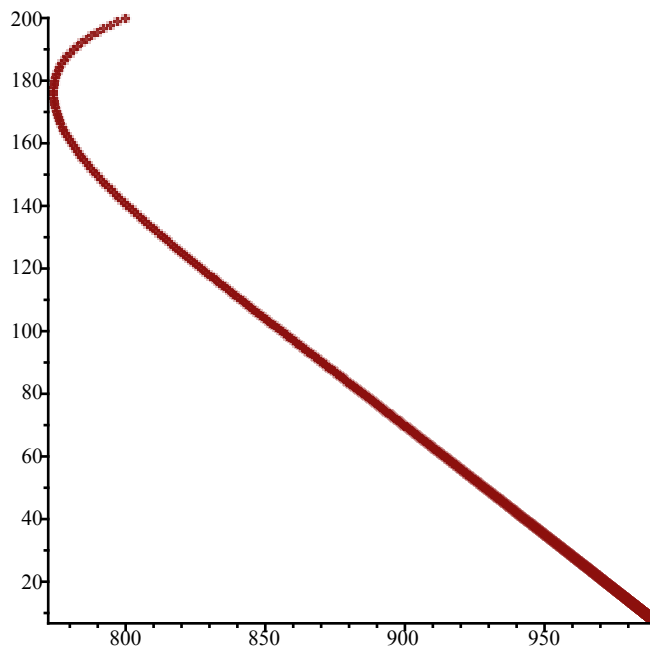
$$\{[1000., 0.]\}$$

(10)

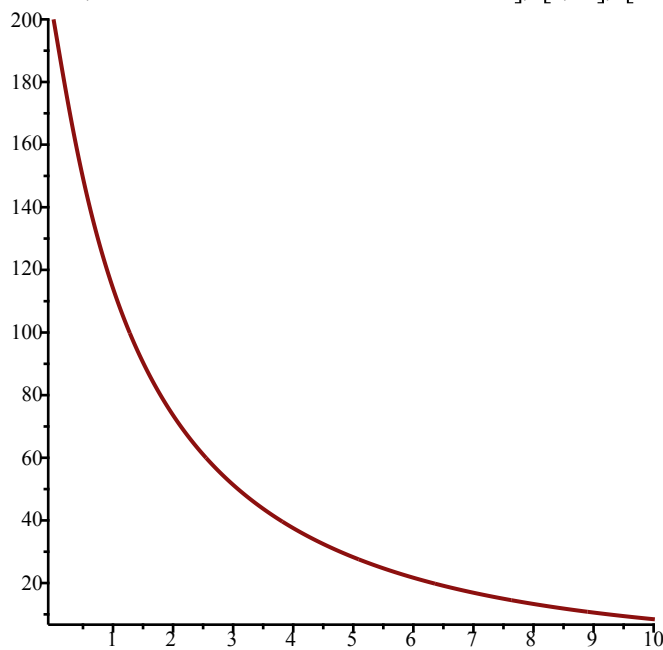
> TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1)



> PhaseDiag(F, [s, i], [800, 200], 0.01, 10)



> TimeSeries ([0.0018 s i - 2 i, -0.0018 s i + 5000 - 5 s - 5 i], [i, s], [200, 800], 0.01, 10, 1)



> #Ii (beta = 3.9 $\frac{\text{nu}}{1000}$)

> F := SIRS(s, i, 0.0078, 5, 2, 1000)

F := [-0.0078 s i + 5000 - 5 s - 5 i, 0.0078 s i - 2 i] (11)

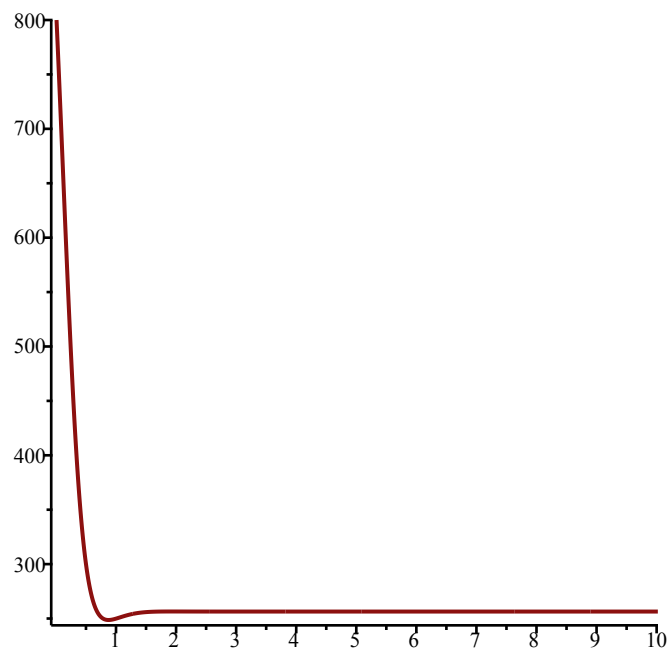
> EquP(F, [s, i])

{[256.4102564, 531.1355311], [1000., 0.]} (12)

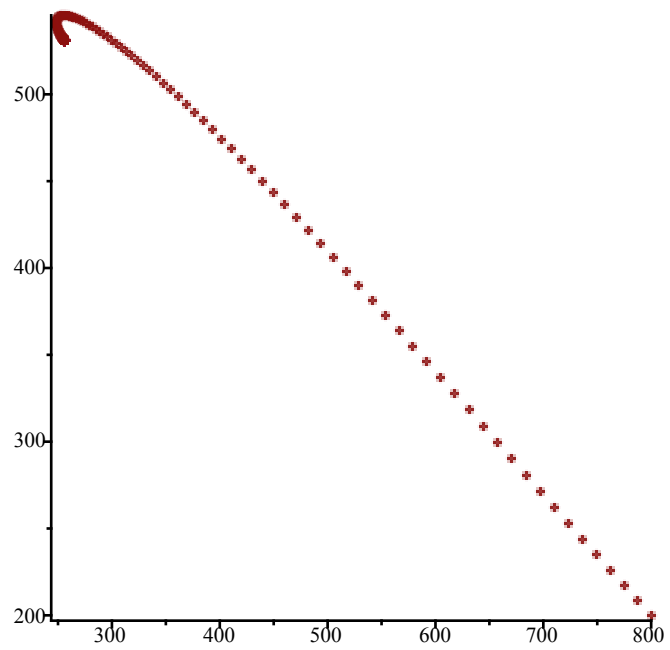
> SEquP(F, [s, i])

{[256.4102564, 531.1355311]} (13)

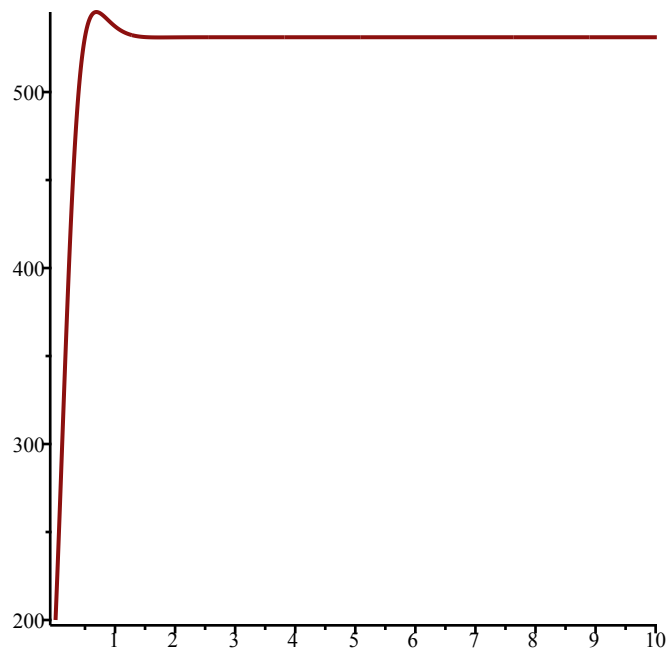
> TimeSeries (F, [s, i], [800, 200], 0.01, 10, 1)



> *PhaseDiag*(*F*, [*s*, *i*], [800, 200], 0.01, 10)



> *TimeSeries*([0.0078 *s i* - 2 *i*, -0.0078 *s i* + 5000 - 5 *s* - 5 *i*], [*i*, *s*], [200, 800], 0.01, 10, 1)



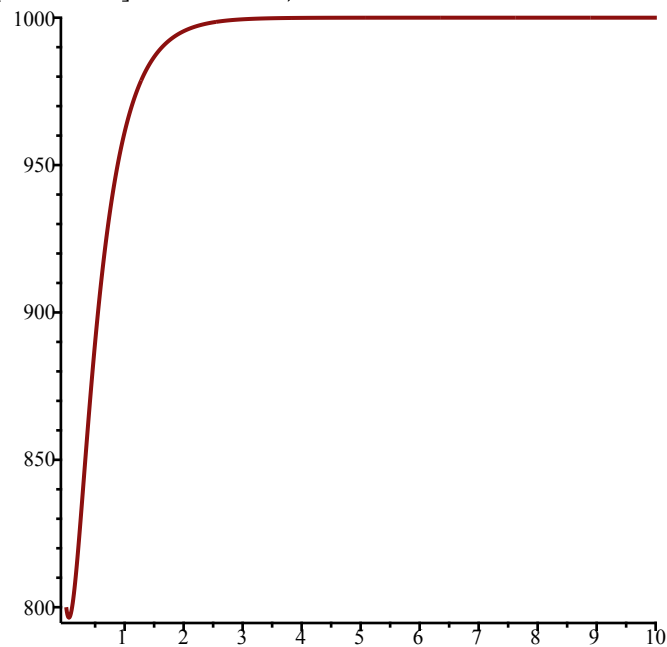
> #1ii(beta = 0.3 $\frac{\text{nu}}$ 1000)

> F := SIRS(s, i, 0.0009, 6, 3, 1000)
 $F := [-0.0009 s i + 6000 - 6 s - 6 i, 0.0009 s i - 3 i]$ (14)

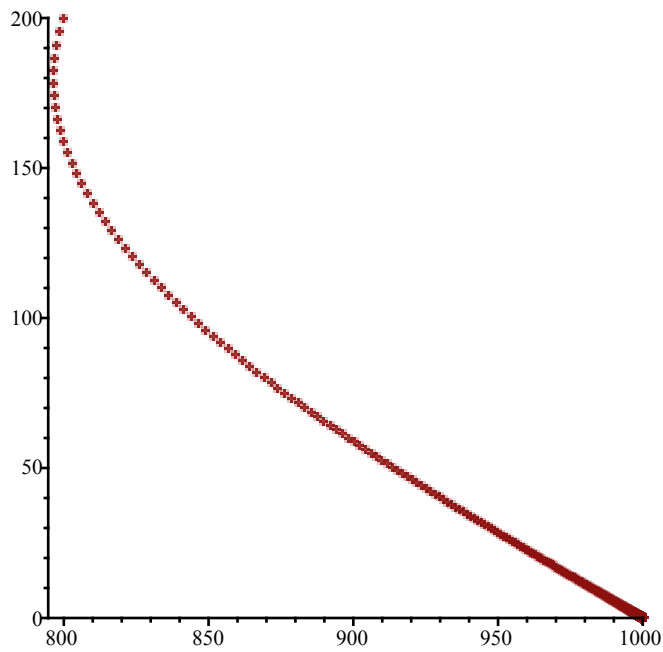
> EquP(F, [s, i])
 $\{[1000., 0.], [3333.333333, -1555.555556]\}$ (15)

> SEquP(F, [s, i])
 $\{[1000., 0.]\}$ (16)

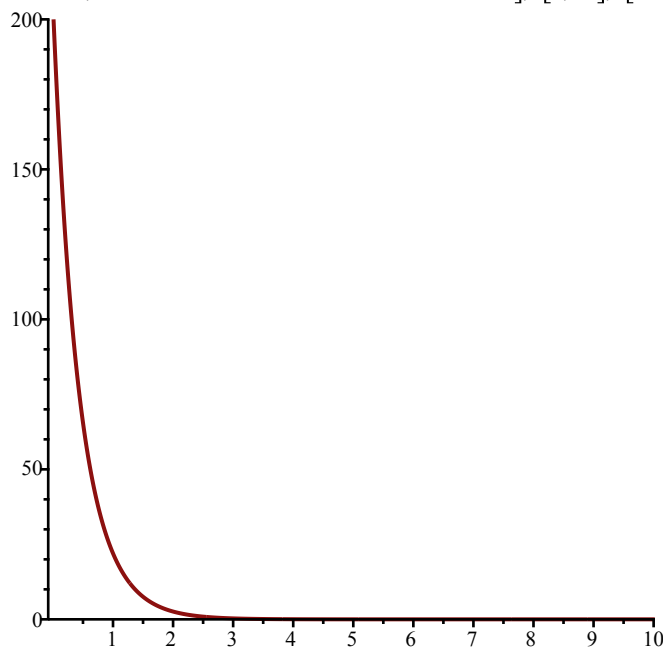
> TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1)



> PhaseDiag(F, [s, i], [800, 200], 0.01, 10)



> *TimeSeries* ([0.0009 *s i* - 3 *i*, -0.0009 *s i* + 6000 - 6 *s* - 6 *i*], [*i*, *s*], [200, 800], 0.01, 10, 1)



> #1ii(beta = 0.9 $\frac{\text{nu}}{1000}$).

> *F* := *SIRS*(*s*, *i*, 0.0027, 6, 3, 1000)

F := [-0.0027 *s i* + 6000 - 6 *s* - 6 *i*, 0.0027 *s i* - 3 *i*] (17)

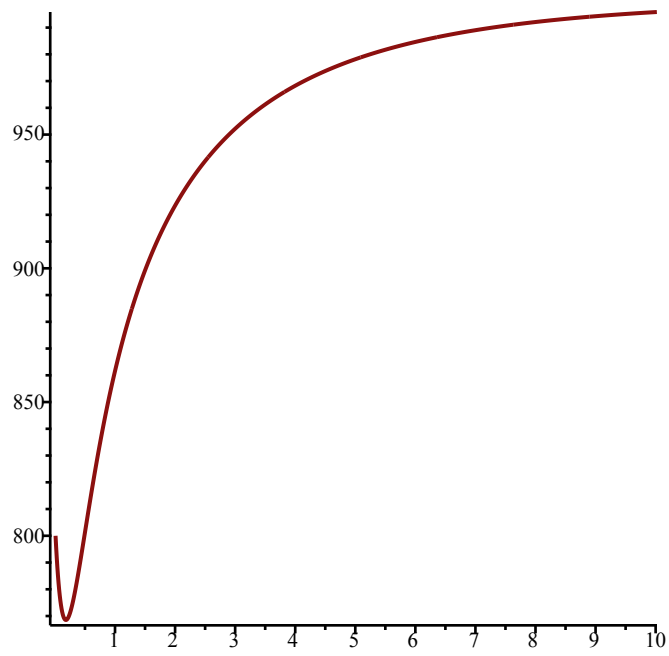
> *EquP*(*F*, [*s*, *i*])

{[1000., 0.], [1111.111111, -74.07407407]} (18)

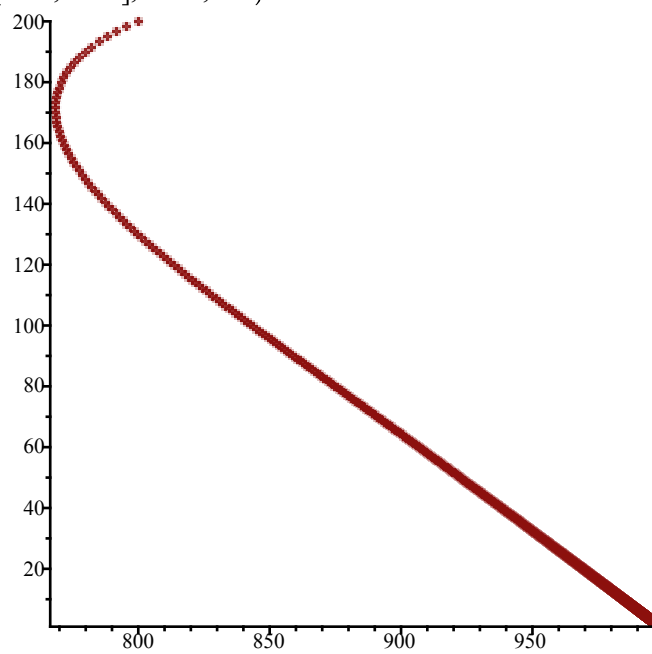
> *SEquP*(*F*, [*s*, *i*])

{[1000., 0.]} (19)

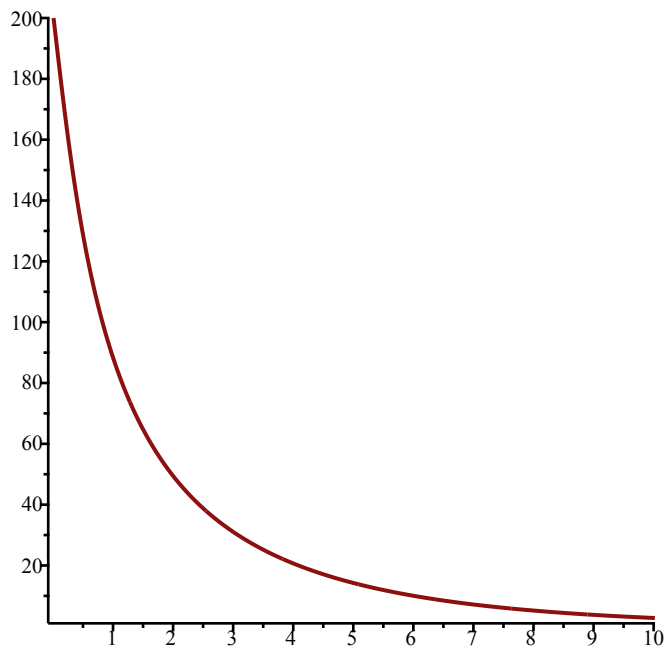
> *TimeSeries* (*F*, [*s*, *i*], [800, 200], 0.01, 10, 1)



> *PhaseDiag*(*F*, [*s*, *i*], [800, 200], 0.01, 10)



> *TimeSeries*([0.0027 *s i* - 3 *i*, -0.0027 *s i* + 6000 - 6 *s* - 6 *i*], [*i*, *s*], [200, 800], 0.01, 10, 1)



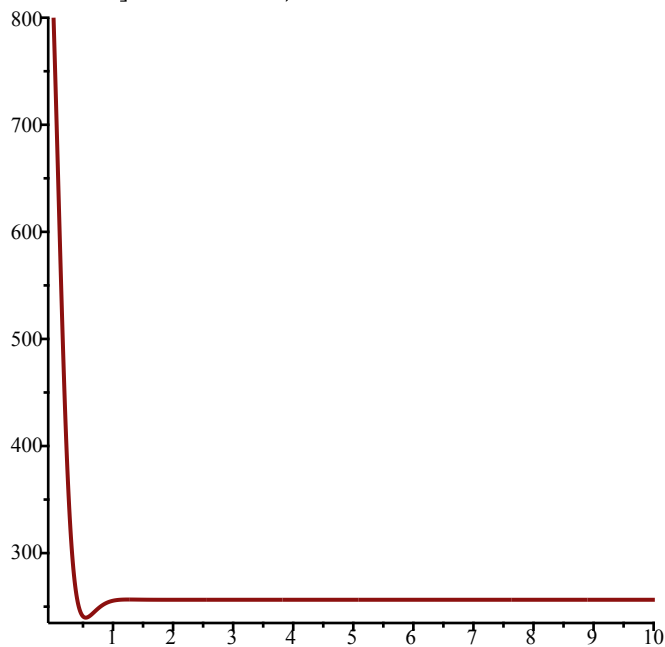
> #1ii (beta = $3.9 \frac{\text{nu}}{1000}$).

> $F := \text{SIRS}(s, i, 0.0117, 6, 3, 1000)$
 $F := [-0.0117 s i + 6000 - 6 s - 6 i, 0.0117 s i - 3 i]$ (20)

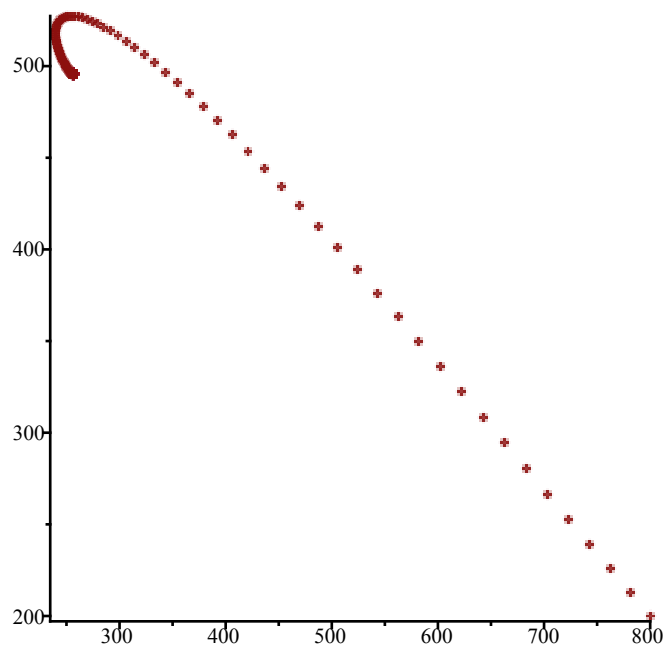
> $\text{EquP}(F, [s, i])$
 $\{[256.4102564, 495.7264957], [1000., 0.]\}$ (21)

> $\text{SEquP}(F, [s, i])$
 $\{[256.4102564, 495.7264957]\}$ (22)

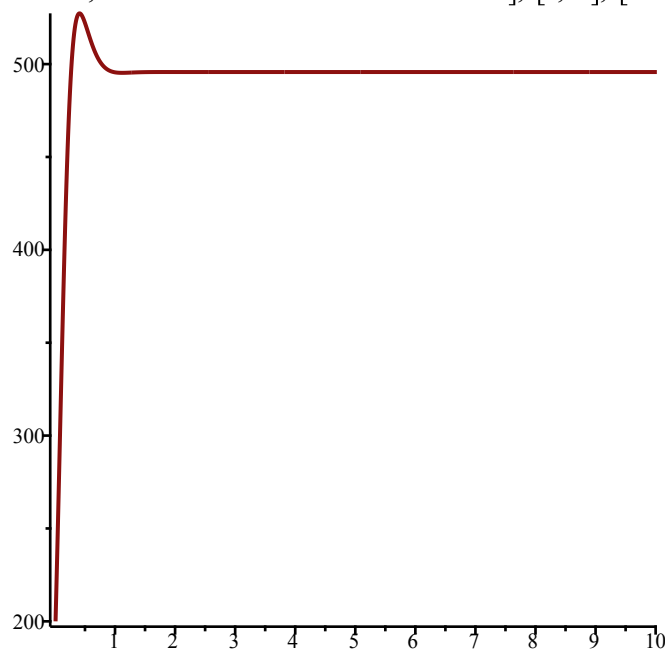
> $\text{TimeSeries}(F, [s, i], [800, 200], 0.01, 10, 1)$



> $\text{PhaseDiag}(F, [s, i], [800, 200], 0.01, 10)$



> *TimeSeries* ([0.0117 *s i* - 3 *i*, -0.0117 *s i* + 6000 - 6 *s* - 6 *i*], [*i*, *s*], [200, 800], 0.01, 10, 1)



> #1iii (beta = 0.3 $\frac{\text{nu}}{1000}$).

> *F* := *SIRS*(*s*, *i*, 0.0012, 1, 4, 1000)

$$F := [-0.0012 s i + 1000 - s - i, 0.0012 s i - 4 i] \quad (23)$$

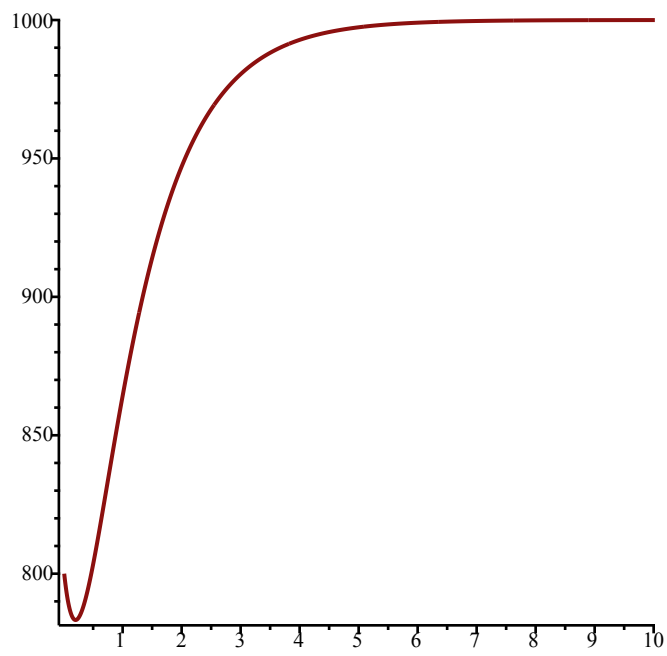
> *EquP*(*F*, [*s*, *i*])

$$\{[1000., 0.], [3333.333333, -466.6666667]\} \quad (24)$$

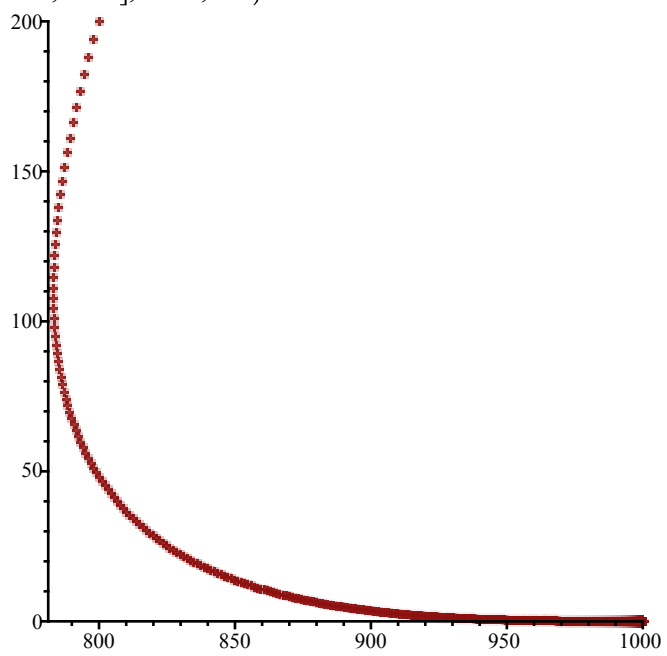
> *SEquP*(*F*, [*s*, *i*])

$$\{[1000., 0.]\} \quad (25)$$

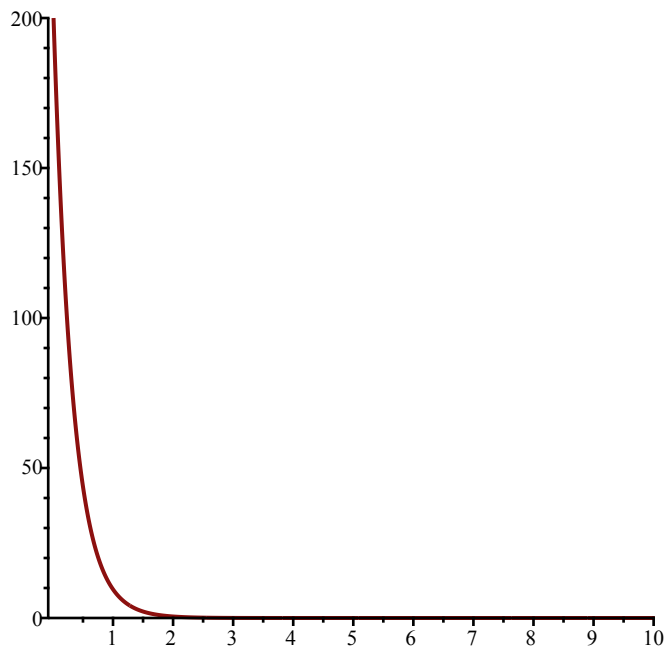
> *TimeSeries*(*F*, [*s*, *i*], [800, 200], 0.01, 10, 1)



> *PhaseDiag*(*F*, [*s*, *i*], [800, 200], 0.01, 10)



> *TimeSeries*([0.0012 *s i* - 4 *i*, -0.0012 *s i* + 1000 - *s* - *i*], [*i*, *s*], [200, 800], 0.01, 10, 1)



> #liii(beta = 0.9 $\frac{\text{nu}}{1000}$).

> $F := \text{SIRS}(s, i, 0.0036, 1, 4, 1000)$

$$F := [-0.0036 s i + 1000 - s - i, 0.0036 s i - 4 i]$$

(26)

> $\text{EquP}(F, [s, i])$

$$\{[1000., 0.], [1111.111111, -22.22222222]\}$$

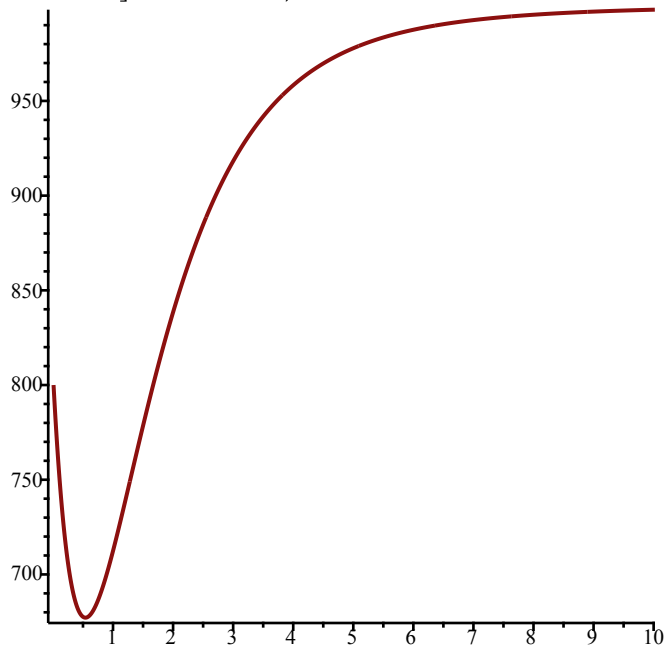
(27)

> $\text{SEquP}(F, [s, i])$

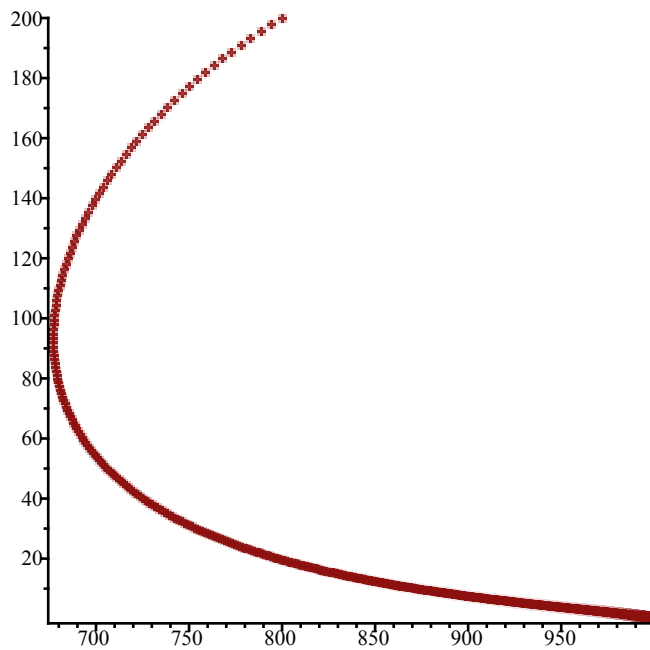
$$\{[1000., 0.]\}$$

(28)

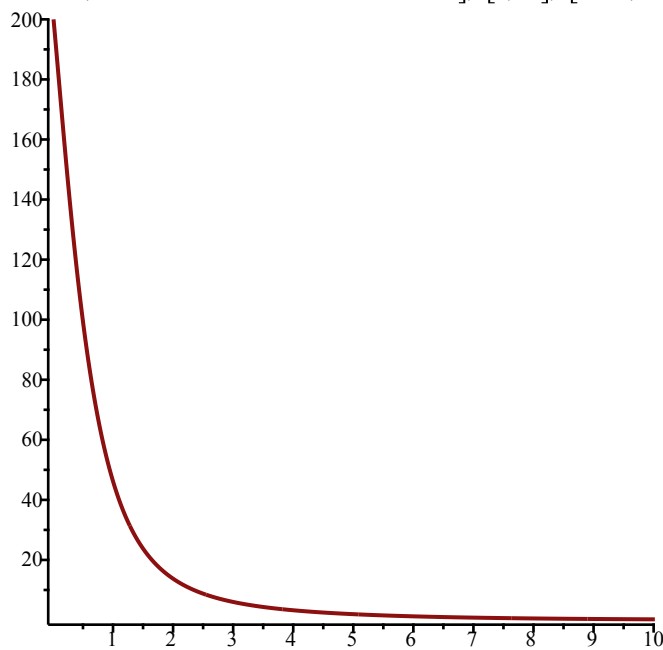
> $\text{TimeSeries}(F, [s, i], [800, 200], 0.01, 10, 1)$



> $\text{PhaseDiag}(F, [s, i], [800, 200], 0.01, 10)$



> TimeSeries ([0.0036 s i - 4 i, -0.0036 s i + 1000 - s - i], [i, s], [200, 800], 0.01, 10, 1)



> #Iiii(beta = 3.9 $\frac{\text{nu}}{1000}$).

> F := SIRS(s, i, 0.0156, 1, 4, 1000)

$$F := [-0.0156 s i + 1000 - s - i, 0.0156 s i - 4 i]$$

(29)

> EquP(F, [s, i])

$$\{[256.4102564, 148.7179487], [1000., 0.]\}$$

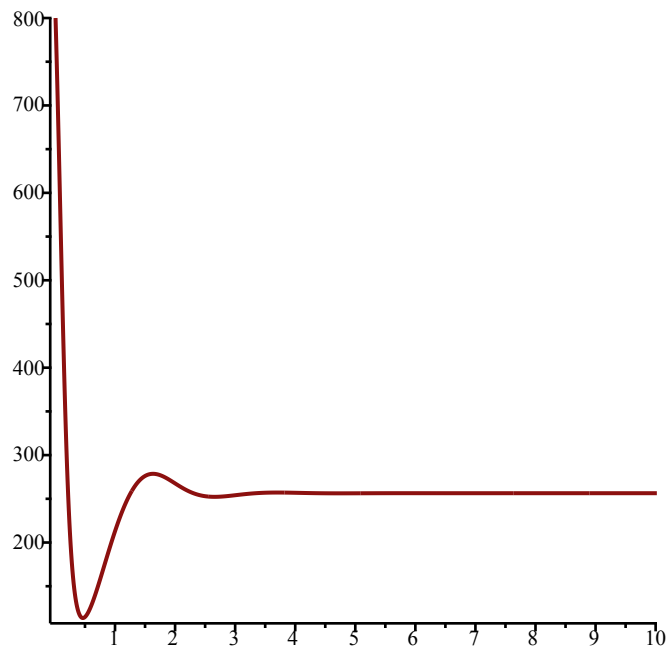
(30)

> SEquP(F, [s, i])

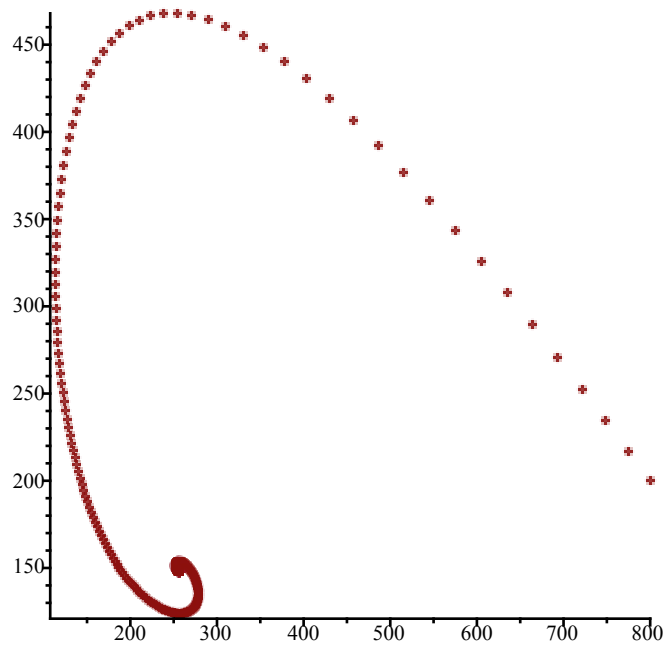
$$\{[256.4102564, 148.7179487]\}$$

(31)

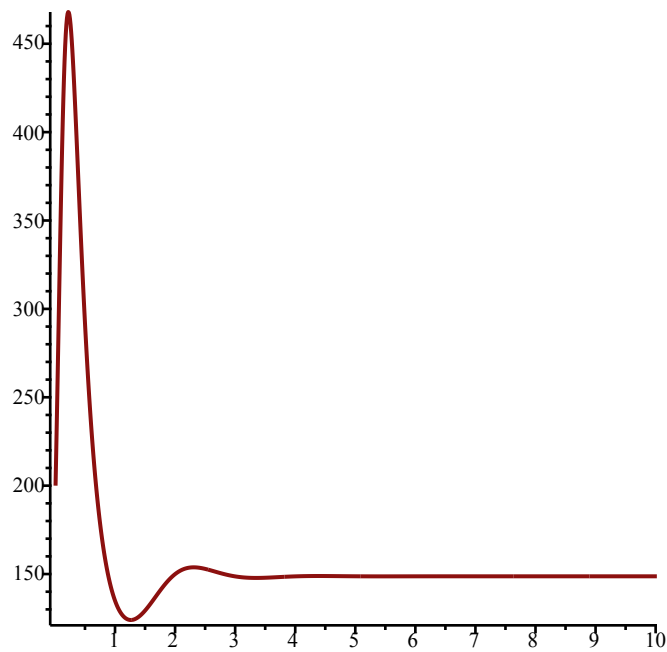
> TimeSeries (F, [s, i], [800, 200], 0.01, 10, 1)



> *PhaseDiag*(*F*, [*s*, *i*], [800, 200], 0.01, 10)



> *TimeSeries*([0.0156 *s* *i* - 4 *i*, -0.0156 *s* *i* + 1000 - *s* - *i*], [*i*, *s*], [200, 800], 0.01, 10, 1)



> #liv(beta = 0.3 $\frac{\text{nu}}{1000}$).

> F := SIRS(s, i, 0.0021, 10, 7, 1000)

$$F := [-0.0021 s i + 10000 - 10 s - 10 i, 0.0021 s i - 7 i]$$

(32)

> EquP(F, [s, i])

$$\{[1000., 0.], [3333.333333, -1372.549020]\}$$

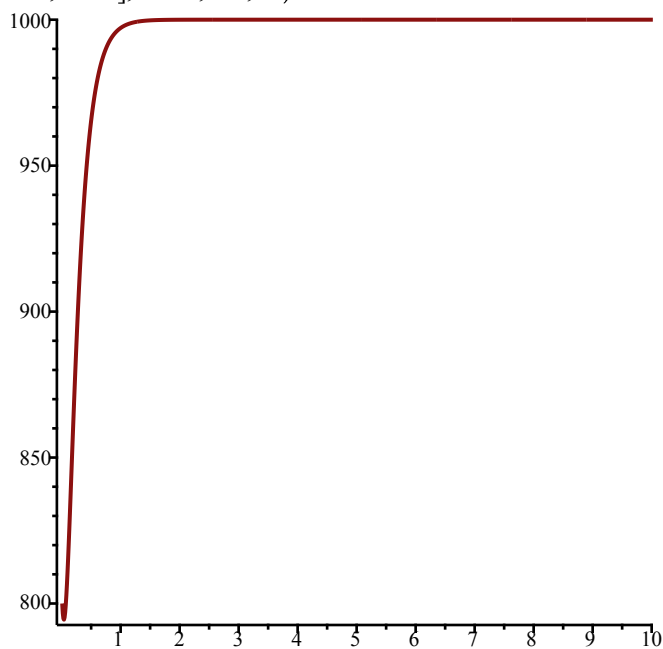
(33)

> SEquP(F, [s, i])

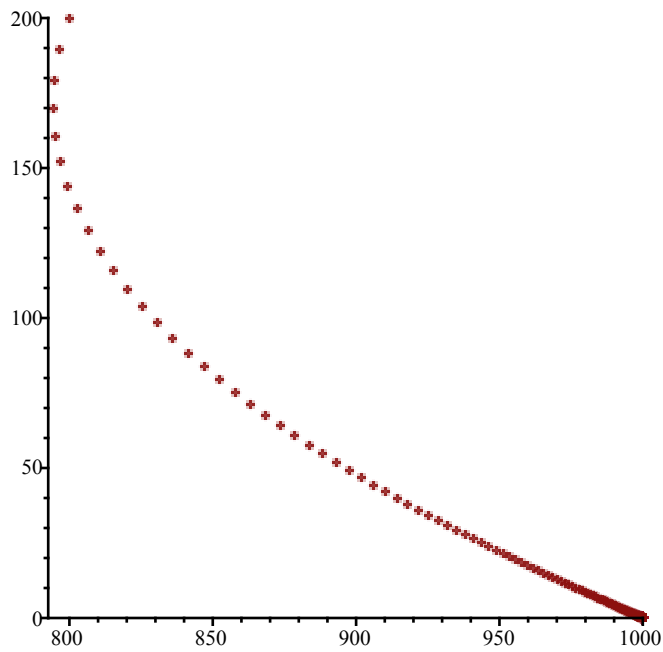
$$\{[1000., 0.]\}$$

(34)

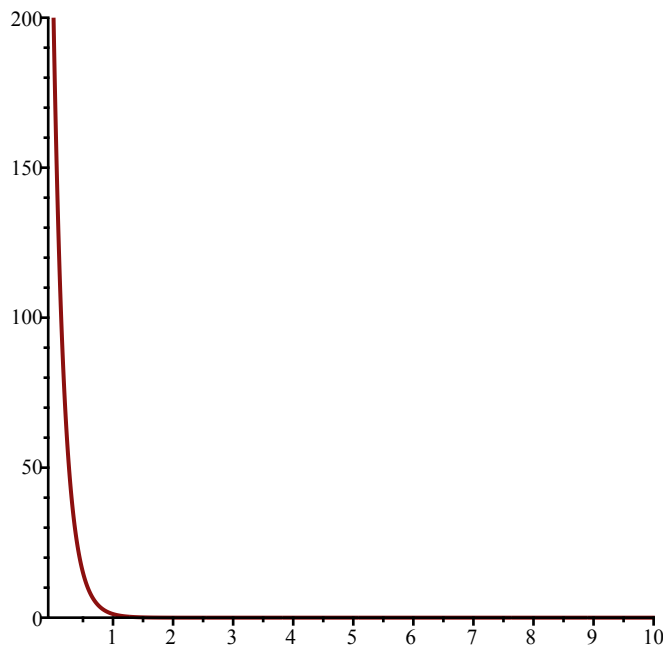
> TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1)



> PhaseDiag(F, [s, i], [800, 200], 0.01, 10)



> TimeSeries([0.0021 s i - 7 i, -0.0021 s i + 10000 - 10 s - 10 i], [i, s], [200, 800], 0.01, 10, 1)



> #Iiv(beta = 0.9 $\frac{\text{nu}}{1000}$).

> F := SIRS(s, i, 0.0063, 10, 7, 1000)

$F := [-0.0063 s i + 10000 - 10 s - 10 i, 0.0063 s i - 7 i]$ (35)

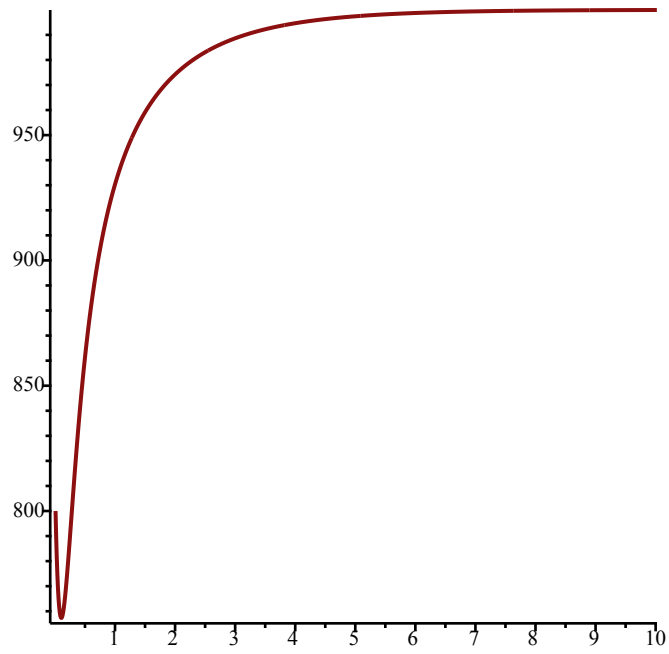
> EquP(F, [s, i])

{[1000., 0.], [1111.111111, -65.35947712]} (36)

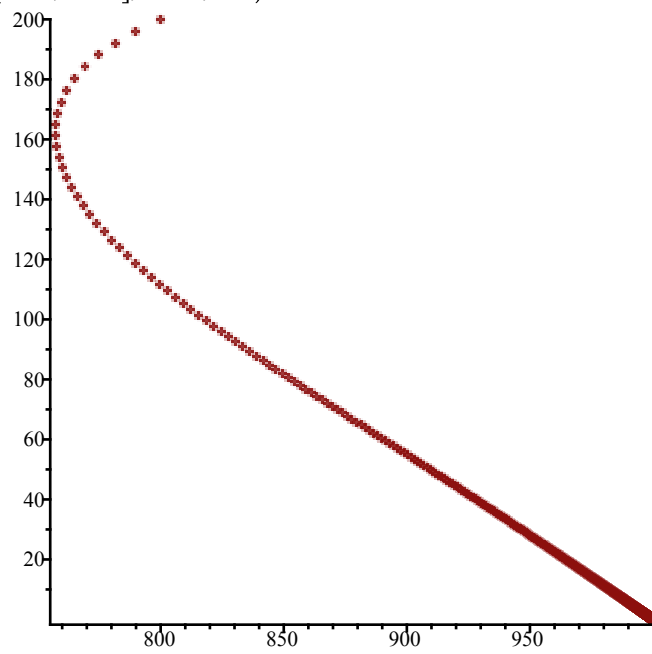
> SEquP(F, [s, i])

{[1000., 0.]} (37)

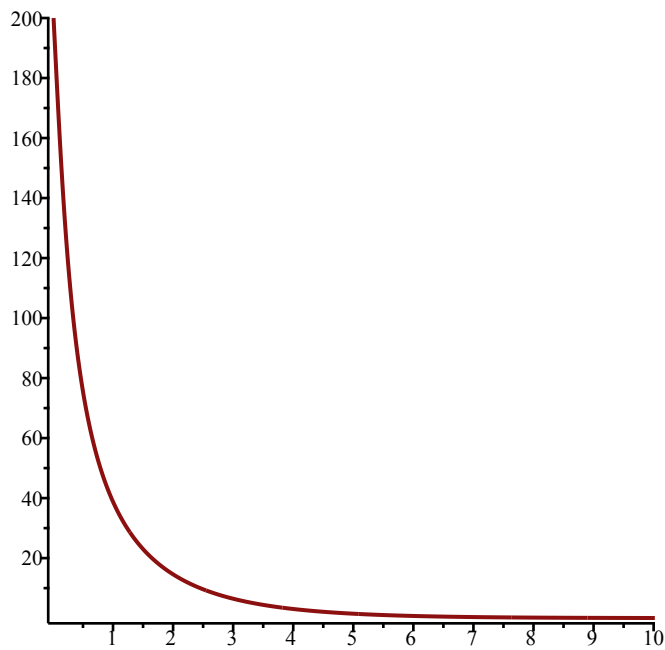
> TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1)



> *PhaseDiag*(*F*, [*s*, *i*], [800, 200], 0.01, 10)



> *TimeSeries*([0.0063 *s* *i* - 7 *i*, -0.0063 *s* *i* + 10000 - 10 *s* - 10 *i*], [*i*, *s*], [200, 800], 0.01, 10,
1)



> #liv (beta = 3.9 $\frac{\text{nu}}{1000}$).

> F := SIRS(s, i, 0.0273, 10, 7, 1000)

$$F := [-0.0273 s i + 10000 - 10 s - 10 i, 0.0273 s i - 7 i]$$

(38)

> EquP(F, [s, i])

$$\{[256.4102564, 437.4057315], [1000., 0.]\}$$

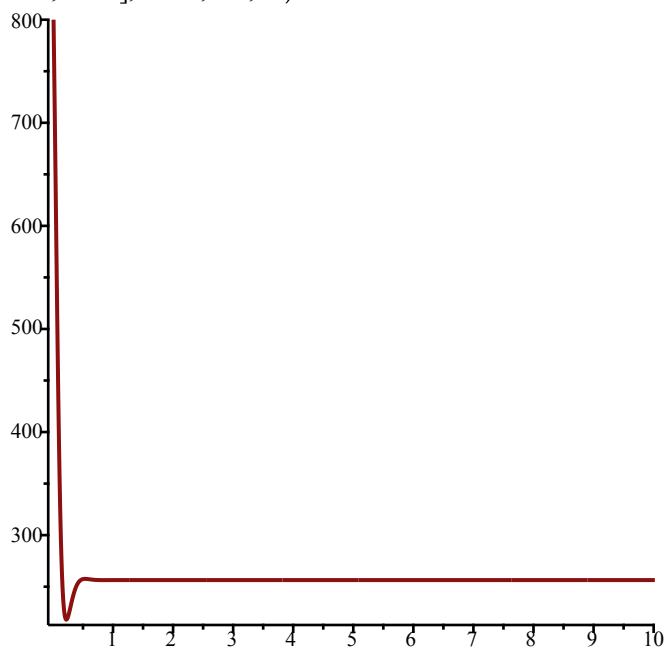
(39)

> SEquP(F, [s, i])

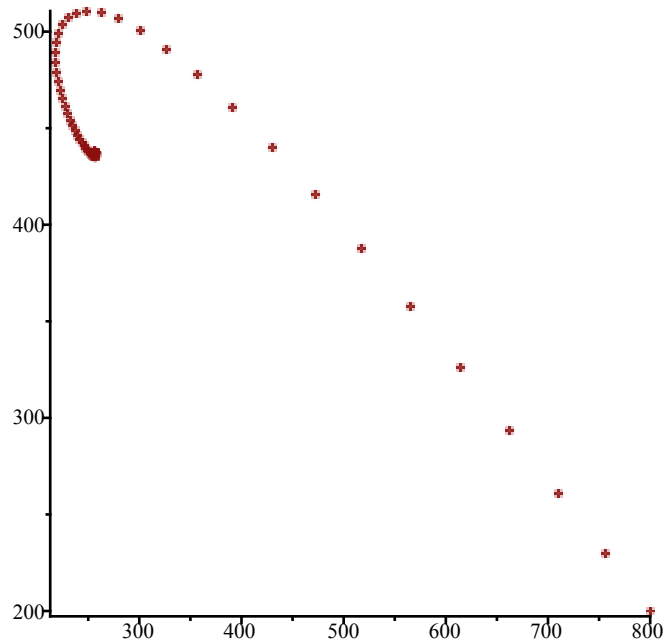
$$\{[256.4102564, 437.4057315]\}$$

(40)

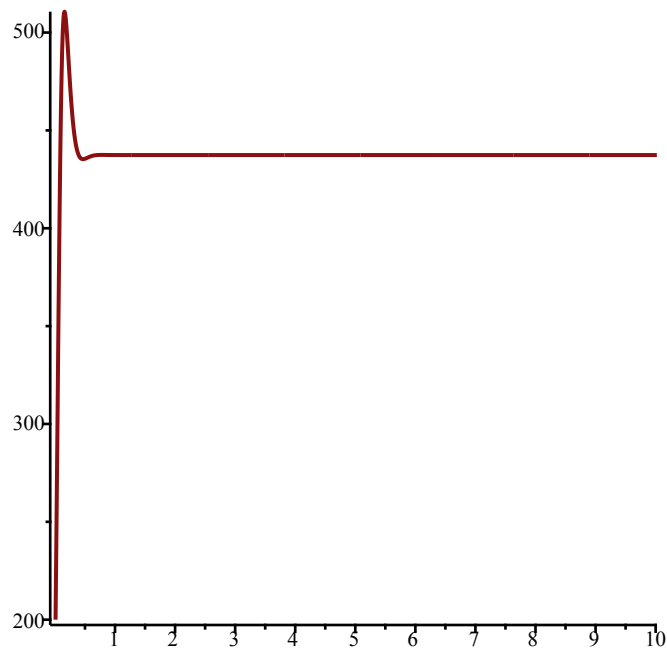
> TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1)



> PhaseDiag(F, [s, i], [800, 200], 0.01, 10)



```
> TimeSeries([0.0273 s i - 7 i, -0.0273 s i + 10000 - 10 s - 10 i], [i, s], [200, 800], 0.01, 10, 1)
```



```
> #2.
```

```
F := RandNice([x, y], 3)
```

$$F := [(3 - 3x - y)(2 - x - y), (2 - 2x - 3y)(3 - 2x - 2y)] \quad (41)$$

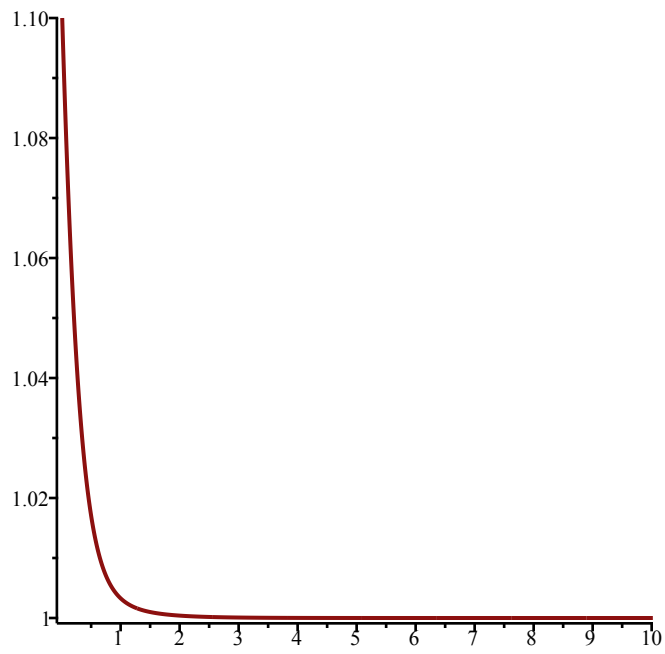
```
> EquP(F, [x, y])
```

$$\left\{ [1, 0], [4, -2], \left[\frac{3}{4}, \frac{3}{4} \right] \right\} \quad (42)$$

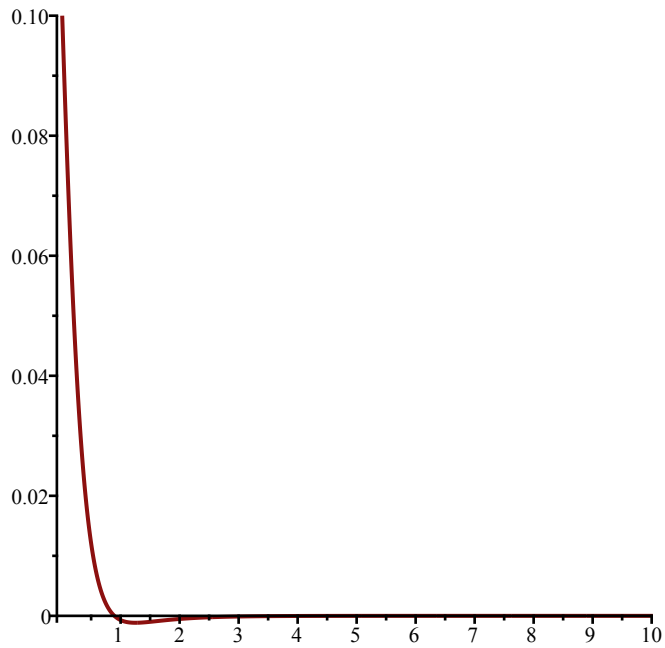
```
> SEquP(F, [x, y])
```

$$\{[1., 0.]\} \quad (43)$$

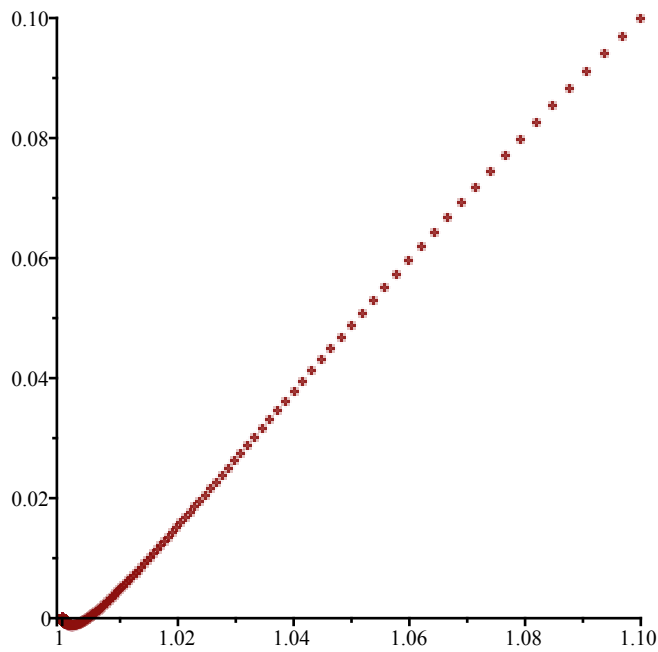
```
> TimeSeries(F, [x, y], [1.1, 0.1], 0.01, 10, 1)
```

> *TimeSeries* ([(2 - 2 x - 3 y) (3 - 2 x - 2 y), (3 - 3 x - y) (2 - x - y)], [y, x], [0.1, 1.1], 0.01, 10, 1)



> *PhaseDiag*(F, [x, y], [1.1, 0.1], 0.01, 10)

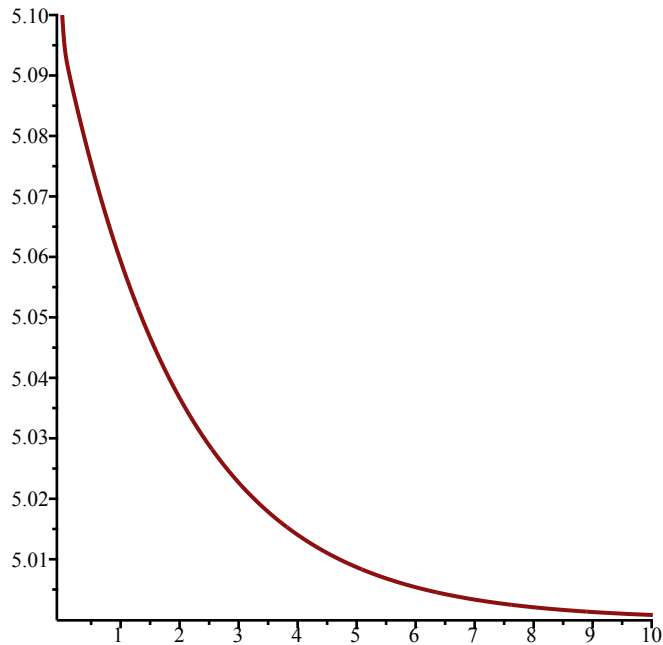


> $F := \text{RandNice}([x, y], 3)$
 $F := [(2 - 3x - 2y)(3 - 2x - y), (1 - x - 3y)(1 - 3x - 2y)]$ (44)

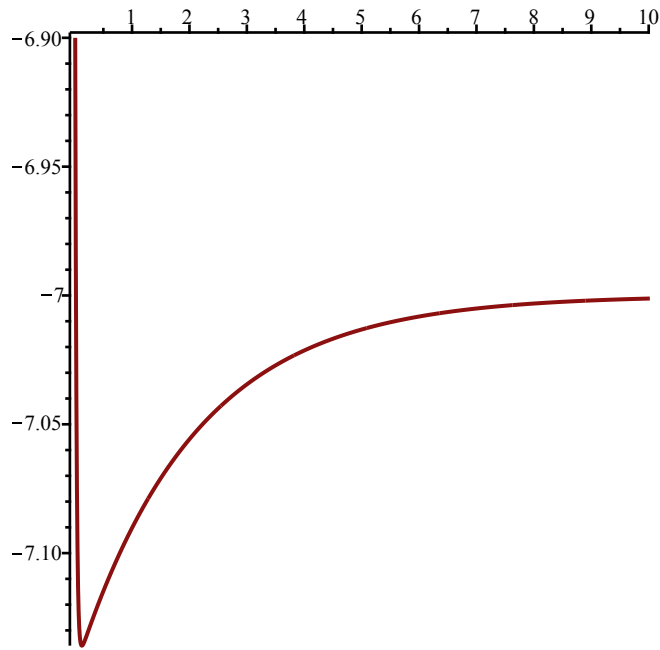
> $\text{EquP}(F, [x, y])$
 $\left\{ [5, -7], \left[\frac{4}{7}, \frac{1}{7} \right], \left[\frac{8}{5}, -\frac{1}{5} \right] \right\}$ (45)

> $\text{SEquP}(F, [x, y])$
 $\{[5., -7.]\}$ (46)

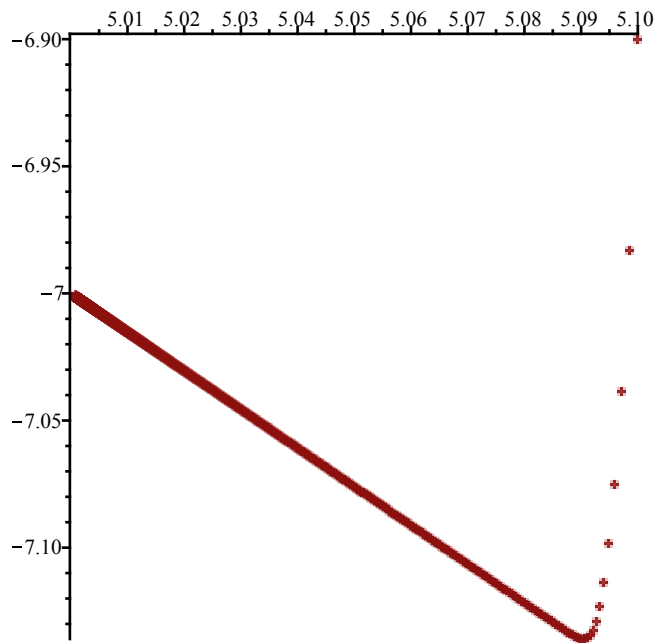
> $\text{TimeSeries}(F, [x, y], [5.1, -6.9], 0.01, 10, 1)$



> $\text{TimeSeries}([(1 - x - 3y)(1 - 3x - 2y), (2 - 3x - 2y)(3 - 2x - y)], [y, x], [-6.9, 5.1], 0.01, 10, 1)$



> *PhaseDiag*(*F*, [*x*, *y*], [5.1, -6.9], 0.01, 10)



> *F* := *RandNice*([*x*, *y*], 3)

$$F := [(3 - x - 2y)(1 - 3x - y), (3 - x - y)(2 - 2x - 3y)] \quad (47)$$

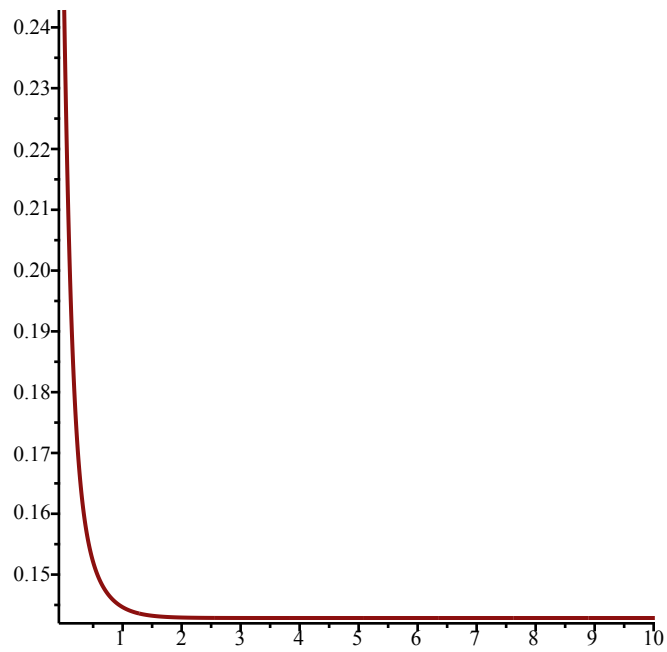
> *EquP*(*F*, [*x*, *y*])

$$\left\{ [-5, 4], [-1, 4], [3, 0], \left[\frac{1}{7}, \frac{4}{7} \right] \right\} \quad (48)$$

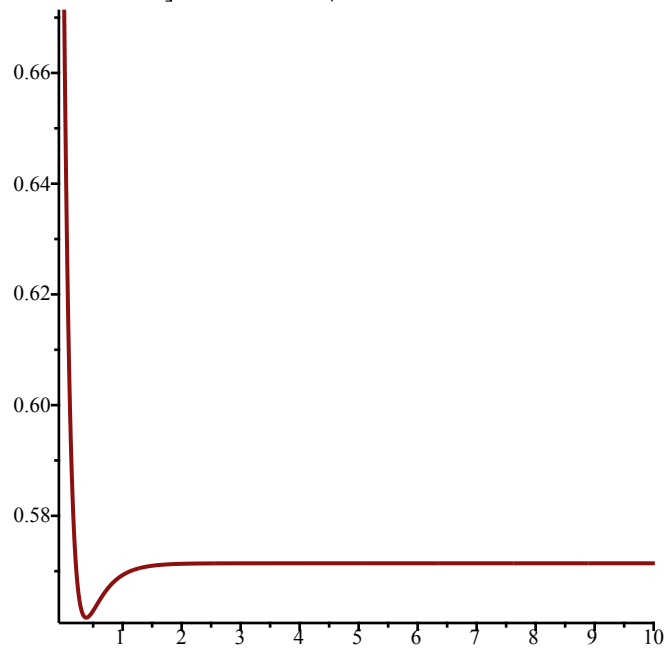
> *SEquP*(*F*, [*x*, *y*])

$$\{ [0.1428571429, 0.5714285714] \} \quad (49)$$

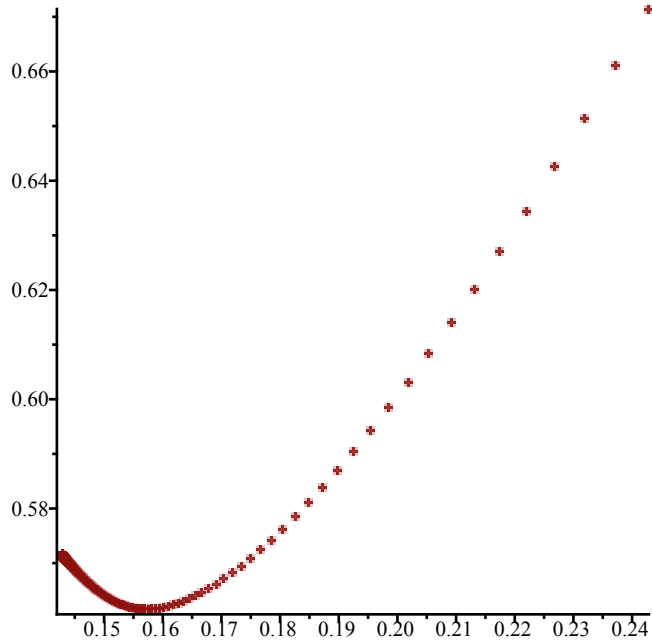
> *TimeSeries*(*F*, [*x*, *y*], [0.2428571429, 0.6714285714], 0.01, 10, 1)



> *TimeSeries* ([(3 - x - y) (2 - 2 x - 3 y), (3 - x - 2 y) (1 - 3 x - y)], [y, x], [0.6714285714, 0.2428571429], 0.01, 10, 1)



> *PhaseDiag*(F, [x, y], [0.2428571429, 0.6714285714], 0.01, 10)



> #3.

> $evalf\left(Orbk\left(4, z, \frac{3 + z[2] + z[3] + z[4]}{1 + z[1] + z[3]}, [1, 1, 1, 1], 1000, 1010\right)\right)$

> #Orbk taking a long time to compute

> $ToSys\left(4, z, \frac{3 + z[2] + z[3] + z[4]}{1 + z[1] + z[3]}\right)$
 $\left[\frac{3 + z_2 + z_3 + z_4}{1 + z_1 + z_3}, z_1, z_2, z_3\right], [z_1, z_2, z_3, z_4]$

(50)

> $SFP\left(\left[\frac{3 + z_2 + z_3 + z_4}{1 + z_1 + z_3}, z_1, z_2, z_3\right], [z_1, z_2, z_3, z_4]\right)$
 $\{[1.822875656, 1.822875656, 1.822875656, 1.822875656]\}$

(51)

>