```
> read "/Users/jjj104/Documents/DMB.txt" :
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*First Written: Nov. 2021*

*This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)*
*accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)*

*The most current version is available on WWW at:*
*http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .*
*Please report all bugs to: DoronZeil at gmail dot com .*

*For general help, and a list of the MAIN functions,*
*type "Help();". For specific help type "Help(procedure_name);"*

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*For a list of the supporting functions type: Help1();*
*For help with any of them type: Help(ProcedureName);*

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*For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();*
*For help with any of them type: Help(ProcedureName);*

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*For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();*
*For help with any of them type: Help(ProcedureName);*

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**(1)**

```
> #1i
  N := 1000;
  v := 2;
  g := 5;
  b1 := 0.3 * (v/N);
  b2 := 0.9 * (v/N);
  b3 := 3.9 * (v/N);
  F := SIRS(s, i, b1, g, v, N);
  EquP(F, [s, i]);
```

```
SEquP(F, [s, i]);
TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1);
TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2);
PhaseDiag(F, [s, i], [800, 200], 0.01, 10);

F := SIRS(s, i, b2, g, v, N);
EquP(F, [s, i]);
SEquP(F, [s, i]);
TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1);
TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2);
PhaseDiag(F, [s, i], [800, 200], 0.01, 10);

F := SIRS(s, i, b3, g, v, N);
EquP(F, [s, i]);
SEquP(F, [s, i]);
TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1);
TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2);
PhaseDiag(F, [s, i], [800, 200], 0.01, 10);
```

$$N := 1000$$

$$v := 2$$

$$g := 5$$

$$b1 := 0.0006000000000$$

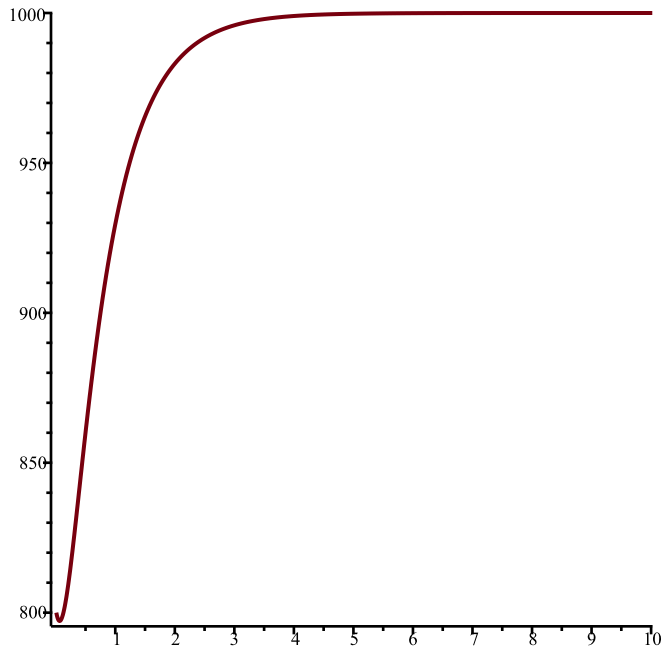
$$b2 := 0.001800000000$$

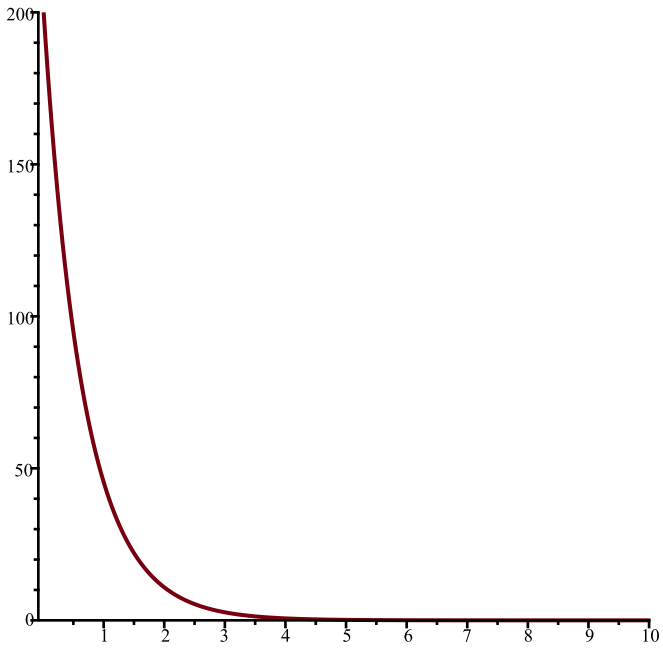
$$b3 := 0.007800000000$$

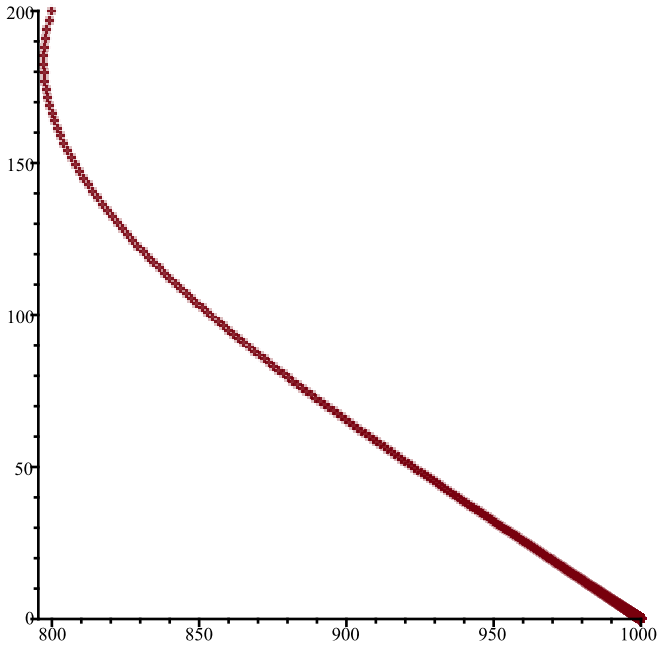
$$F := [-0.0006000000000\, s\, i + 5000 - 5\, s - 5\, i,\ 0.0006000000000\, s\, i - 2\, i]$$

$$\{[1000., 0.], [3333.333333, -1666.666667]\}$$

$$\{[1000., 0.]\}$$



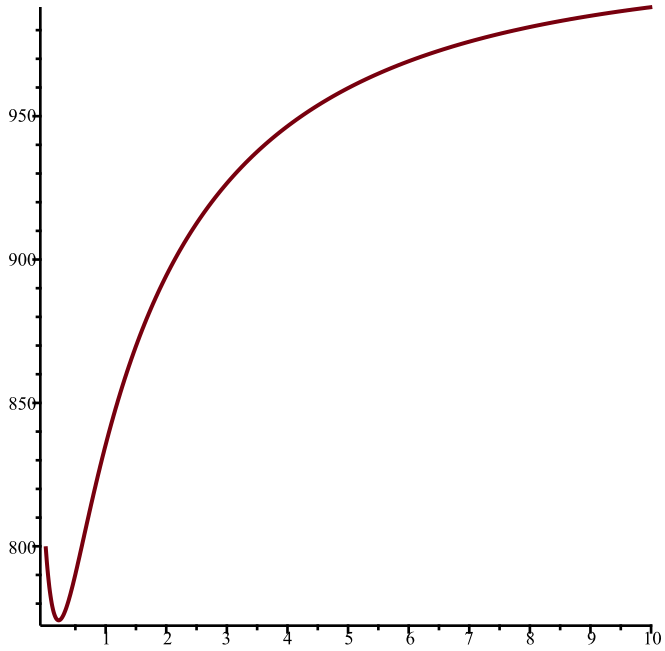



$$F := [-0.001800000000\, s\, i + 5000 - 5\, s - 5\, i,\ 0.001800000000\, s\, i - 2\, i]$$

$$\{[1000., 0.], [1111.111111, -79.36507937]\}$$

$$\{[1000., 0.]\}$$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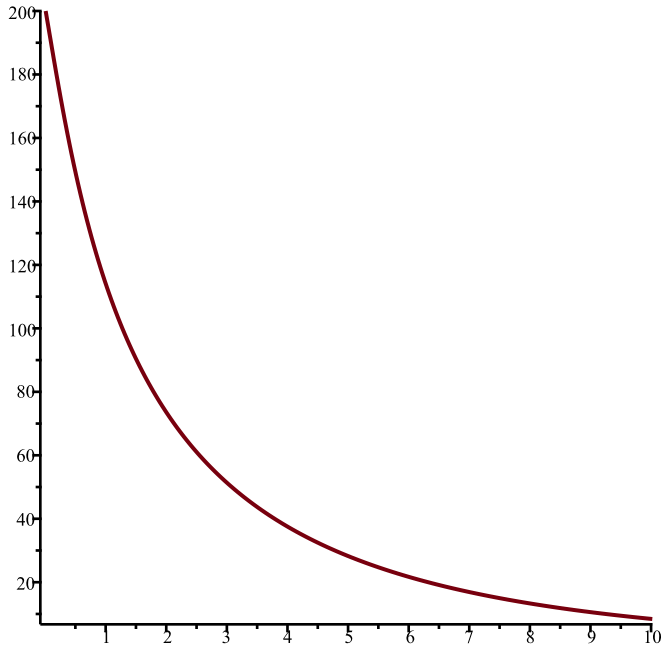
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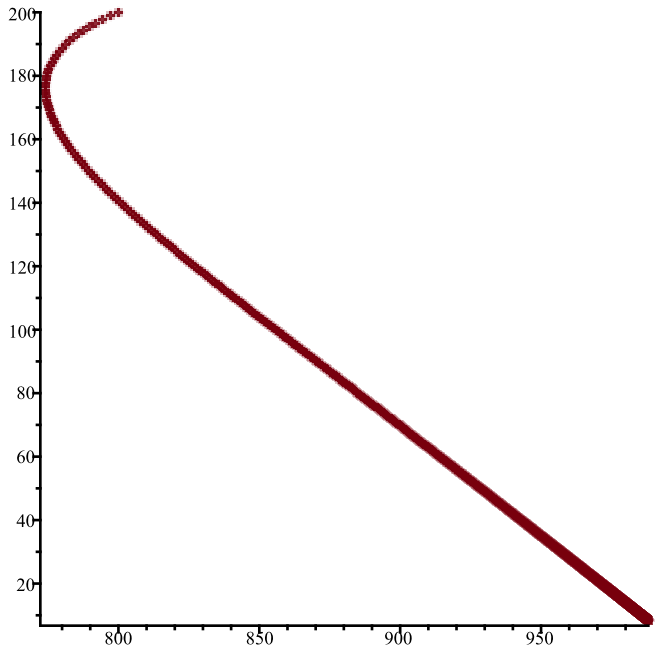
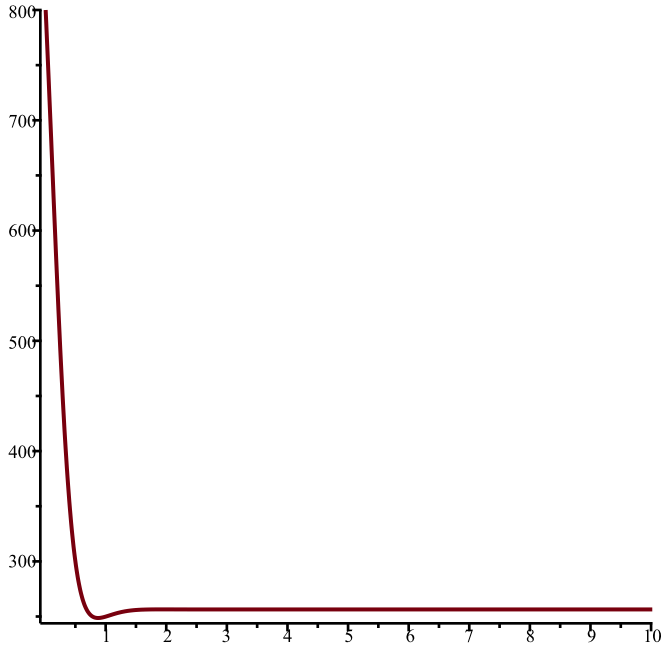
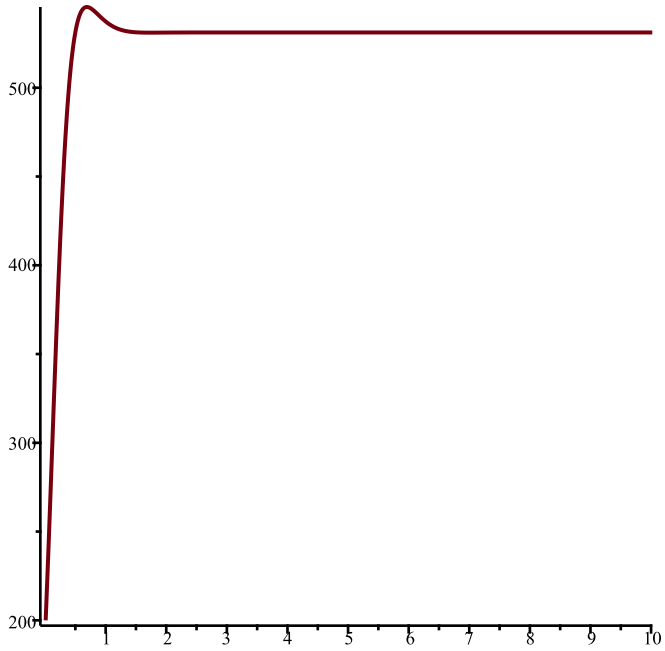


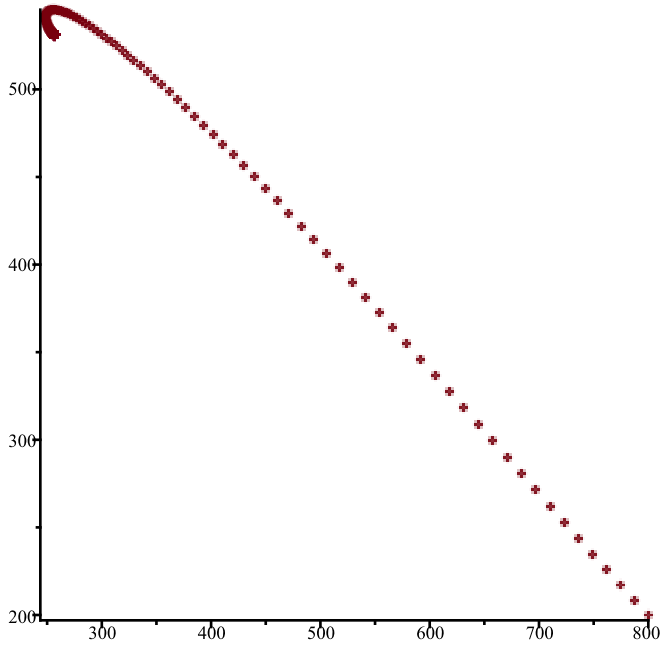

$$F := [-0.007800000000\, s\, i + 5000 - 5\, s - 5\, i,\ 0.007800000000\, s\, i - 2\, i]$$

$$\{[256.4102564,\ 531.1355311],\ [1000.,\ 0.]\}$$

$$\{[256.4102564,\ 531.1355311]\}$$






```
> #1ii
  N := 1000;
  v := 3;
  g := 6;
   b1 := 0.3 * (v/N);
   b2 := 0.9 * (v/N);
   b3 := 3.9 * (v/N) ;

  F := SIRS(s, i, b1, g, v, N);
  EquP(F, [s, i]);
  SEquP(F, [s, i]);
  TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1);
```

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TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2);
PhaseDiag(F, [s, i], [800, 200], 0.01, 10);

F := SIRS(s, i, b2, g, v, N);
EquP(F, [s, i]);
SEquP(F, [s, i]);
TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1);
TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2);
PhaseDiag(F, [s, i], [800, 200], 0.01, 10);

F := SIRS(s, i, b3, g, v, N);
EquP(F, [s, i]);
SEquP(F, [s, i]);
TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1);
TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2);
PhaseDiag(F, [s, i], [800, 200], 0.01, 10);
```

$$N := 1000$$

$$v := 3$$

$$g := 6$$

$$b1 := 0.0009000000000$$

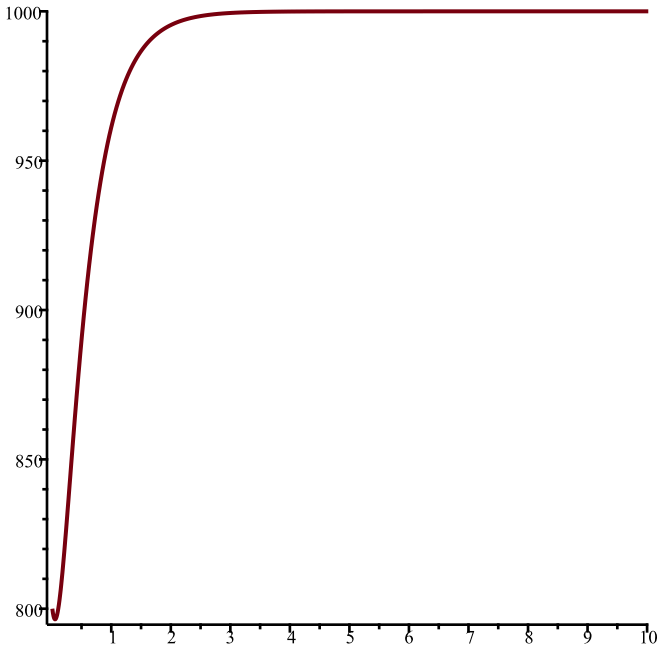
$$b2 := 0.002700000000$$

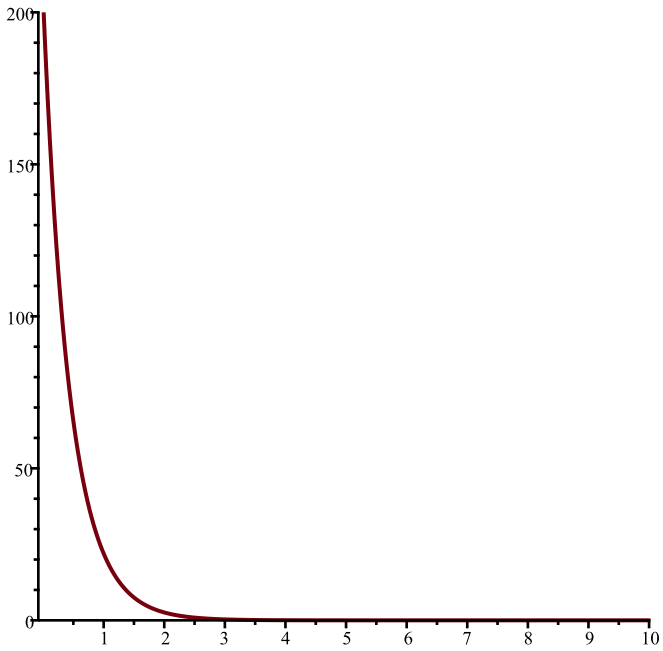
$$b3 := 0.01170000000$$

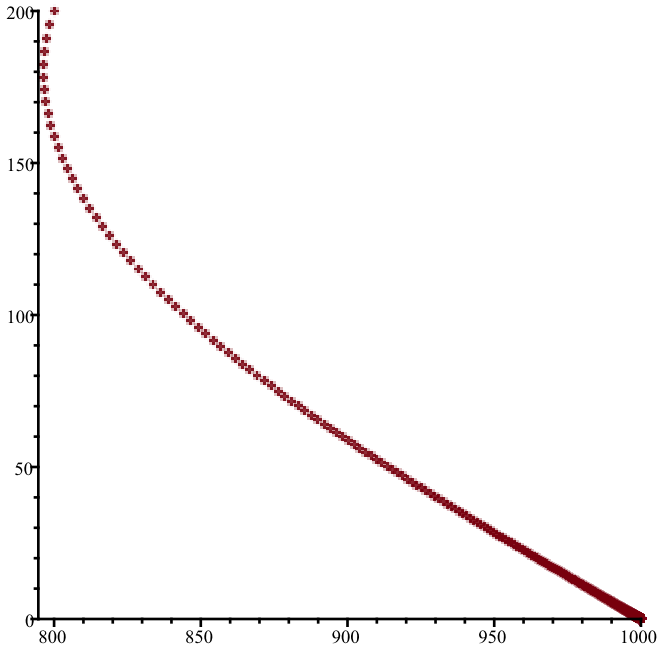
$$F := [-0.0009000000000\, s\, i + 6000 - 6\, s - 6\, i,\ 0.0009000000000\, s\, i - 3\, i]$$

$$\{[1000., 0.], [3333.333333, -1555.555556]\}$$

$$\{[1000., 0.]\}$$



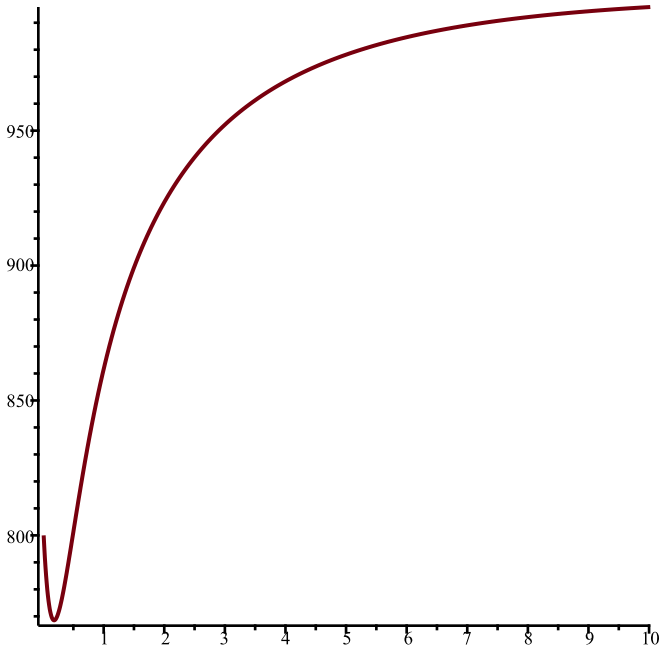



$$F := [-0.002700000000\, s\, i + 6000 - 6\, s - 6\, i,\ 0.002700000000\, s\, i - 3\, i]$$

$$\{[1000., 0.], [1111.111111, -74.07407407]\}$$

$$\{[1000., 0.]\}$$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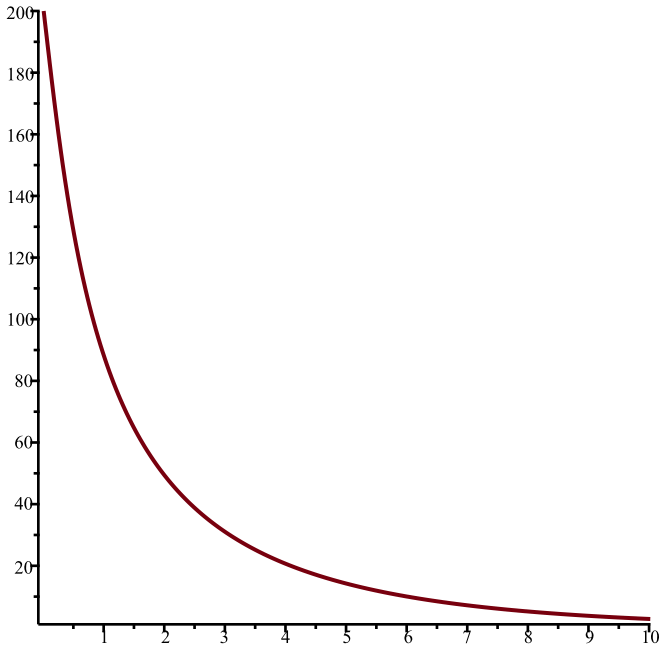
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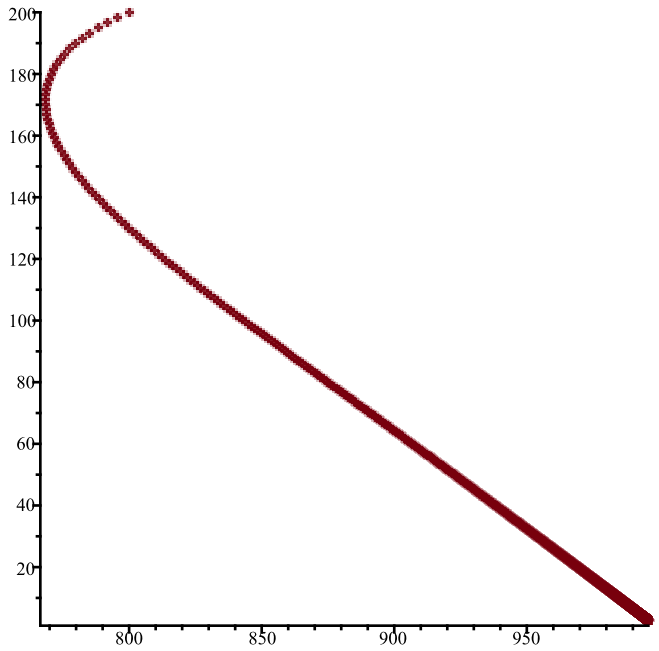
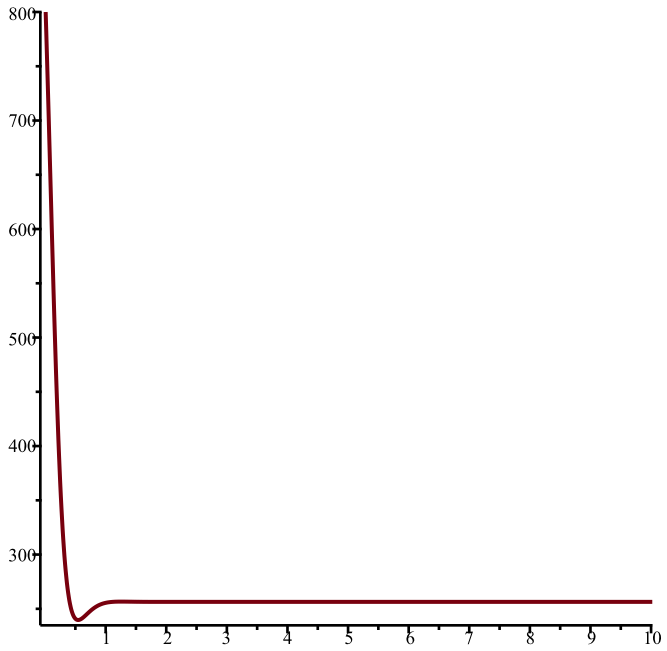
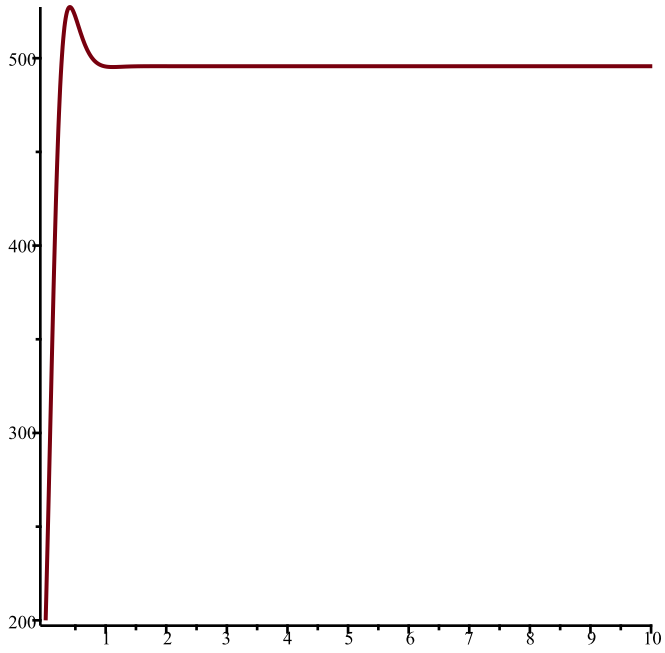


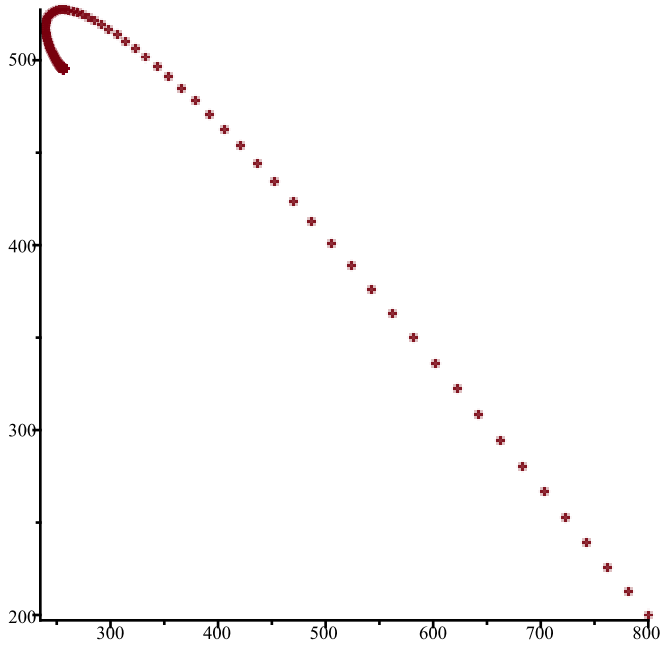

$$F := [-0.01170000000\, s\, i + 6000 - 6\, s - 6\, i,\ 0.01170000000\, s\, i - 3\, i]$$

$$\{[256.4102564,\ 495.7264957],\ [1000.,\ 0.]\}$$

$$\{[256.4102564,\ 495.7264957]\}$$






```
> #1iii
  N := 1000;
  v := 4;
  g := 1;
   b1 := 0.3 * (v/N);
   b2 := 0.9 * (v/N);
   b3 := 3.9 * (v/N);

  F := SIRS(s, i, b1, g, v, N);
  EquP(F, [s, i]);
  SEquP(F, [s, i]);
  TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1);
```

$$TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2);$$
$$PhaseDiag(F, [s, i], [800, 200], 0.01, 10);$$

$$F := SIRS(s, i, b2, g, v, N);$$
$$EquP(F, [s, i]);$$
$$SEquP(F, [s, i]);$$
$$TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1);$$
$$TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2);$$
$$PhaseDiag(F, [s, i], [800, 200], 0.01, 10);$$

$$F := SIRS(s, i, b3, g, v, N);$$
$$EquP(F, [s, i]);$$
$$SEquP(F, [s, i]);$$
$$TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1);$$
$$TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2);$$
$$PhaseDiag(F, [s, i], [800, 200], 0.01, 10);$$

$$N := 1000$$

$$v := 4$$

$$g := 1$$

$$b1 := 0.001200000000$$

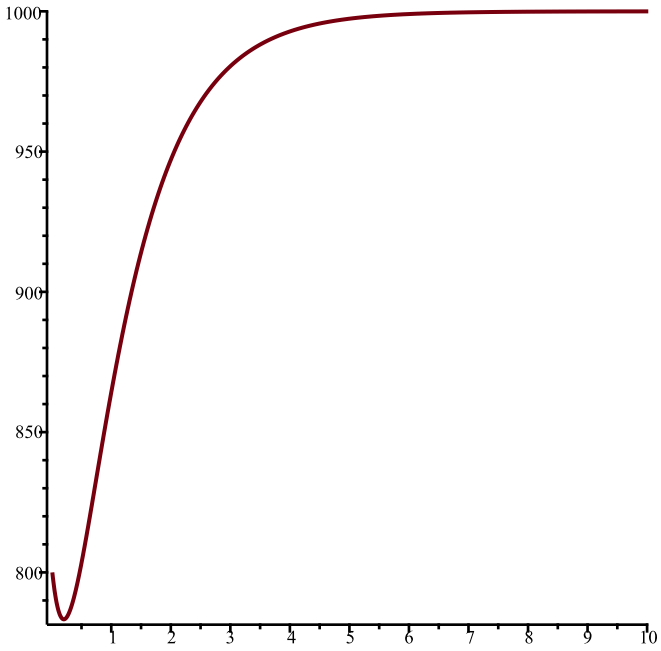
$$b2 := 0.003600000000$$

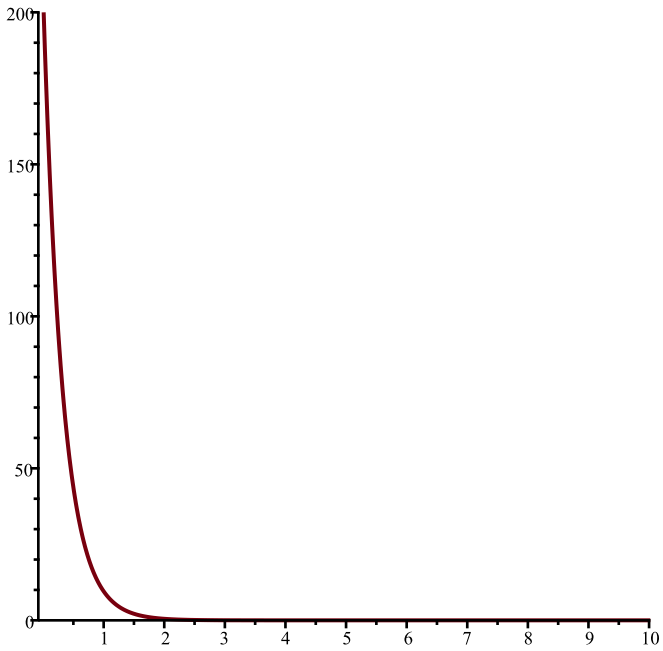
$$b3 := 0.01560000000$$

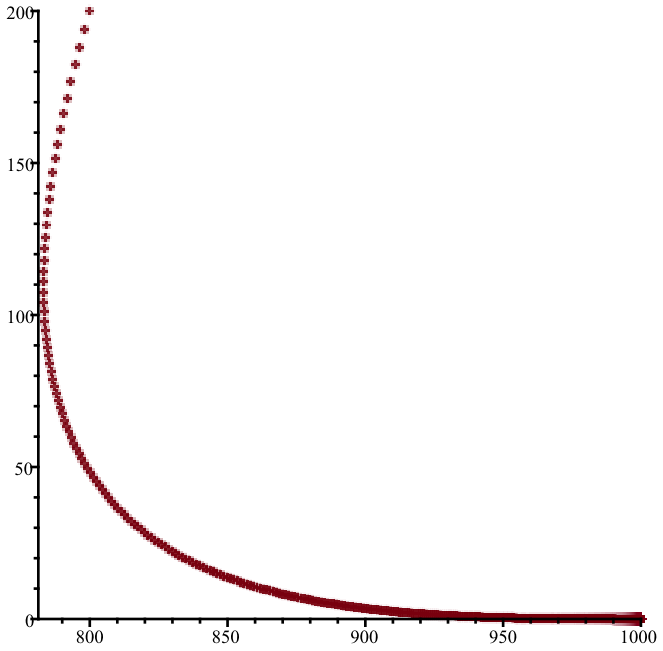
$$F := [-0.001200000000\, s\, i + 1000 - s - i,\ 0.001200000000\, s\, i - 4\, i]$$

$$\{[1000., 0.], [3333.333333, -466.6666667]\}$$

$$\{[1000., 0.]\}$$



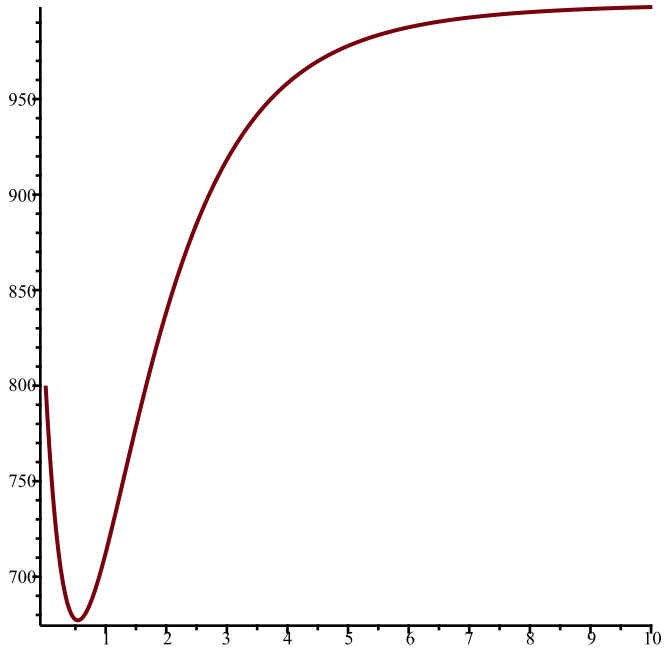



$$F := [-0.003600000000\, s\, i + 1000 - s - i,\ 0.003600000000\, s\, i - 4\, i]$$

$$\{[1000., 0.], [1111.111111, -22.22222222]\}$$

$$\{[1000., 0.]\}$$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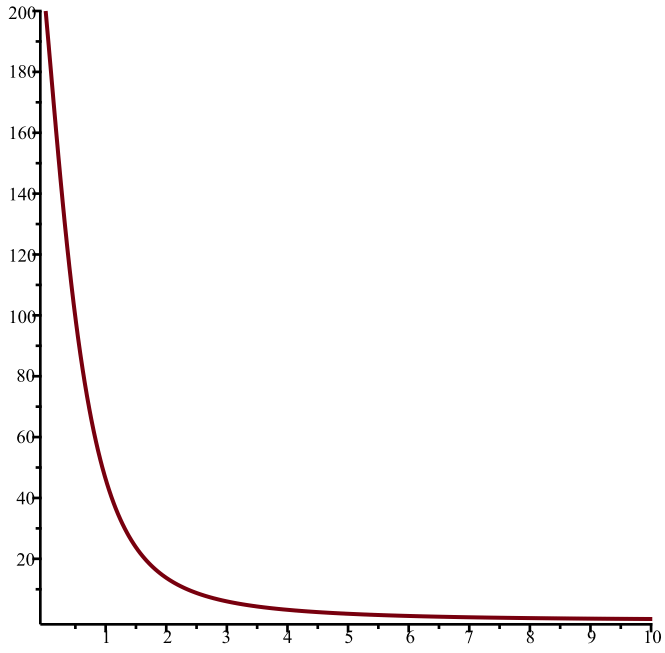
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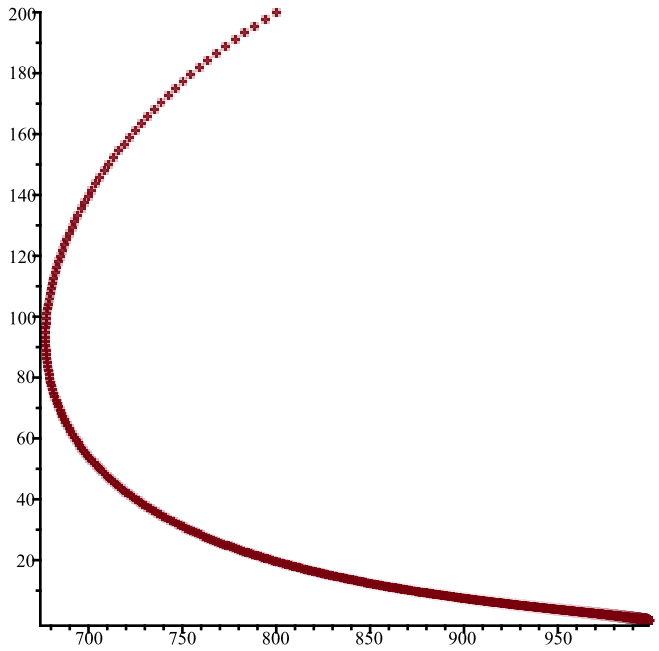
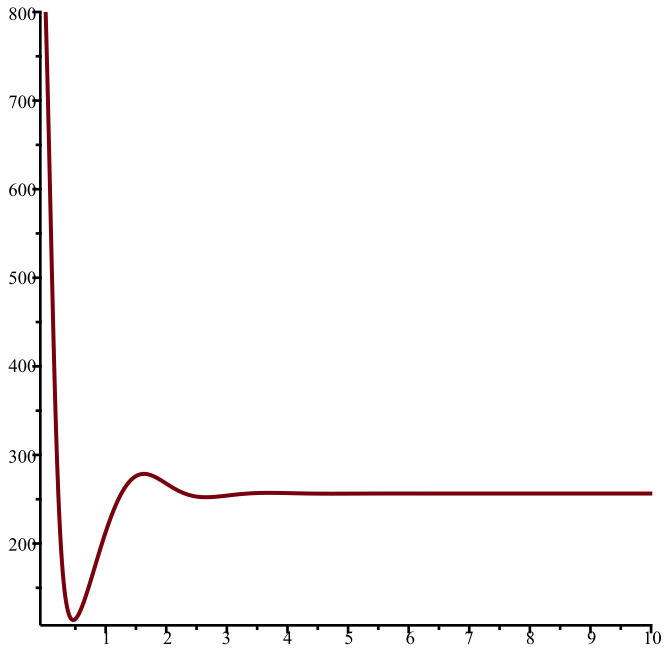
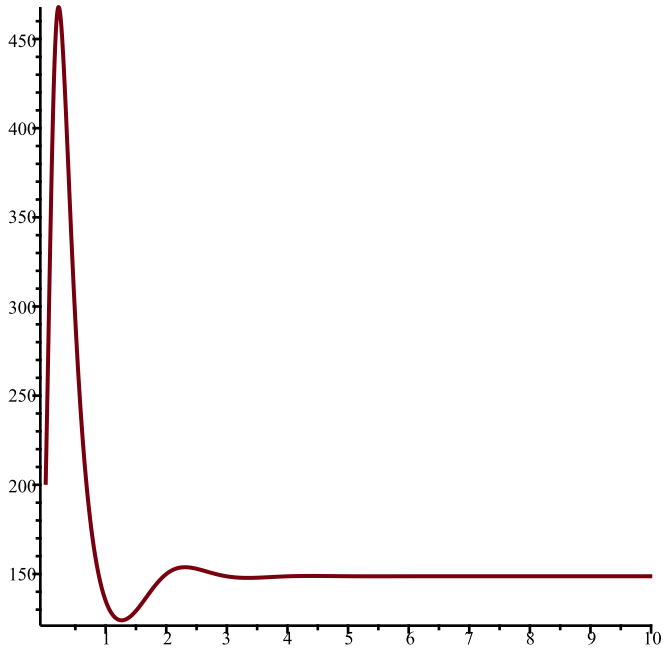


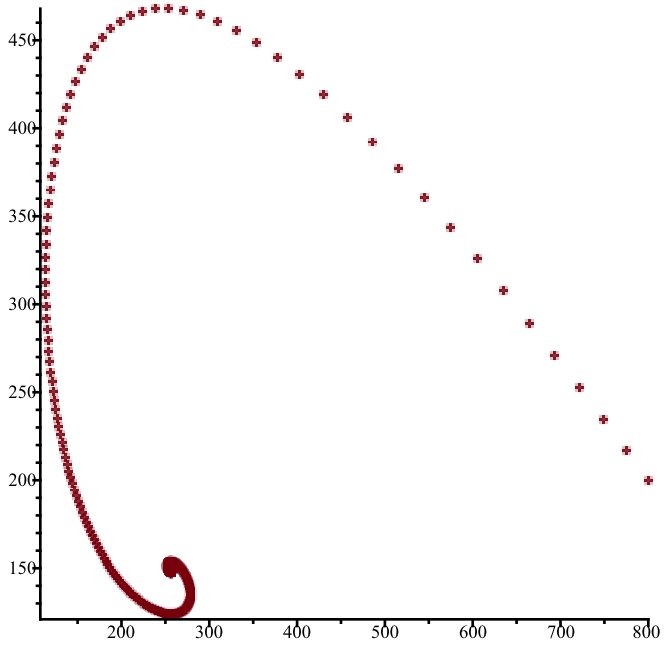

$$F := [-0.01560000000\, s\, i + 1000 - s - i,\ 0.01560000000\, s\, i - 4\, i]$$

$$\{[256.4102564,\ 148.7179487],\ [1000.,\ 0.]\}$$

$$\{[256.4102564,\ 148.7179487]\}$$






```
> #1iv
  N := 1000;
  v := 7;
  g := 10;
   b1 := 0.3 * (v/N);
   b2 := 0.9 * (v/N);
   b3 := 3.9 * (v/N);

  F := SIRS(s, i, b1, g, v, N);
  EquP(F, [s, i]);
  SEquP(F, [s, i]);
  TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1);
```

$$TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2);$$
$$PhaseDiag(F, [s, i], [800, 200], 0.01, 10);$$

$$F := SIRS(s, i, b2, g, v, N);$$
$$EquP(F, [s, i]);$$
$$SEquP(F, [s, i]);$$
$$TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1);$$
$$TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2);$$
$$PhaseDiag(F, [s, i], [800, 200], 0.01, 10);$$

$$F := SIRS(s, i, b3, g, v, N);$$
$$EquP(F, [s, i]);$$
$$SEquP(F, [s, i]);$$
$$TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1);$$
$$TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2);$$
$$PhaseDiag(F, [s, i], [800, 200], 0.01, 10);$$

$$N := 1000$$

$$v := 7$$

$$g := 10$$

$$b1 := 0.002100000000$$

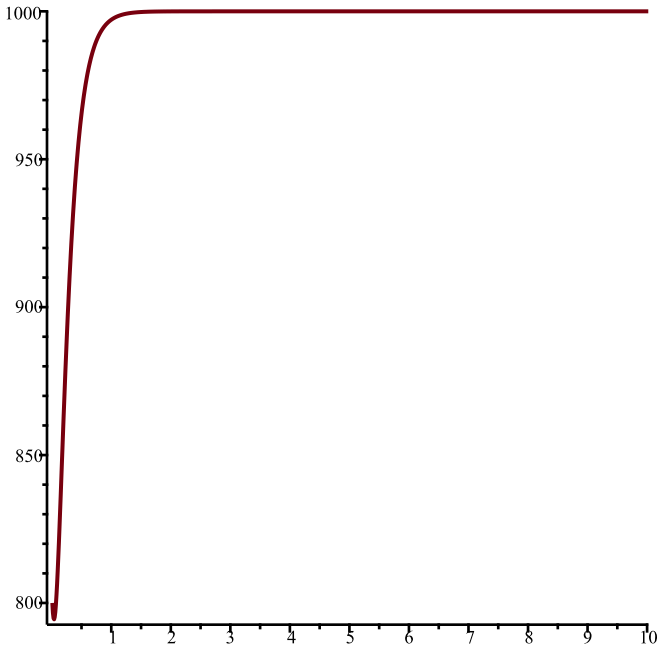
$$b2 := 0.006300000000$$

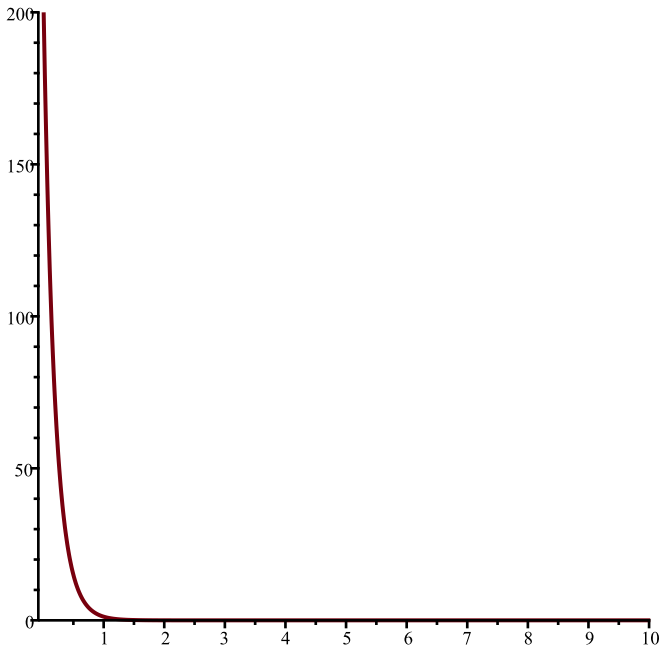
$$b3 := 0.02730000000$$

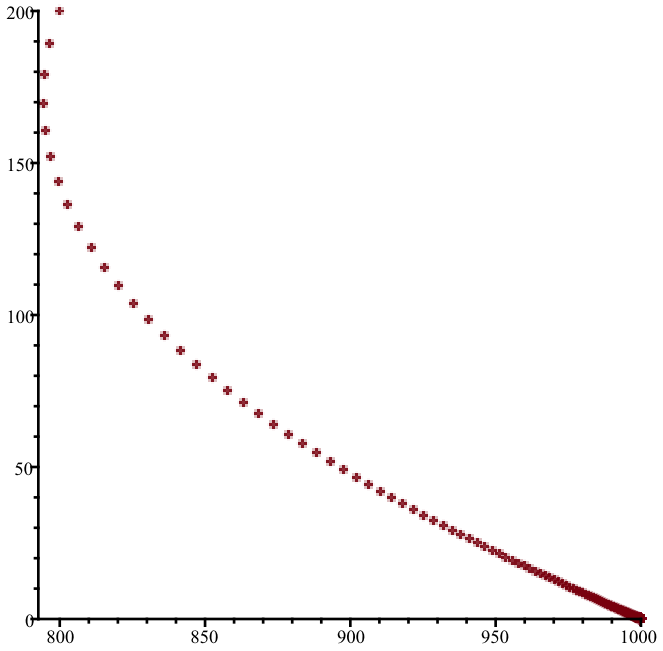
$$F := [-0.002100000000\, s\, i + 10000 - 10\, s - 10\, i,\ 0.002100000000\, s\, i - 7\, i]$$

$$\{[1000., 0.], [3333.333333, -1372.549020]\}$$

$$\{[1000., 0.]\}$$



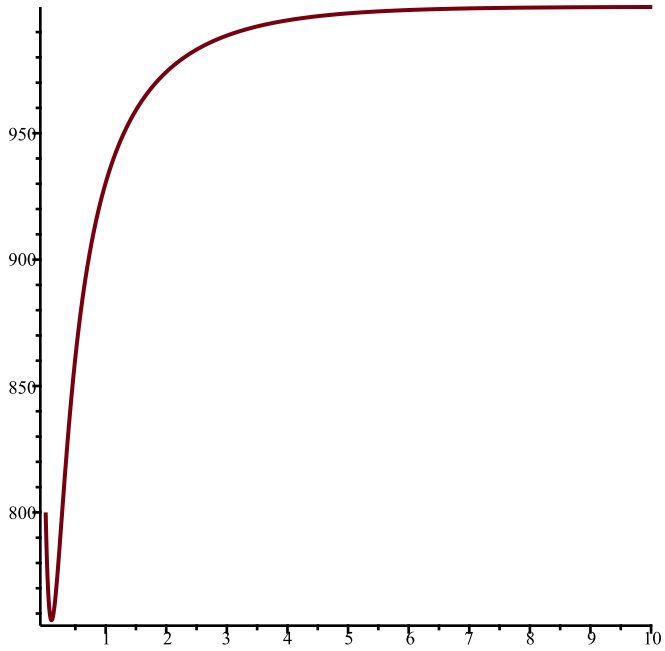



$$F := [-0.006300000000\, s\, i + 10000 - 10\, s - 10\, i,\ 0.006300000000\, s\, i - 7\, i]$$

$$\{[1000., 0.], [1111.111111, -65.35947712]\}$$

$$\{[1000., 0.]\}$$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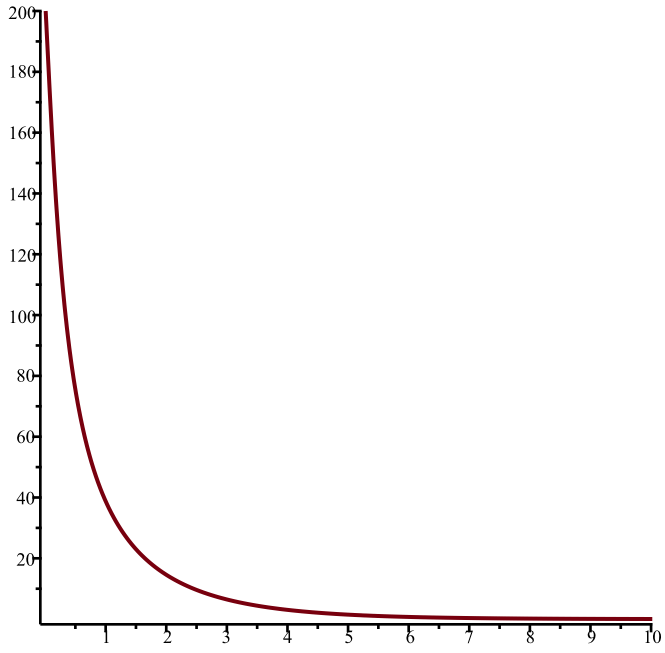
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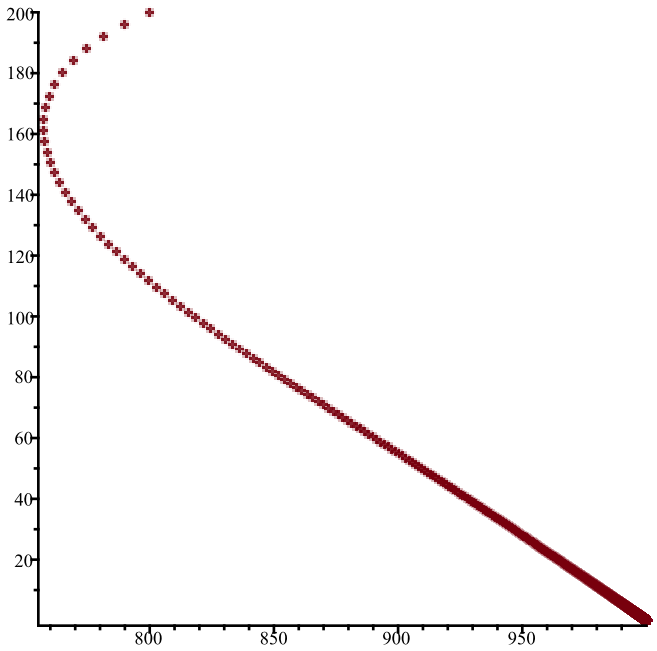
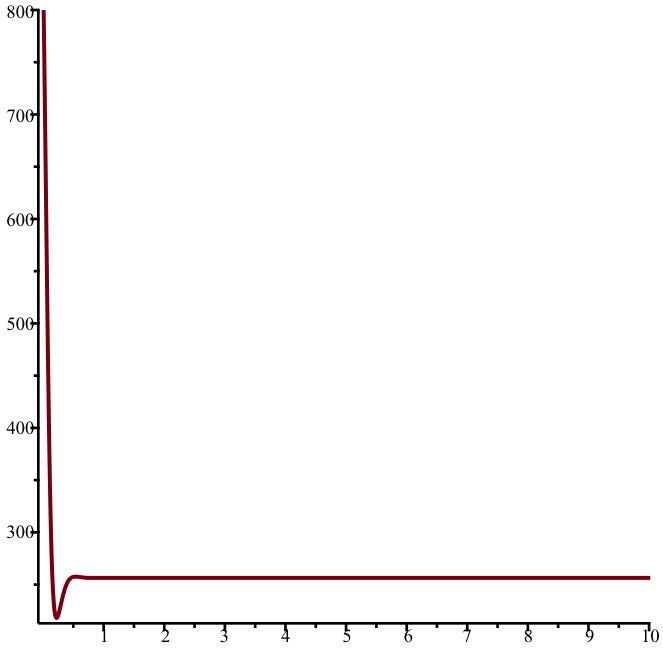
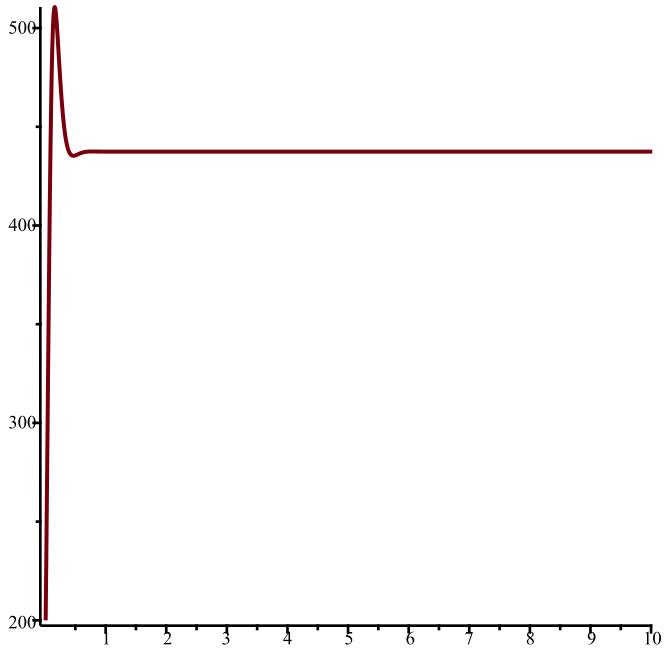


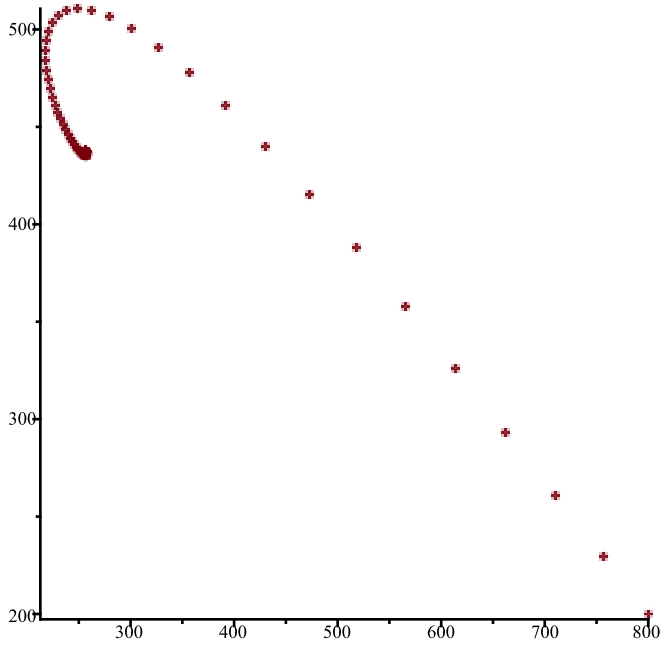

$$F := [-0.02730000000\, s\, i + 10000 - 10\, s - 10\, i,\ 0.02730000000\, s\, i - 7\, i]$$

$$\{[256.4102564,\ 437.4057315],\ [1000.,\ 0.]\}$$

$$\{[256.4102564,\ 437.4057315]\}$$






```
> #2i
  F1 := RandNice([x, y], 3);
  EquP(F1, [x, y]);
  SEquP(F1, [x, y]);
  F2 := RandNice([x, y], 3);
  EquP(F2, [x, y]);
  SEquP(F2, [x, y]);
  F3 := RandNice([x, y], 3);
  EquP(F3, [x, y]);
  SEquP(F3, [x, y]);
```

$$F1 := [(2 - 2x - 3y)(2 - x - 3y), (1 - x - 2y)(3 - 2x - 2y)]$$

$$\left\{[-1, 1], [1, 0], \left[\frac{5}{2}, -1\right], \left[\frac{5}{4}, \frac{1}{4}\right]\right\}$$

$$\{[1., 0.]\}$$

$$F2 := [(1 - 2x - y)\ (1 - 2x - 2y), (3 - 3x - 2y)\ (2 - x - y)]$$

$$\left\{[-1, 3], \left[2, -\frac{3}{2}\right]\right\}$$

$$\{[2., -1.500000000]\}$$

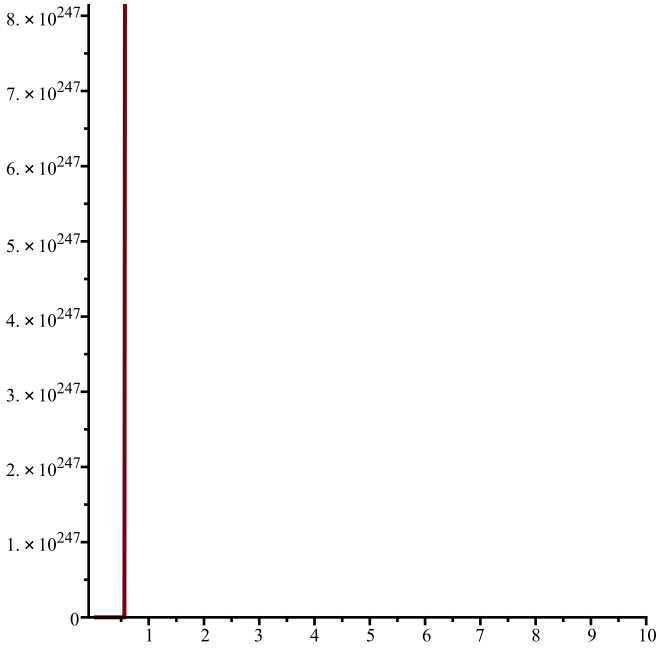
$$F3 := [(3 - x - 2y)\ (1 - 2x - 3y), (3 - 3x - 2y)\ (1 - 3x - 3y)]$$

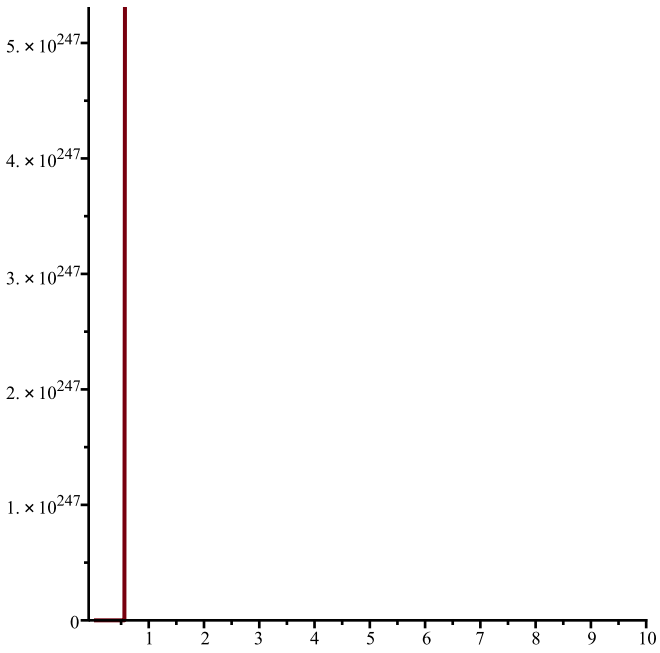
$$\left\{\left[0, \frac{1}{3}\right], \left[0, \frac{3}{2}\right], \left[-\frac{7}{3}, \frac{8}{3}\right], \left[\frac{7}{5}, -\frac{3}{5}\right]\right\}$$

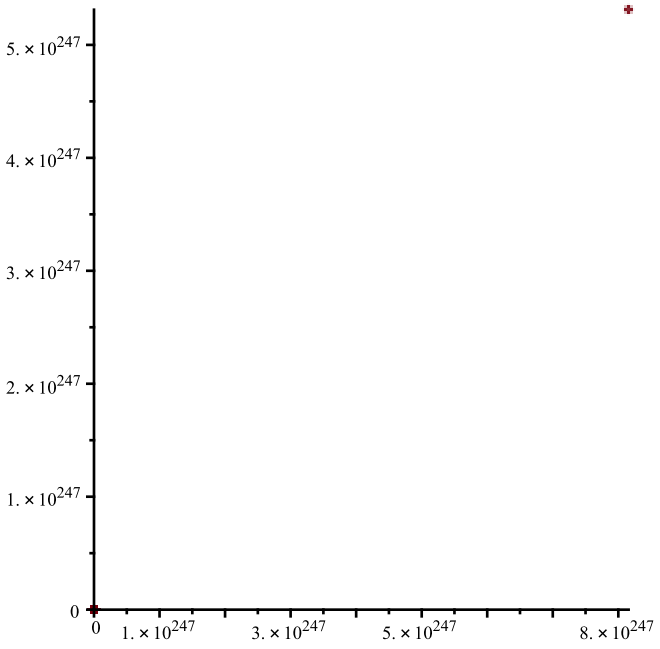
$$\{[-2.333333333, 2.666666667], [1.400000000, -0.6000000000]\}$$ **(2)**

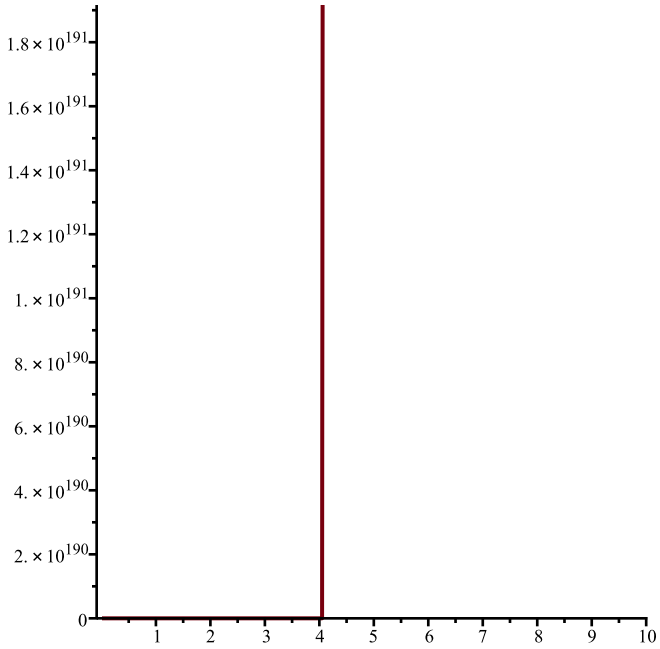
> *#2ii*

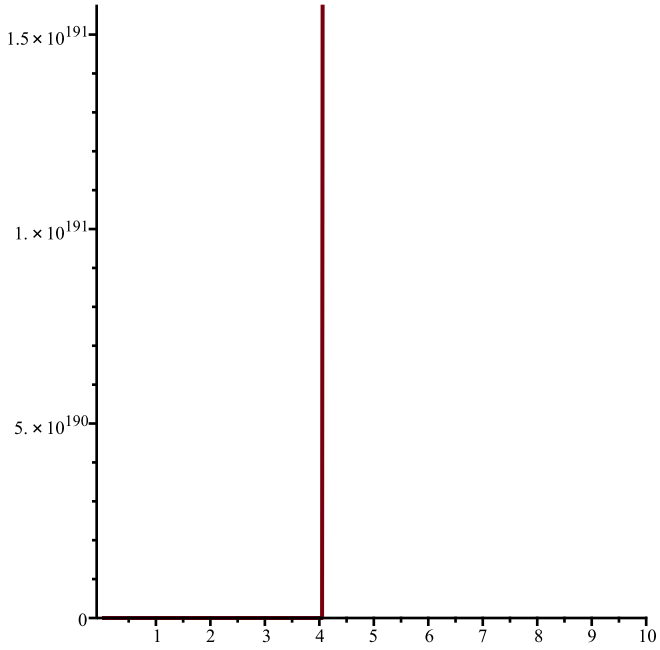
```
TimeSeries(F1, [x, y], [1, 0.5], 0.01, 10, 1);
TimeSeries(F1, [x, y], [1, 0.5], 0.01, 10, 2);
PhaseDiag(F1, [x, y], [1, 0.5], 0.01, 10);
TimeSeries(F2, [x, y], [-2, 2], 0.01, 10, 1);
TimeSeries(F2, [x, y], [-2, 2], 0.01, 10, 2);
PhaseDiag(F2, [x, y], [-2, 2], 0.01, 10);
TimeSeries(F3, [x, y], [-2, 3], 0.01, 10, 1);
TimeSeries(F3, [x, y], [-2, 3], 0.01, 10, 2);
PhaseDiag(F3, [x, y], [-2, 3], 0.01, 10);
```

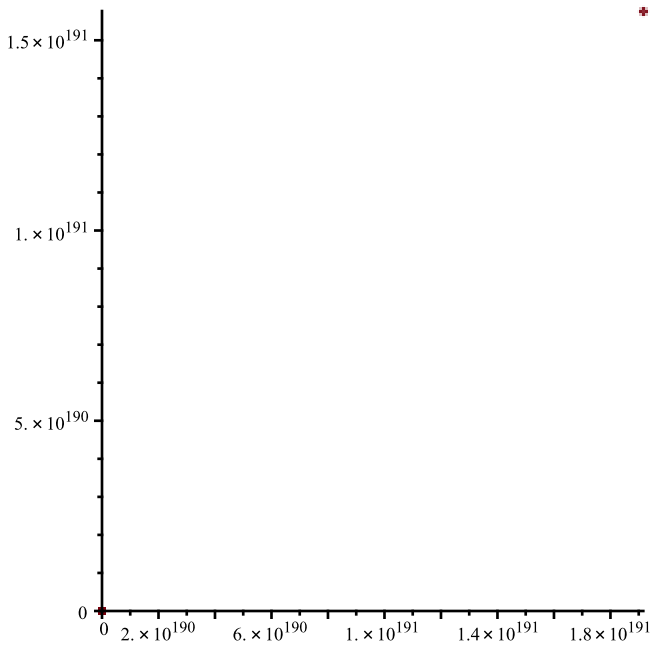


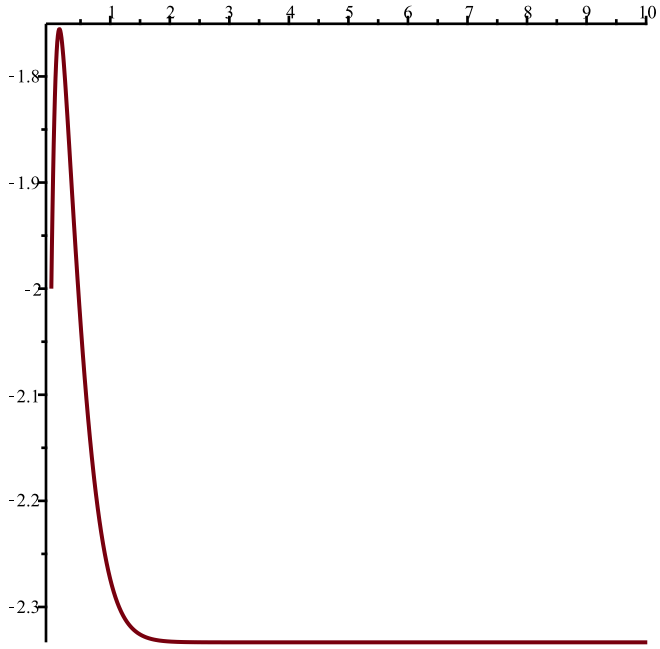

5. × 10^247
4. × 10^247
3. × 10^247
2. × 10^247
1. × 10^247
0
1
2
3
4
5
6
7
8
9
10

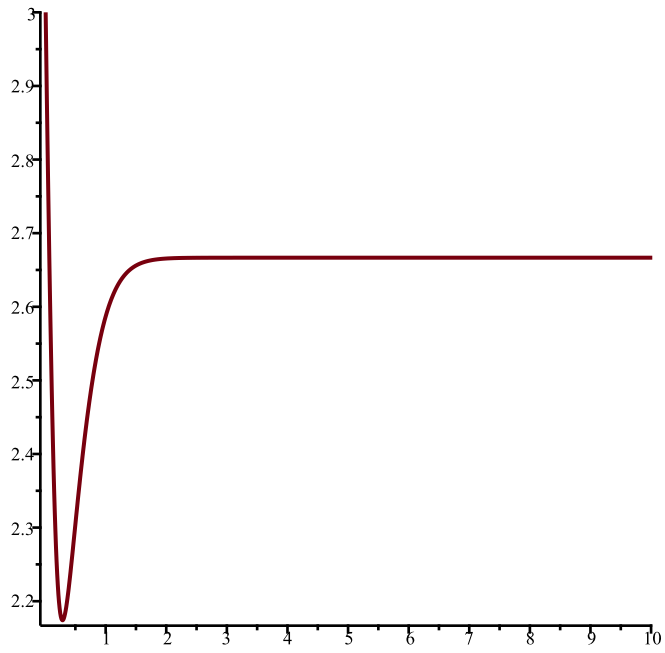

5. × 10^247
4. × 10^247
3. × 10^247
2. × 10^247
1. × 10^247
0
0
1. × 10^247
3. × 10^247
5. × 10^247
8. × 10^247

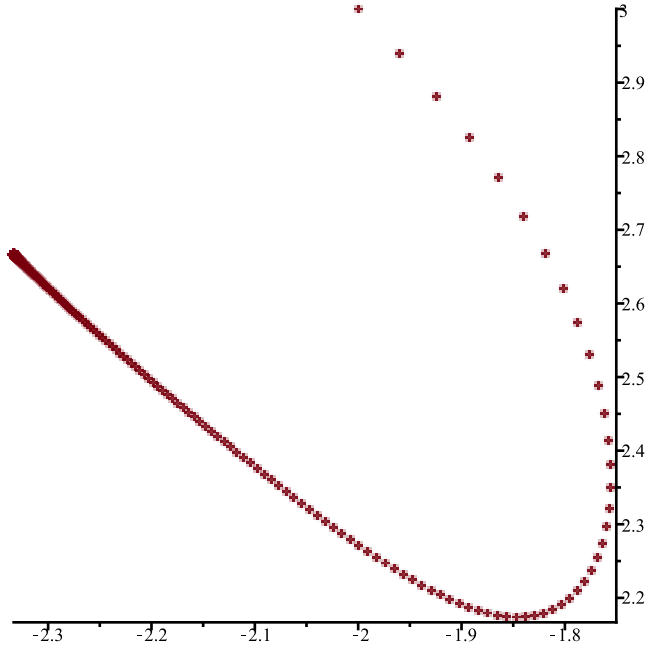

1.8 × 10^191
1.6 × 10^191
1.4 × 10^191
1.2 × 10^191
1. × 10^191
8. × 10^190
6. × 10^190
4. × 10^190
2. × 10^190
0
1
2
3
4
5
6
7
8
9
10


1.5 × 10^191
1. × 10^191
5. × 10^190
0
1
2
3
4
5
6
7
8
9
10


1.5 × 10^191
1. × 10^191
5. × 10^190
0
0
2. × 10^190
6. × 10^190
1. × 10^191
1.4 × 10^191
1.8 × 10^191


1
2
3
4
5
6
7
8
9
10
-1.8
-1.9
-2
-2.1
-2.2
-2.3




```
> #3
  #x(n) = (3 + x(n-2) +x(n-3) +x(n-4))/(1 +x(n-1) +x(n-3));
  Orbk(4, z, (3 + z[2] + z[3] + z[4]) / (1 + z[1] + z[3]), [0., 3., 6., 7.], 2000,
   2005);
  Orbk(4, z, (3 + z[2] + z[3] + z[4]) / (1 + z[1] + z[3]), [0., 3., 4., 1.], 2000,
  2005);
  Orbk(4, z, (3 + z[2] + z[3] + z[4]) / (1 + z[1] + z[3]), [1., 2., 7., 6.], 2000,
  2005);#Closest Iteration however The value at the orbit
  oscillates.
  g := (3 + z[2] + z[3] + z[4]) / (1 + z[1] + z[3]);
  G := ToSys(4, z, g);
  SFP(G);
```

$$[2.943675310, 1.216134420, 2.943675310, 1.216134420, 2.943675310, 1.216134420]$$
$$[1.365333221, 2.522342327, 1.365333221, 2.522342327, 1.365333221, 2.522342327]$$
$$[2.288145505, 1.478667561, 2.288145505, 1.478667561, 2.288145505, 1.478667561]$$

$$oscillates \cdot 10 := \frac{3 + z_2 + z_3 + z_4}{1 + z_1 + z_3}$$

$$G := \left[10, z_1, z_2, z_3\right], \left[z_1, z_2, z_3, z_4\right]$$

$$\{[10., 10., 10., 10.]\} \quad \textbf{(3)}$$