#OK to post
#Julian Herman, Assignment 20, November 15th, 2021

```
> read `/Users/julianherman/Documents/Rutgers/Fall 2021/Dynamical Models In Biology/HW/DMB.txt`
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

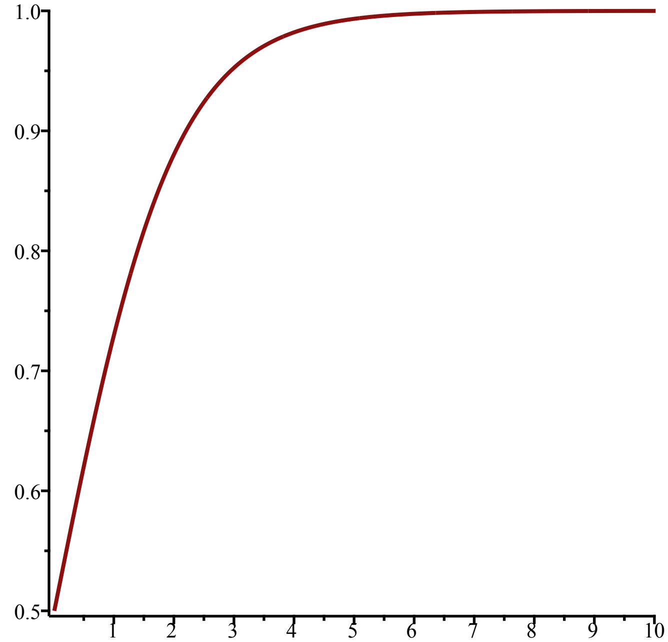
For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

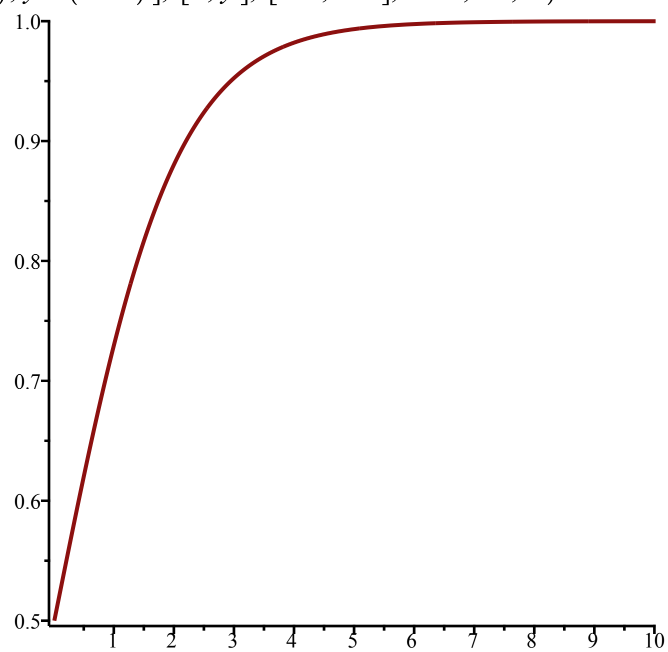
```
> Help(TimeSeries)
```

TimeSeries(F,x,pt,h,A,i): Inputs a transformation F in the list of variables x
The time-series of x[i] vs. time of the Dynamical system approximating the the autonomous continuous dynamical process
dx/dt=F[1](x(t)) by a discrete time dynamical system with step-size h from t=0 to t=A
Try:
TimeSeries([x(1-y),y*(1-x)],[x,y],[0.5,0.5], 0.01, 10,1);* **(2)**

> $TimeSeries([x*(1-y), y*(1-x)], [x, y], [0.5, 0.5], 0.01, 10, 1)$



> $TimeSeries([x*(1-y), y*(1-x)], [x, y], [0.5, 0.5], 0.01, 10, 2)$



> $Help(PhaseDiag)$

PhaseDiag(F,x,pt,h,A): Inputs a transformation F in the list of variables x (of length 2), i.e. a mapping from R^2 to R^2 gives the

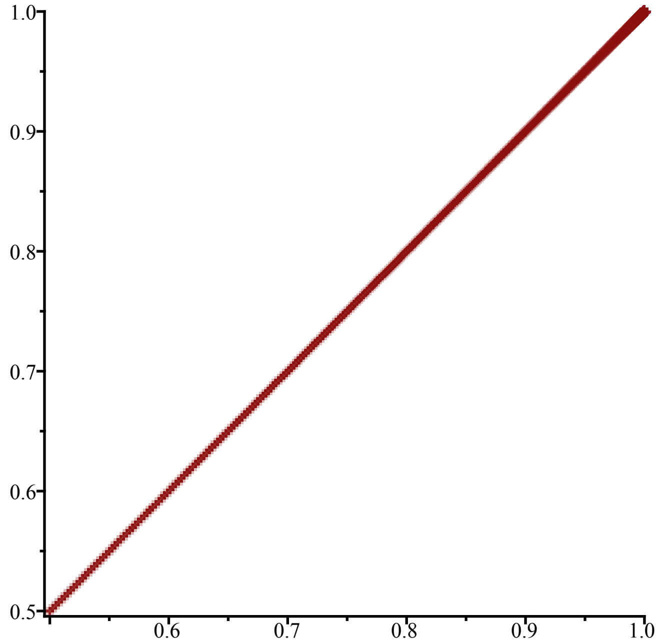
The phase diagram of the solution with initial condition x(0)=pt

dx/dt=F[1](x(t)) by a discrete time dynamical system with step-size h from t=0 to t=A

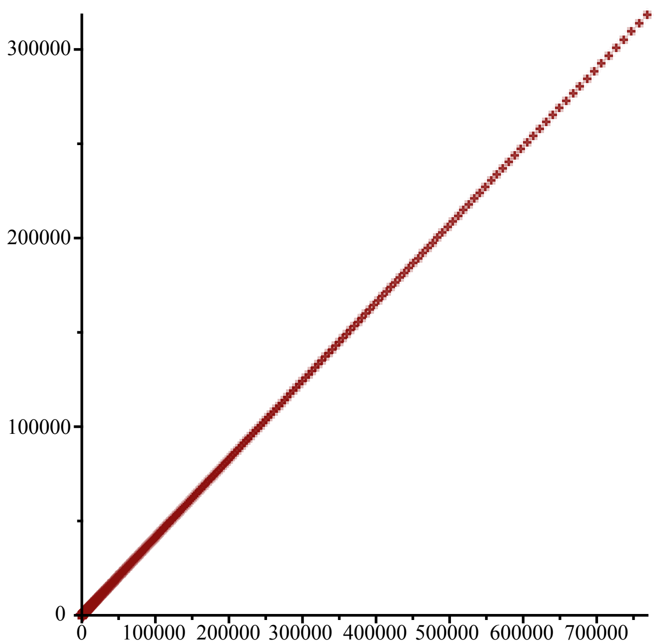
Try:

PhaseDiag([x(1-y),y*(1-x)],[x,y],[0.5,0.5], 0.01, 10);* **(3)**

> $PhaseDiag([x*(1-y), y*(1-x)], [x, y], [0.5, 0.5], 0.01, 10)$



> $PhaseDiag([x+y, x-y], [x, y], [0.5, 0.5], 0.01, 10)$



> #1)

> #i) $\nu = 2, \gamma = 5, \beta = 0.3 \cdot \frac{\nu}{1000} = 0.3 \cdot \left(\frac{2}{1000}\right) = 0.0006000000000$

> $F := subs(\{\nu = 2, \gamma = 5, \beta = 0.0006000000000\}, SIRS(s, i, \text{beta}, \text{gamma}, \text{nu}, 1000))$

$$F := [-0.0006000000000\, s\, i + 5000 - 5\, s - 5\, i, 0.0006000000000\, s\, i - 2\, i] \quad \textbf{(4)}$$

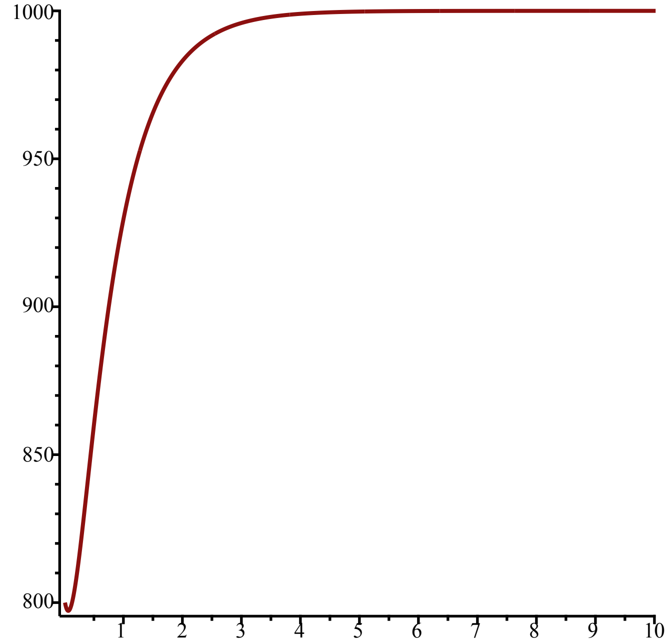
> $EquP(F, [s, i])$

$$\{[1000., 0.], [3333.333333, -1666.666667]\} \quad \textbf{(5)}$$

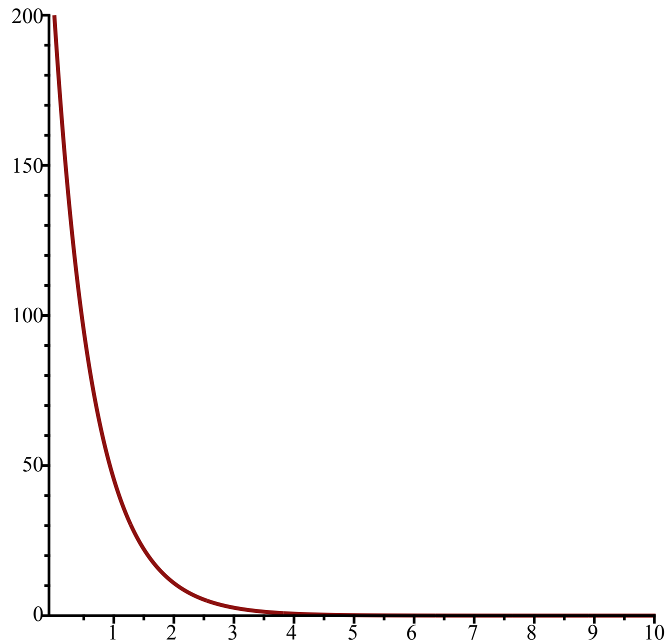
> $SEquP(F, [s, i])$

$$\{[1000., 0.]\} \quad \textbf{(6)}$$

> $TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1)$

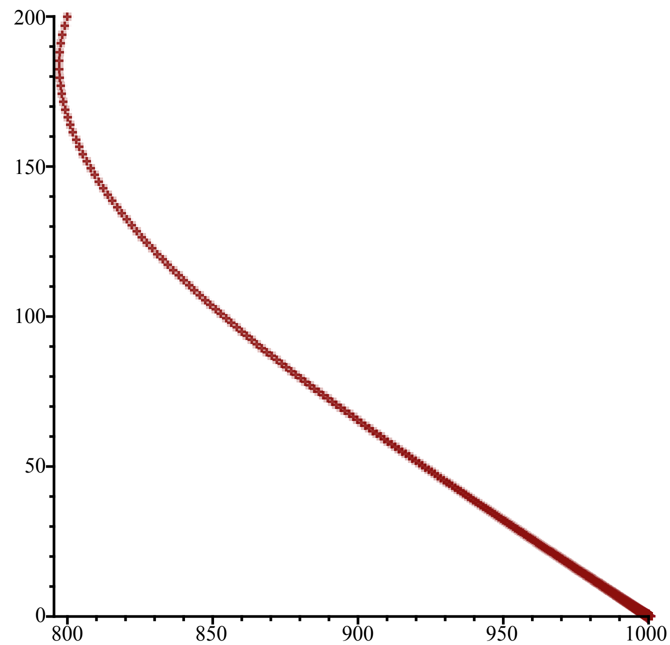


> $TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2)$



> *#The horiztonal asymptote of the susceptible occurs at S=1000 and the horizontal asymptote of the infected occurs at I=0, which corresponds to the STABLE EQ point [1000,0].*

> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10)$



> $\#\nu = 2, \gamma = 5, \beta = 0.9 \cdot \frac{\nu}{1000} = \frac{0.9}{500} = 0.001800000000$

> $F := subs(\{\nu = 2, \gamma = 5, \beta = 0.001800000000\}, SIRS(s, i, \text{beta}, \text{gamma}, \text{nu}, 1000))$

$$F := [-0.001800000000\, s\, i + 5000 - 5\, s - 5\, i, 0.001800000000\, s\, i - 2\, i] \quad \textbf{(7)}$$

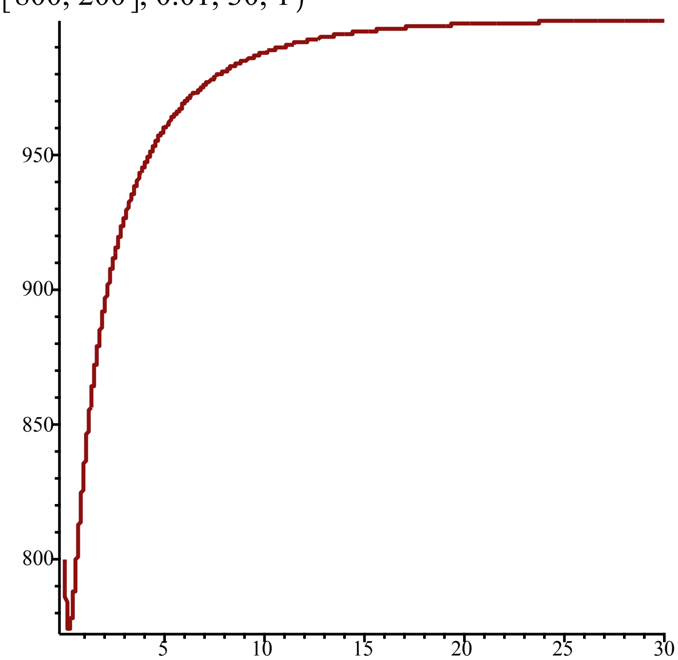
> $EquP(F, [s, i])$

$$\{[1000., 0.], [1111.111111, -79.36507937]\} \quad \textbf{(8)}$$

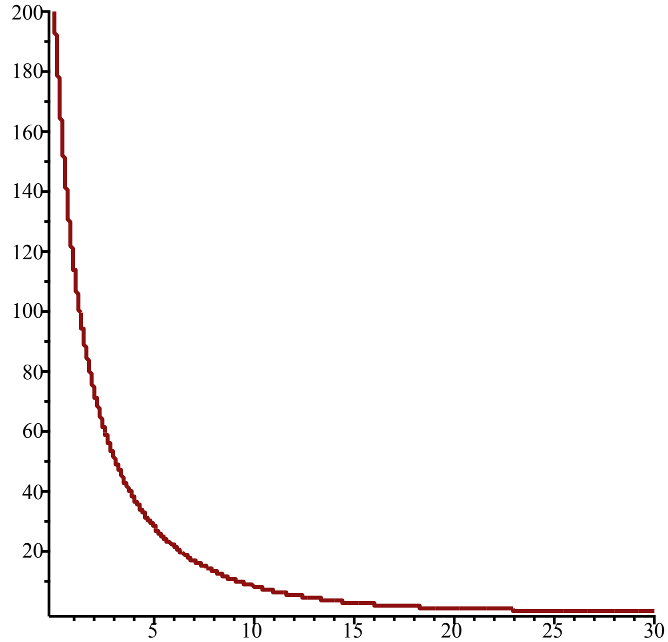
> $SEquP(F, [s, i])$

$$\{[1000., 0.]\} \quad \textbf{(9)}$$

> $TimeSeries(F, [s, i], [800, 200], 0.01, 30, 1)$

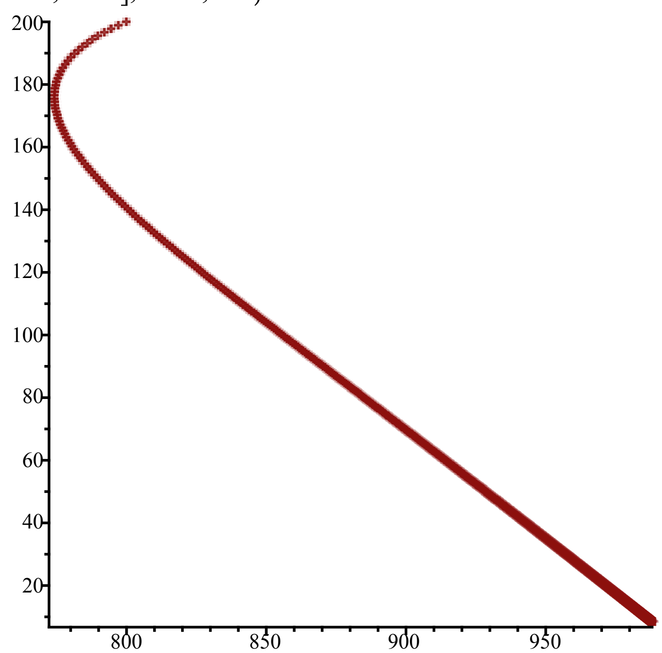


> $TimeSeries(F, [s, i], [800, 200], 0.01, 30, 2)$



> *#The horiztonal asymptote of the susceptible occurs at S=1000 and the horizontal asymptote of the infected occurs at I=0, which corresponds to the STABLE EQ point [1000,0].*

> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10)$



> $\#\nu = 2, \gamma = 5, \beta = 3.9 \cdot \frac{\nu}{1000} = \frac{3.9}{500} = 0.007800000000$

> $F := subs(\{\nu = 2, \gamma = 5, \beta = 0.007800000000\}, SIRS(s, i, \text{beta}, \text{gamma}, \text{nu}, 1000))$

$$F := [-0.007800000000\, s\, i + 5000 - 5\, s - 5\, i, 0.007800000000\, s\, i - 2\, i] \quad \textbf{(10)}$$

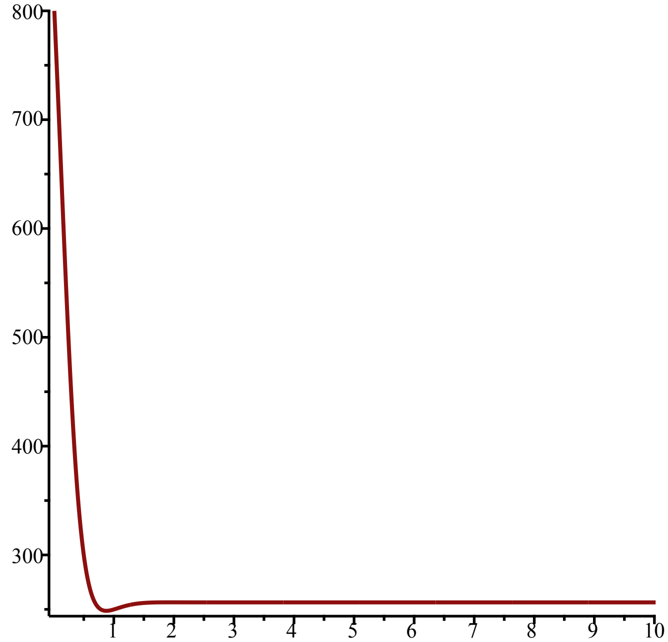
> $EquP(F, [s, i])$

$$\{[256.4102564, 531.1355311], [1000., 0.]\} \quad \textbf{(11)}$$

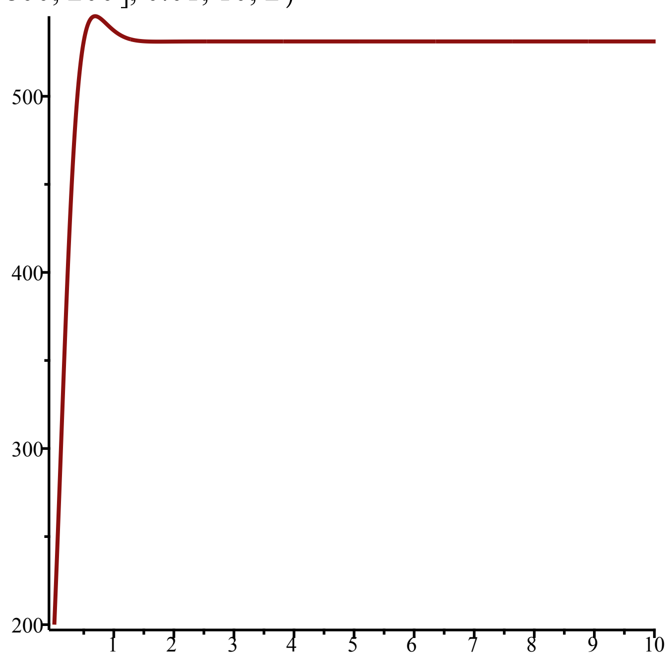
> $SEquP(F, [s, i])$

$$\{[256.4102564, 531.1355311]\} \quad \textbf{(12)}$$

> *TimeSeries*(*F*, [*s*, *i*], [800, 200], 0.01, 10, 1)

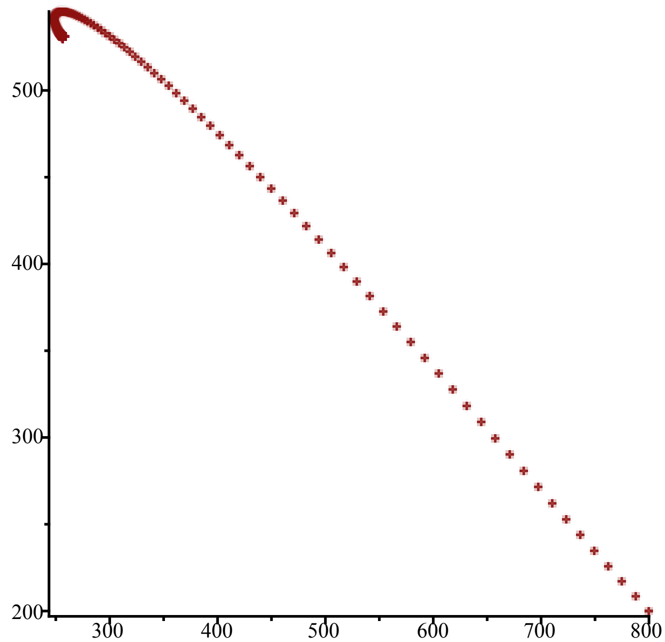


> *TimeSeries*(*F*, [*s*, *i*], [800, 200], 0.01, 10, 2)



> *#The horiztonal asymptote of the susceptible occurs at about S=256 and the horizontal asymptote of the infected occurs at about I=531, which corresponds to the STABLE EQ point* [256.4102564, 531.1355311]

> *PhaseDiag*(*F*, [*s*, *i*], [800, 200], 0.01, 10)



> #ii) $\nu = 3, \gamma = 6, \beta = 0.3 \cdot \frac{\nu}{1000} = 0.0009$

> $F := subs(\{\nu = 3, \gamma = 6, \beta = 0.0009\}, SIRS(s, i, \text{beta}, \text{gamma}, \text{nu}, 1000))$

$$F := [-0.0009\, s\, i + 6000 - 6\, s - 6\, i, 0.0009\, s\, i - 3\, i] \quad \textbf{(13)}$$

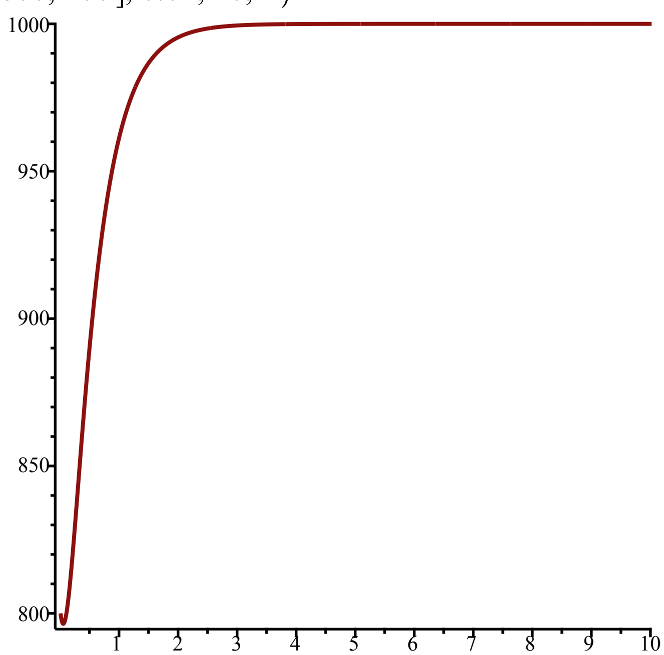
> $EquP(F, [s, i])$

$$\{[1000., 0.], [3333.333333, -1555.555556]\} \quad \textbf{(14)}$$

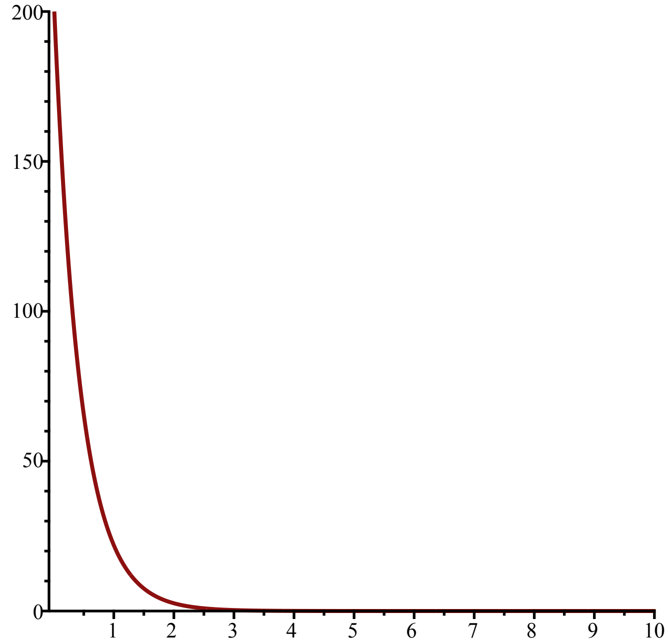
> $SEquP(F, [s, i])$

$$\{[1000., 0.]\} \quad \textbf{(15)}$$

> $TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1)$

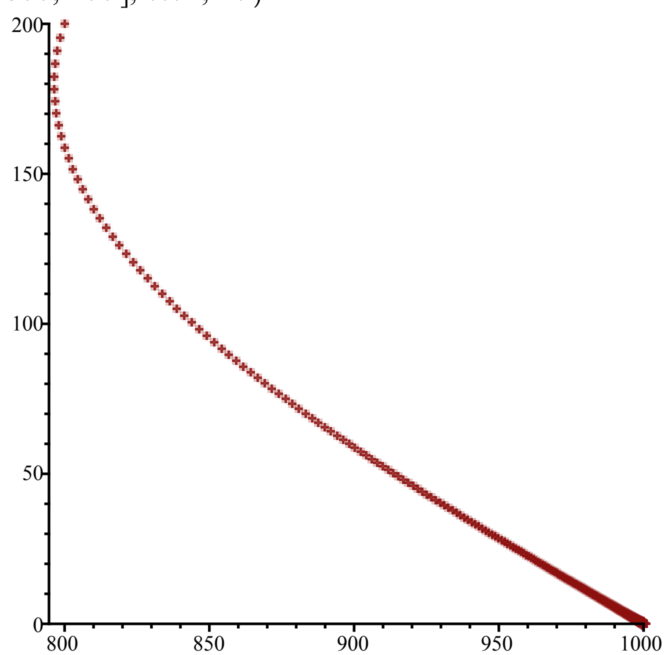


> $TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2)$



> *#The horiztonal asymptote of the susceptible occurs at about S=1000 and the horizontal asymptote of the infected occurs at about I=0, which corresponds to the STABLE EQ point* [1000, 0]

> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10)$



> $\#\nu = 3, \gamma = 6, \beta = 0.9 \cdot \frac{\nu}{1000} = 0.0027$

> $F := subs(\{\nu = 3, \gamma = 6, \beta = 0.0027\}, SIRS(s, i, \text{beta}, \text{gamma}, \text{nu}, 1000))$

$$F := [-0.0027\, s\, i + 6000 - 6\, s - 6\, i, 0.0027\, s\, i - 3\, i] \quad (16)$$

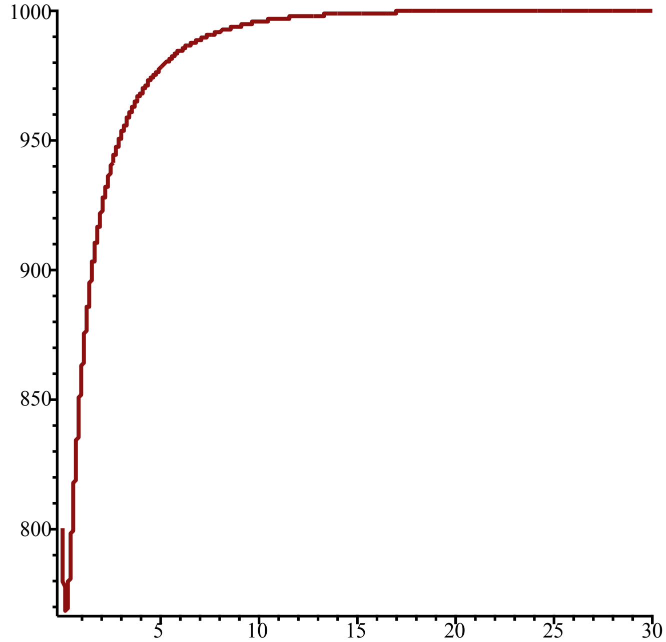
> $EquP(F, [s, i])$

$$\{[1000., 0.], [1111.111111, -74.07407407]\} \quad (17)$$

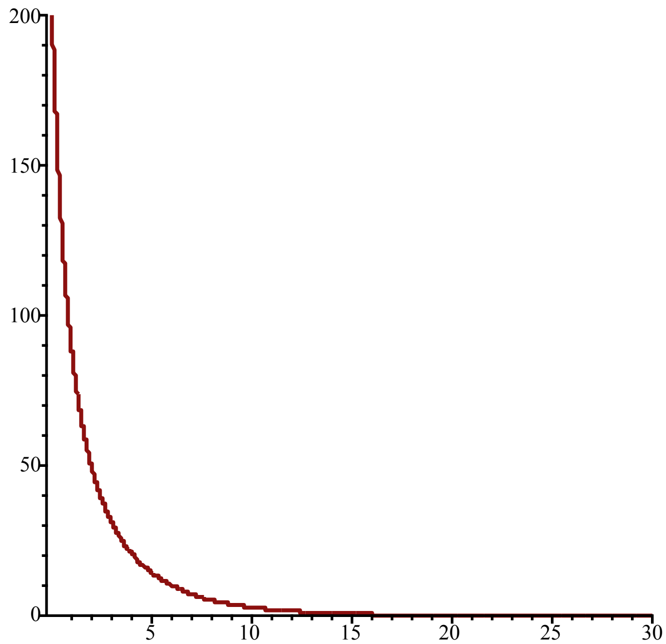
> $SEquP(F, [s, i])$

$$\{[1000., 0.]\} \quad (18)$$

> $TimeSeries(F, [s, i], [800, 200], 0.01, 30, 1)$

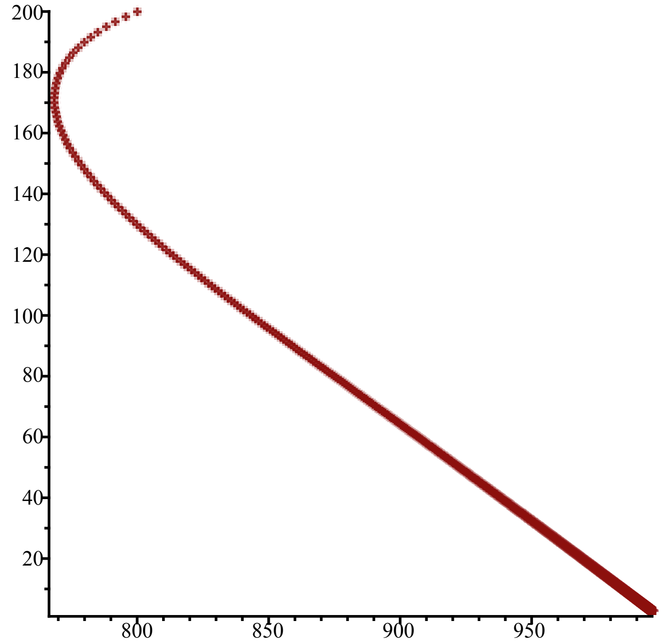


> $TimeSeries(F, [s, i], [800, 200], 0.01, 30, 2)$



> *#The horiztonal asymptote of the susceptible occurs at about S=1000 and the horizontal asymptote of the infected occurs at about I=0, which corresponds to the STABLE EQ point* $[1000, 0]$

> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10)$



> #ν = 3, γ = 6, β = 3.9 · $\frac{\nu}{1000}$ = 0.0117

> F := subs({ν = 3, γ = 6, β = 0.0117}, SIRS(s, i, beta, gamma, nu, 1000))

$$F := [-0.0117\, s\, i + 6000 - 6\, s - 6\, i, 0.0117\, s\, i - 3\, i] \quad \textbf{(19)}$$

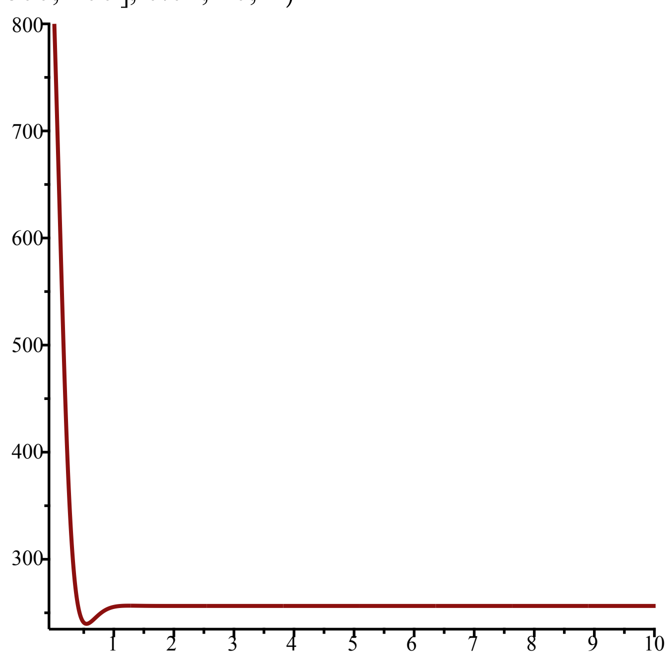
> EquP(F, [s, i])

$$\{[256.4102564, 495.7264957], [1000., 0.]\} \quad \textbf{(20)}$$

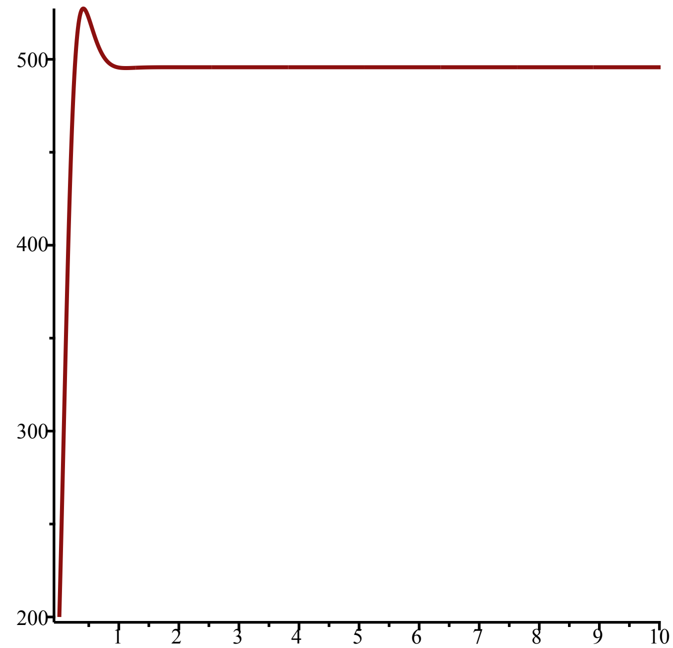
> SEquP(F, [s, i])

$$\{[256.4102564, 495.7264957]\} \quad \textbf{(21)}$$

> TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1)

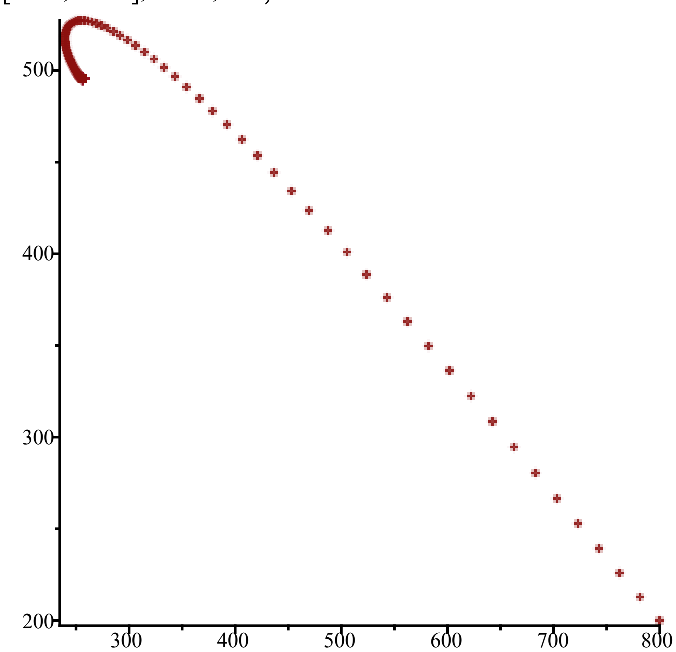


> TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2)



> #The horiztonal asymptote of the susceptible occurs at about S=256 and the horizontal asymptote of the infected occurs at about I=496, which corresponds to the STABLE EQ point [256.4102564, 495.7264957]

> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10)$



> #iii) $\nu = 4, \gamma = 1, \beta = 0.3 \cdot \frac{\nu}{1000} = 0.0012$

> $F := subs(\{\nu = 4, \gamma = 1, \beta = 0.0012\}, SIRS(s, i, \text{beta}, \text{gamma}, \text{nu}, 1000))$

$$F := [-0.0012\, s\, i + 1000 - s - i, 0.0012\, s\, i - 4\, i] \quad \textbf{(22)}$$

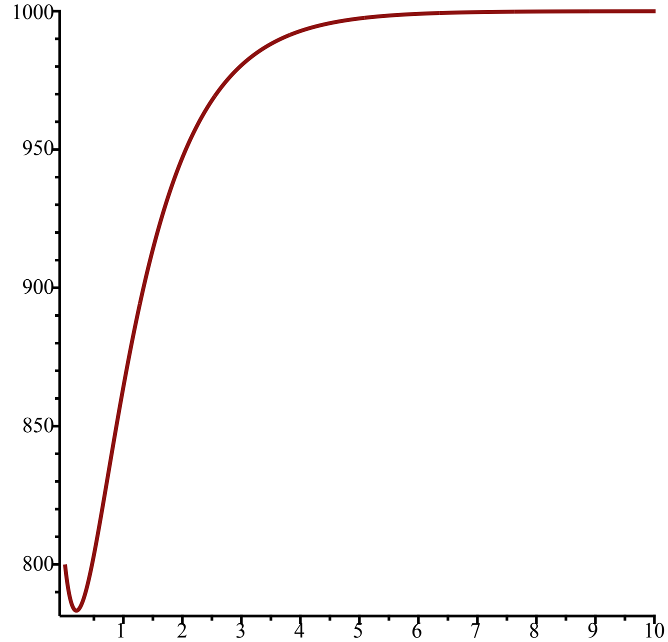
> $EquP(F, [s, i])$

$$\{[1000., 0.], [3333.333333, -466.6666667]\} \quad \textbf{(23)}$$

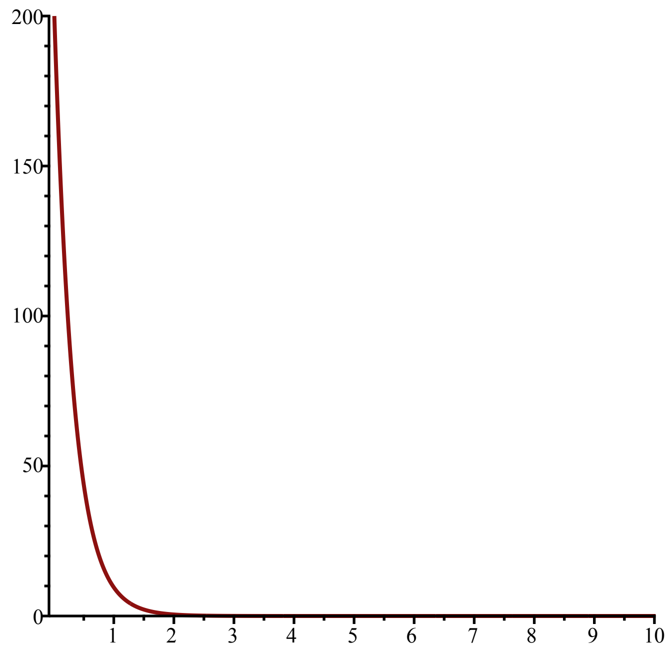
> $SEquP(F, [s, i])$

$$\{[1000., 0.]\}$$ **(24)**

> $TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1)$

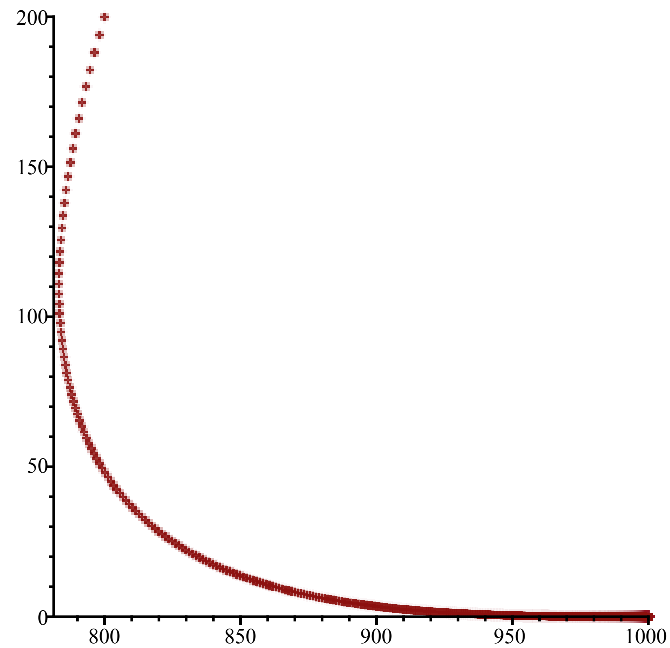


> $TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2)$



> *#The horiztonal asymptote of the susceptible occurs at about S=1000 and the horizontal asymptote of the infected occurs at about I=0, which corresponds to the STABLE EQ point* $[1000, 0]$

> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10)$



> $\#\nu = 4,\ \gamma = 1,\ \beta = 0.9 \cdot \frac{\nu}{1000} = 0.0036$

> $F := subs(\{\nu = 4, \gamma = 1, \beta = 0.0036\},\ SIRS(s, i, \text{beta}, \text{gamma}, \text{nu}, 1000))$

$$F := [-0.0036\, s\, i + 1000 - s - i,\ 0.0036\, s\, i - 4\, i] \quad \textbf{(25)}$$

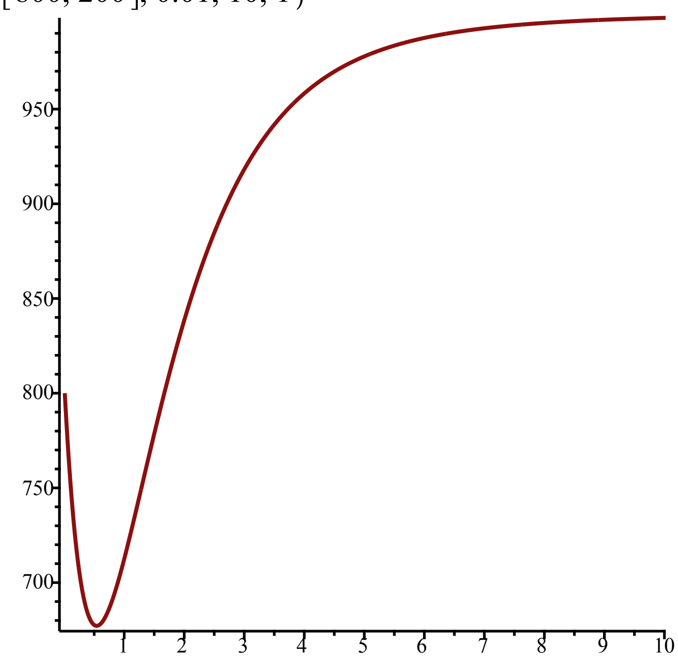
> $EquP(F, [s, i])$

$$\{[1000., 0.], [1111.111111, -22.22222222]\} \quad \textbf{(26)}$$

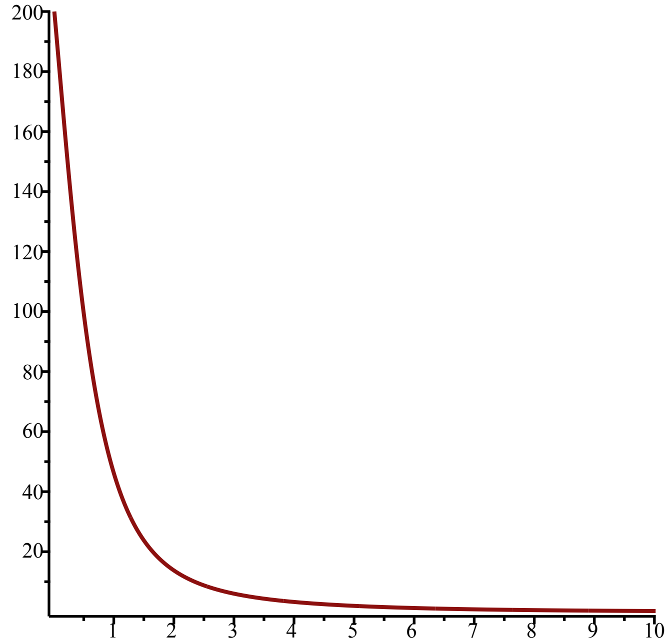
> $SEquP(F, [s, i])$

$$\{[1000., 0.]\} \quad \textbf{(27)}$$

> $TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1)$

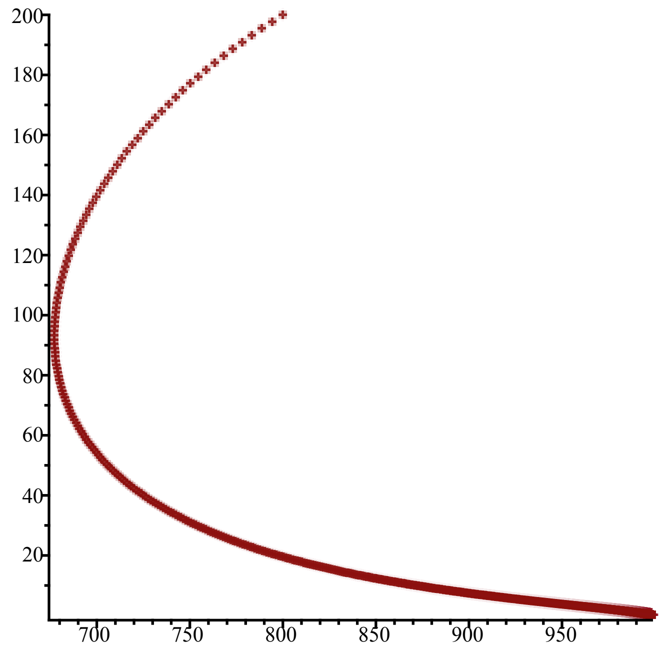


> $TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2)$



> *#The horiztonal asymptote of the susceptible occurs at about S=1000 and the horizontal asymptote of the infected occurs at about I=0, which corresponds to the STABLE EQ point* [1000, 0]

> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10)$



> $\#\nu = 4, \gamma = 1, \beta = 3.9 \cdot \frac{\nu}{1000} = 0.0156$

> $F := subs(\{\nu = 4, \gamma = 1, \beta = 0.0156\}, SIRS(s, i, \text{beta}, \text{gamma}, \text{nu}, 1000))$

$$F := [-0.0156\, s\, i + 1000 - s - i, 0.0156\, s\, i - 4\, i] \quad \textbf{(28)}$$

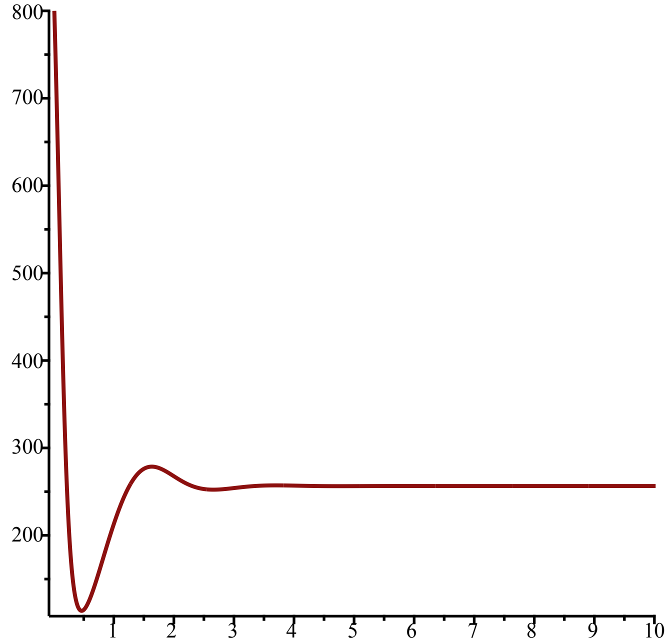
> $EquP(F, [s, i])$

$$\{[256.4102564, 148.7179487], [1000., 0.]\} \quad \textbf{(29)}$$

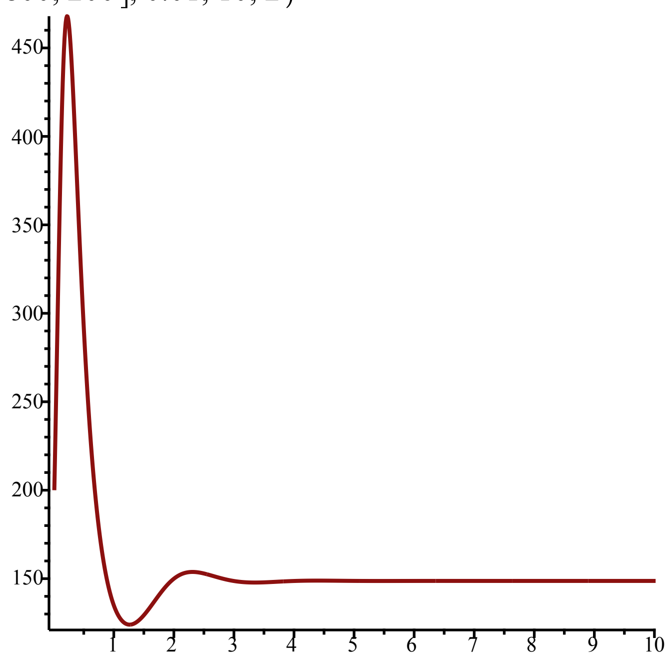
> $SEquP(F, [s, i])$

$$\{[256.4102564, 148.7179487]\} \quad \textbf{(30)}$$

> $TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1)$

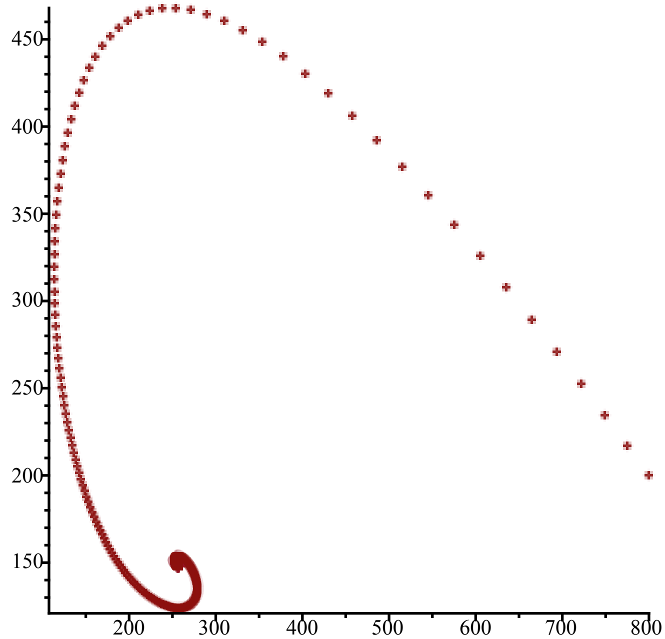


> $TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2)$



> *#The horiztonal asymptote of the susceptible occurs at about S=256 and the horizontal asymptote of the infected occurs at about I=150, which corresponds to the STABLE EQ point* [256.4102564, 148.7179487]

> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10)$



> #iv) $\nu = 7, \gamma = 10, \beta = 0.3 \cdot \frac{\nu}{1000} = 0.0021$

> $F := subs(\{\nu = 7, \gamma = 10, \beta = 0.0021\}, SIRS(s, i, \text{beta}, \text{gamma}, \text{nu}, 1000))$

$$F := [-0.0021\, s\, i + 10000 - 10\, s - 10\, i, 0.0021\, s\, i - 7\, i] \quad \textbf{(31)}$$

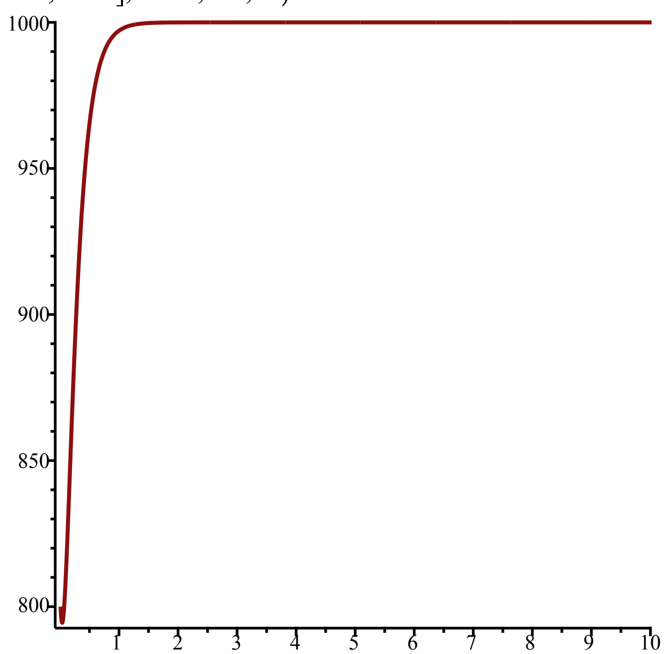
> $EquP(F, [s, i])$

$$\{[1000., 0.], [3333.333333, -1372.549020]\} \quad \textbf{(32)}$$

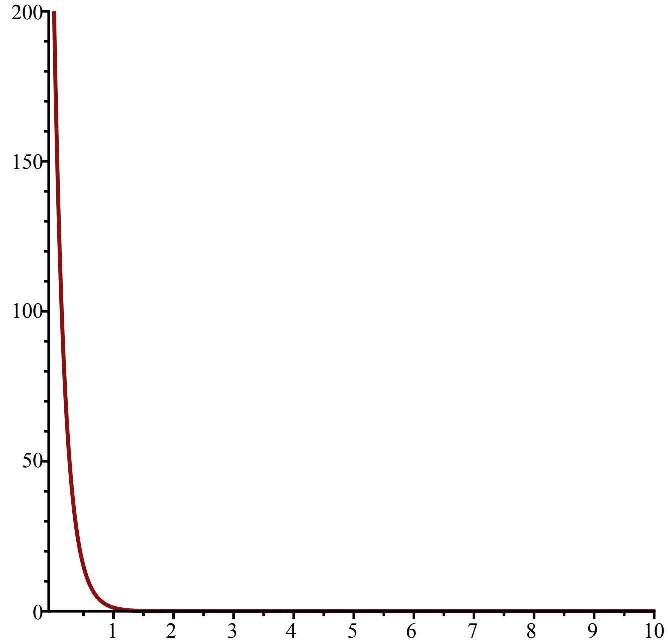
> $SEquP(F, [s, i])$

$$\{[1000., 0.]\} \quad \textbf{(33)}$$

> $TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1)$

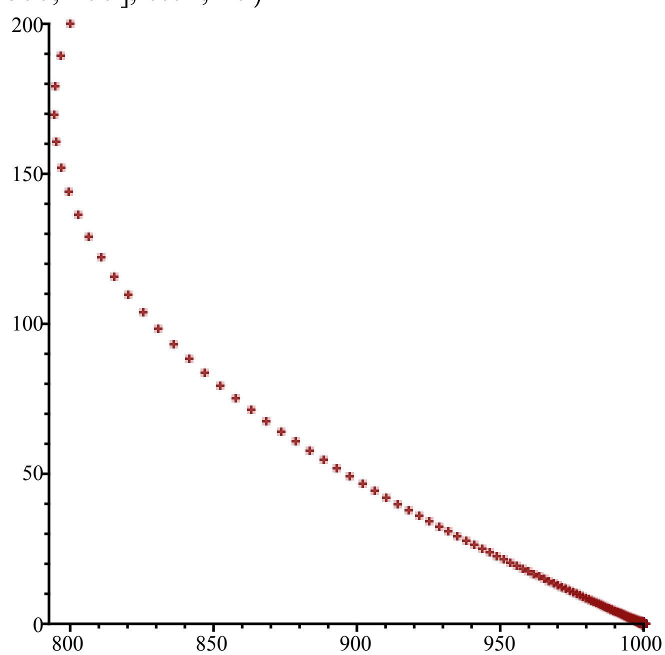


> $TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2)$



> *#The horiztonal asymptote of the susceptible occurs at about S=1000 and the horizontal asymptote of the infected occurs at about I=0, which corresponds to the STABLE EQ point* $[1000, 0]$

> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10)$



> $\#\nu = 7, \gamma = 10, \beta = 0.9 \cdot \frac{\nu}{1000} = 0.0063$

> $F := subs(\{\nu = 7, \gamma = 10, \beta = 0.0063\}, SIRS(s, i, \text{beta}, \text{gamma}, \text{nu}, 1000))$

$$F := [-0.0063\, s\, i + 10000 - 10\, s - 10\, i, 0.0063\, s\, i - 7\, i] \quad \textbf{(34)}$$

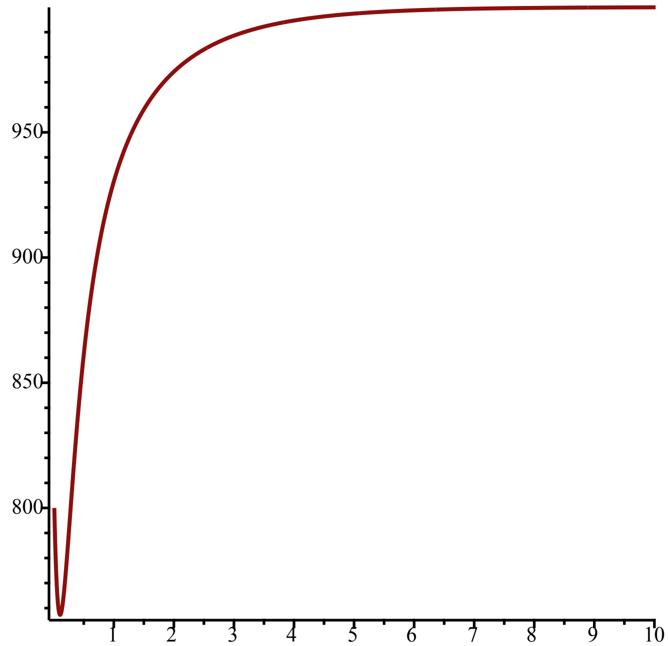
> $EquP(F, [s, i])$

$$\{[1000., 0.], [1111.111111, -65.35947712]\} \quad \textbf{(35)}$$

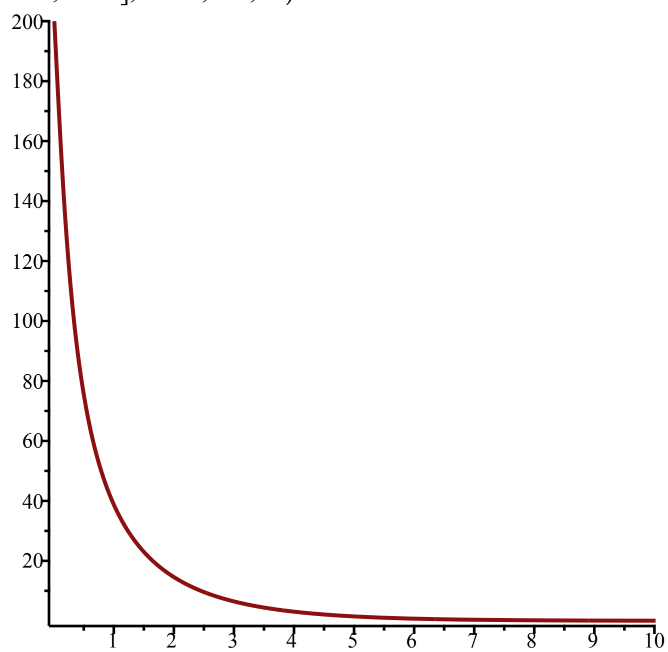
> $SEquP(F, [s, i])$

$$\{[1000., 0.]\} \quad \textbf{(36)}$$

> $TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1)$

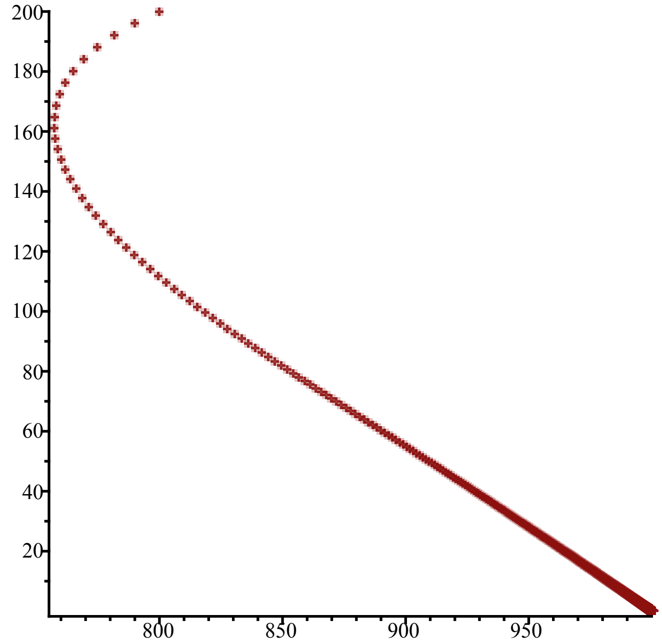


> $TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2)$



> *#The horiztonal asymptote of the susceptible occurs at about S=1000 and the horizontal asymptote of the infected occurs at about I=0, which corresponds to the STABLE EQ point* $[1000, 0]$

> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10)$



> #$\nu = 7, \gamma = 10, \beta = 3.9 \cdot \frac{\nu}{1000} = 0.0273$

> $F := subs(\{\nu = 7, \gamma = 10, \beta = 0.0273\}, SIRS(s, i, \text{beta}, \text{gamma}, \text{nu}, 1000))$

$$F := [-0.0273\, s\, i + 10000 - 10\, s - 10\, i, 0.0273\, s\, i - 7\, i] \quad (37)$$

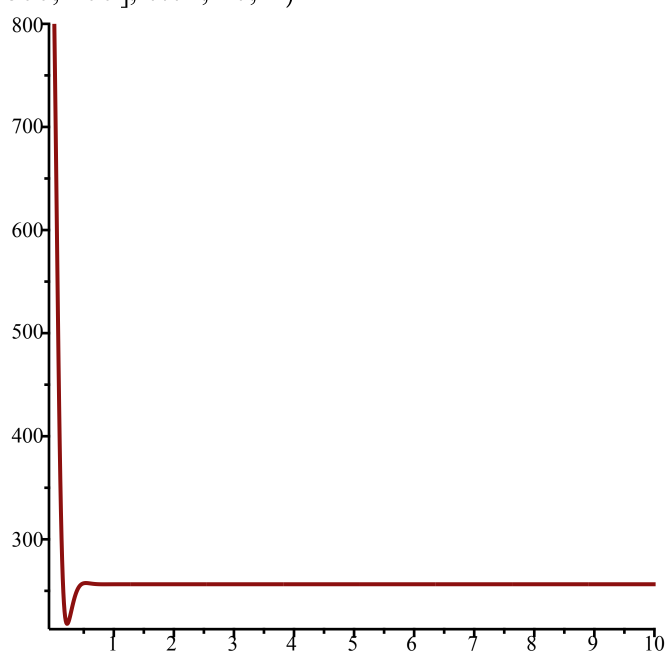
> $EquP(F, [s, i])$

$$\{[256.4102564, 437.4057315], [1000., 0.]\} \quad (38)$$

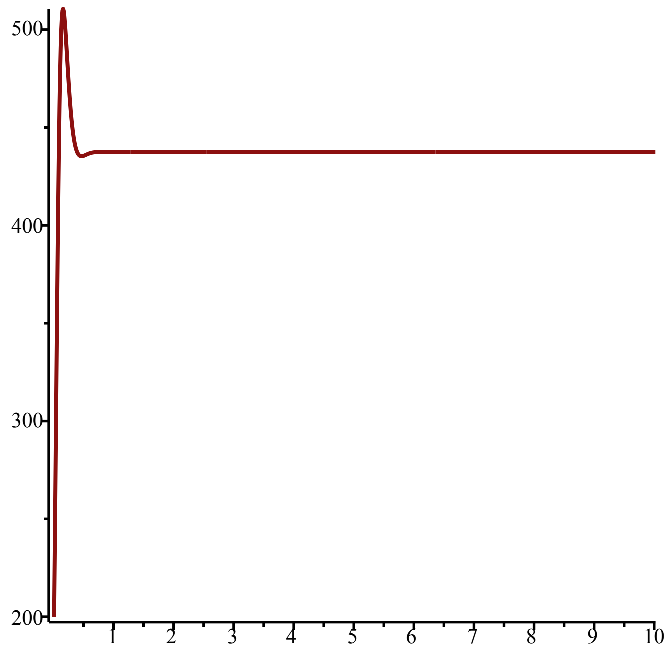
> $SEquP(F, [s, i])$

$$\{[256.4102564, 437.4057315]\} \quad (39)$$

> $TimeSeries(F, [s, i], [800, 200], 0.01, 10, 1)$

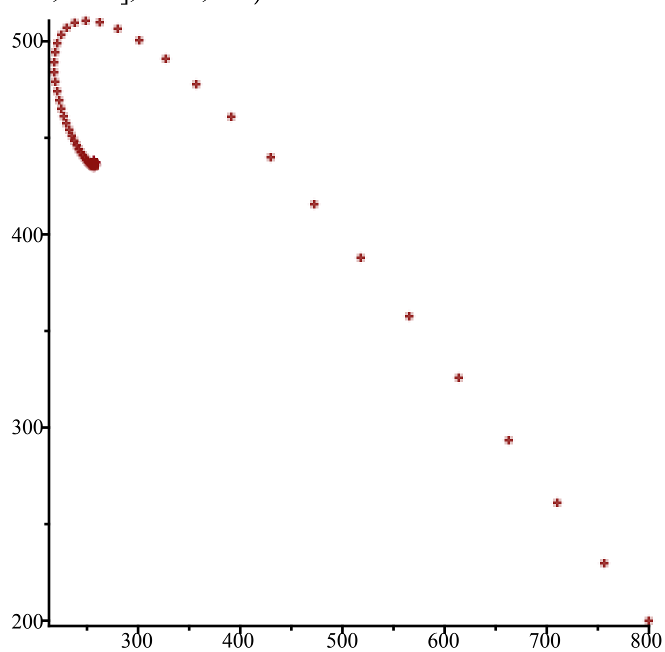


> $TimeSeries(F, [s, i], [800, 200], 0.01, 10, 2)$



> *#The horiztonal asymptote of the susceptible occurs at about S=256 and the horizontal asymptote of the infected occurs at about I=440, which corresponds to the STABLE EQ point* [256.4102564, 437.4057315]

> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10)$



>

>

>

> *#2)*

> $F := RandNice([x, y], 3)$

$$F := [(2 - 2x - 3y)(2 - x - 3y), (1 - x - 2y)(3 - 2x - 2y)] \quad \textbf{(40)}$$

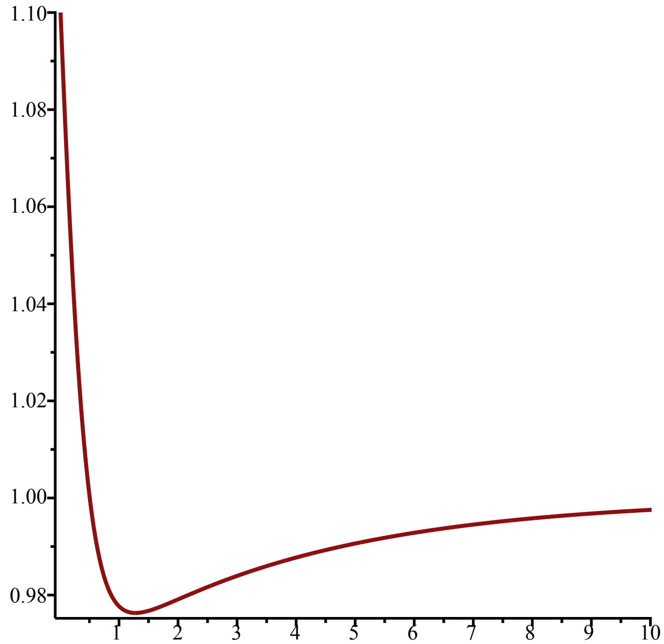
> $EquP(F, [x, y])$

$$\left\{[-1, 1], [1, 0], \left[\frac{5}{2}, -1\right], \left[\frac{5}{4}, \frac{1}{4}\right]\right\} \quad \textbf{(41)}$$

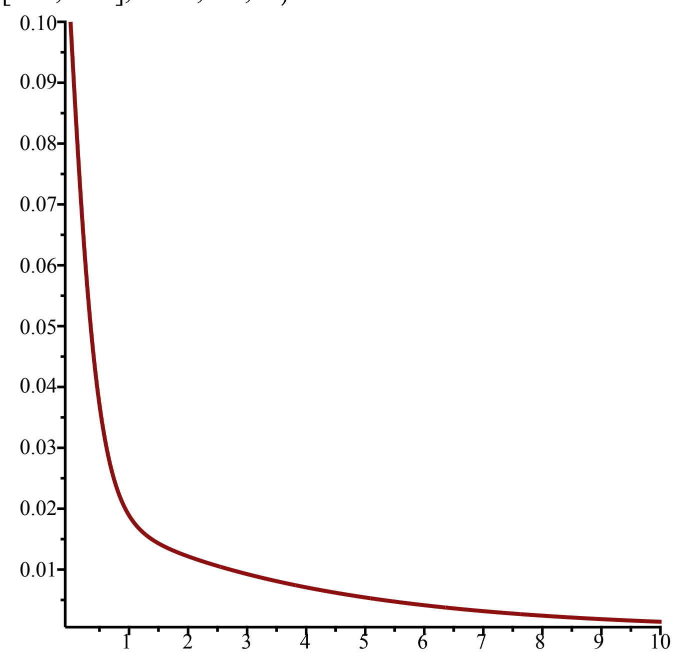
> $SEquP(F, [x, y])$

$$\{[1., 0.]\} \quad \textbf{(42)}$$

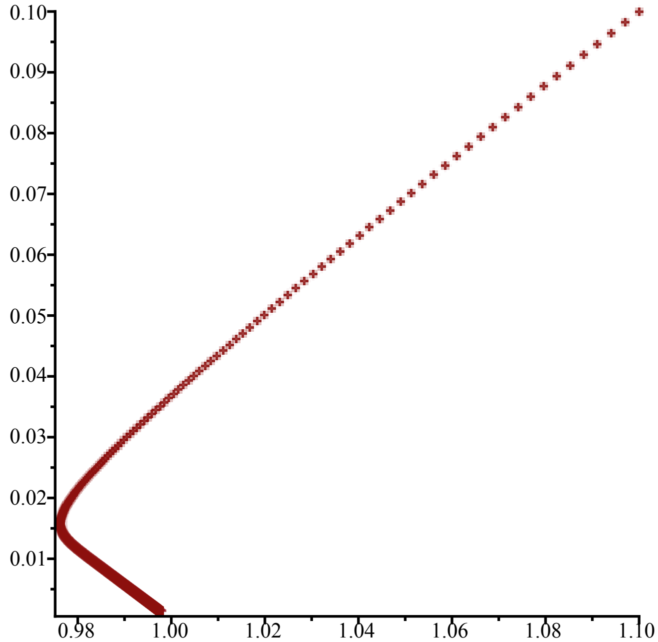
> $TimeSeries(F, [x, y], [1.1, 0.1], 0.01, 10, 1)$



> $TimeSeries(F, [x, y], [1.1, 0.1], 0.01, 10, 2)$



> $PhaseDiag(F, [x, y], [1.1, 0.1], 0.01, 10)$



> *#For the second F:*

> $F := RandNice([x, y], 3)$

$$F := [(3 - x - 2y)(1 - 2x - 3y), (3 - 3x - 2y)(1 - 3x - 3y)] \quad \textbf{(43)}$$

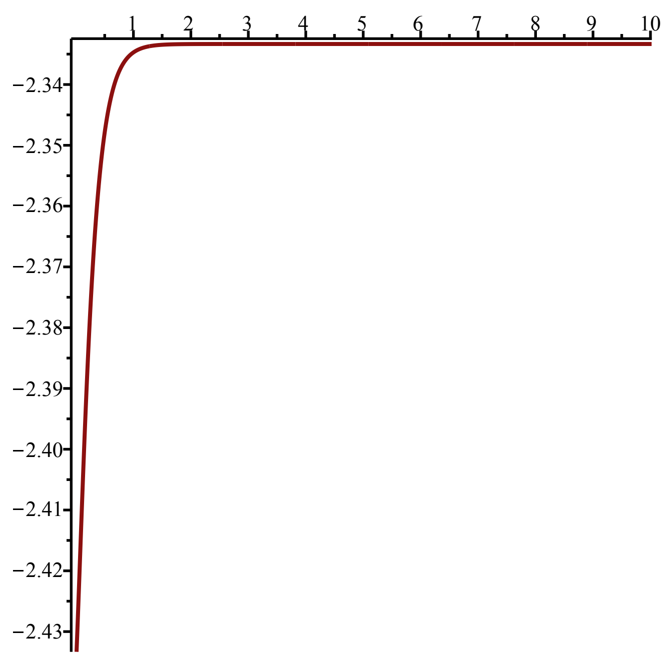
> $EquP(F, [x, y])$

$$\left\{\left[0, \frac{1}{3}\right], \left[0, \frac{3}{2}\right], \left[-\frac{7}{3}, \frac{8}{3}\right], \left[\frac{7}{5}, -\frac{3}{5}\right]\right\} \quad \textbf{(44)}$$

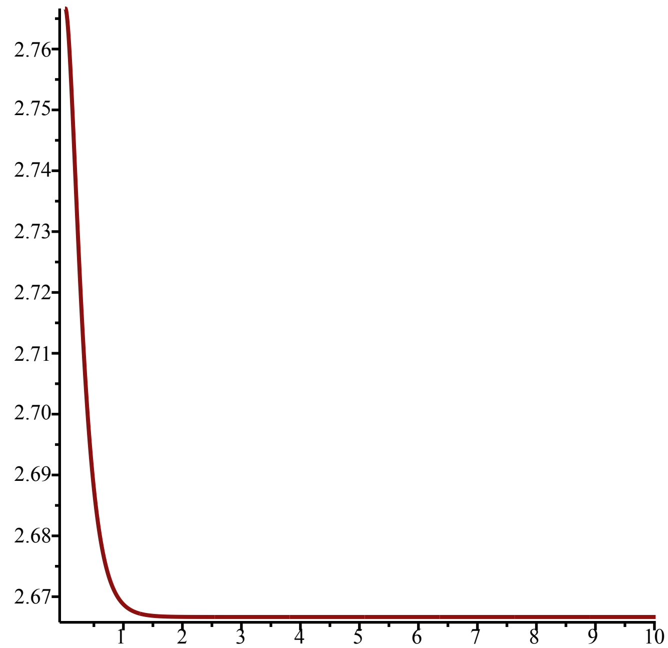
> $SEquP(F, [x, y])$

$$\{[-2.333333333, 2.666666667], [1.400000000, -0.6000000000]\} \quad \textbf{(45)}$$

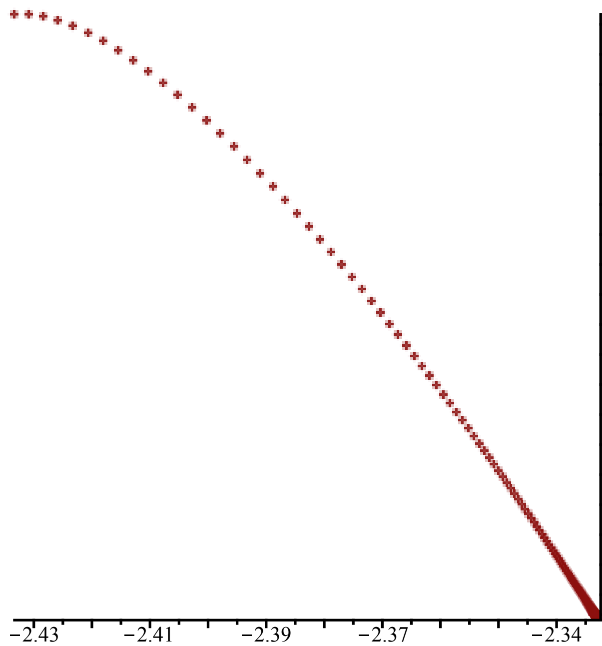
> $TimeSeries(F, [x, y], [-2.43333, 2.76666], 0.01, 10, 1)$



> $TimeSeries(F, [x, y], [-2.43333, 2.76666], 0.01, 10, 2)$

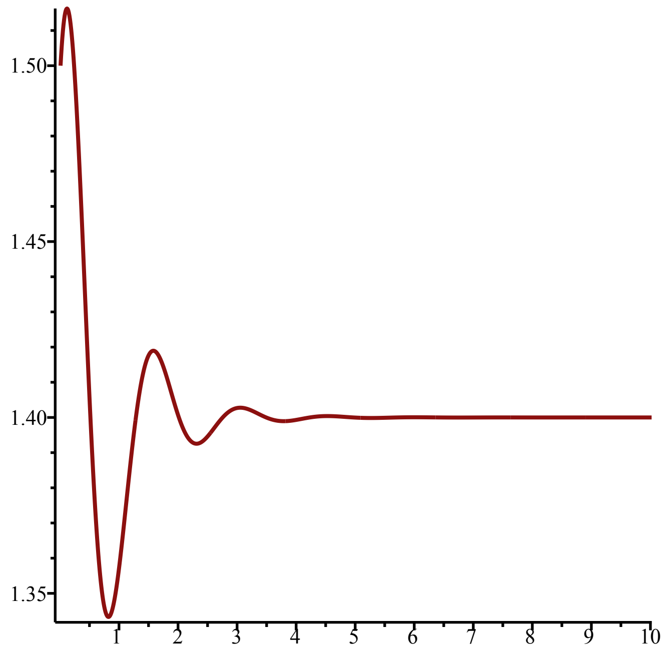


> *PhaseDiag*(*F*, [*x*, *y*], [−2.43333, 2.76666], 0.01, 10)

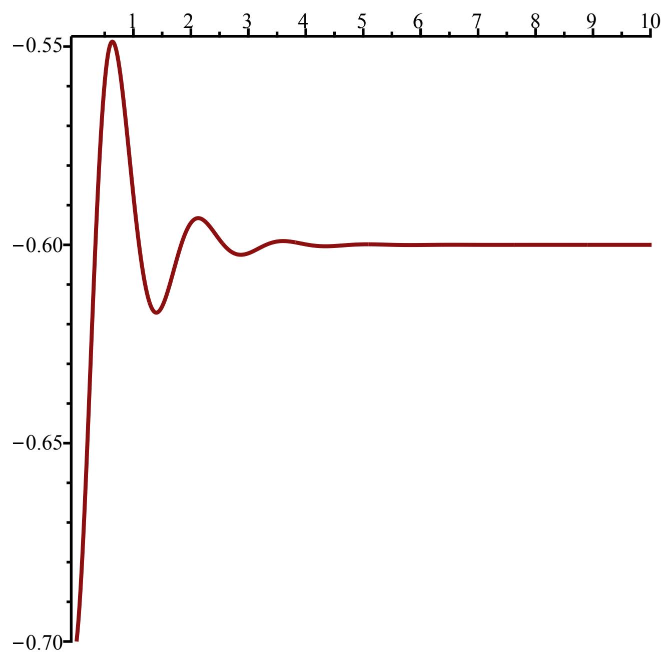


>

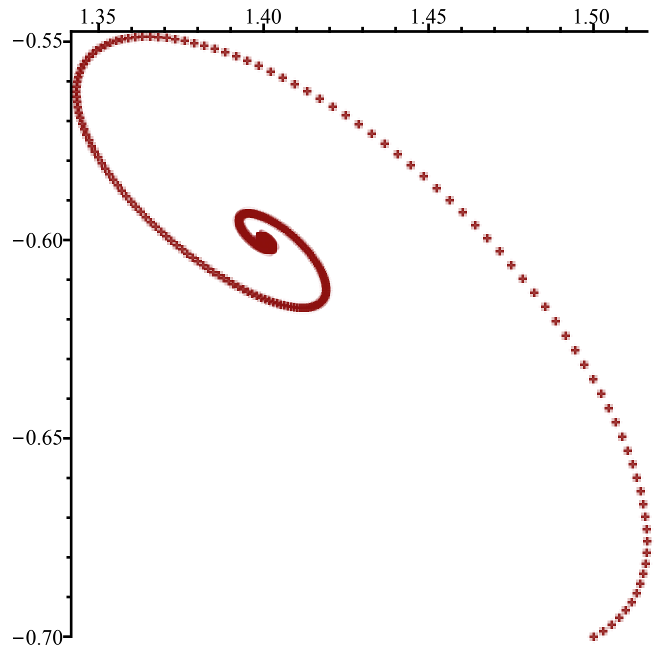
> *TimeSeries*(*F*, [*x*, *y*], [1.5, −0.7], 0.01, 10, 1)



> *TimeSeries*(*F*, [*x*, *y*], [1.5, -0.7], 0.01, 10, 2)



> *PhaseDiag*(*F*, [*x*, *y*], [1.5, -0.7], 0.01, 10)



> #For the third F:

> $F := RandNice([x, y], 3)$

$$F := [(3 - x - 3y)(3 - 2x - 3y), (1 - x - 3y)(1 - x - 2y)] \quad \textbf{(46)}$$

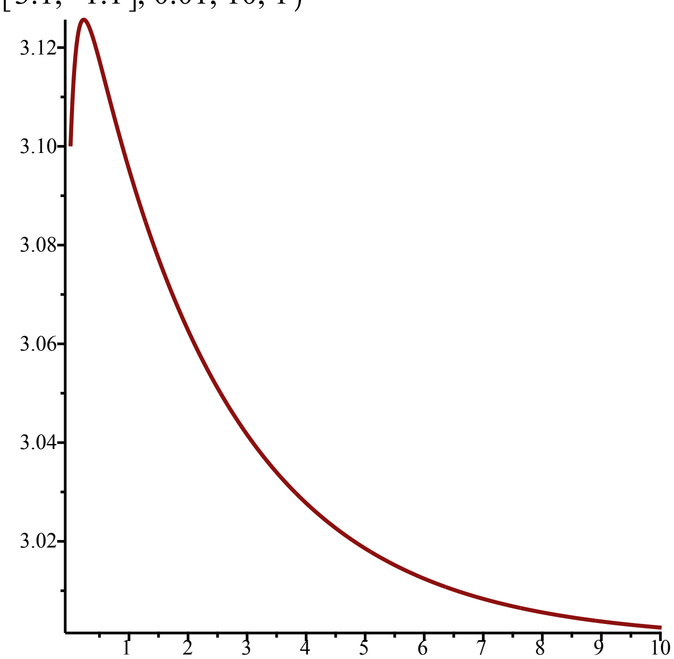
> $EquP(F, [x, y])$

$$\left\{[-3, 2], \left[2, -\frac{1}{3}\right], [3, -1]\right\} \quad \textbf{(47)}$$

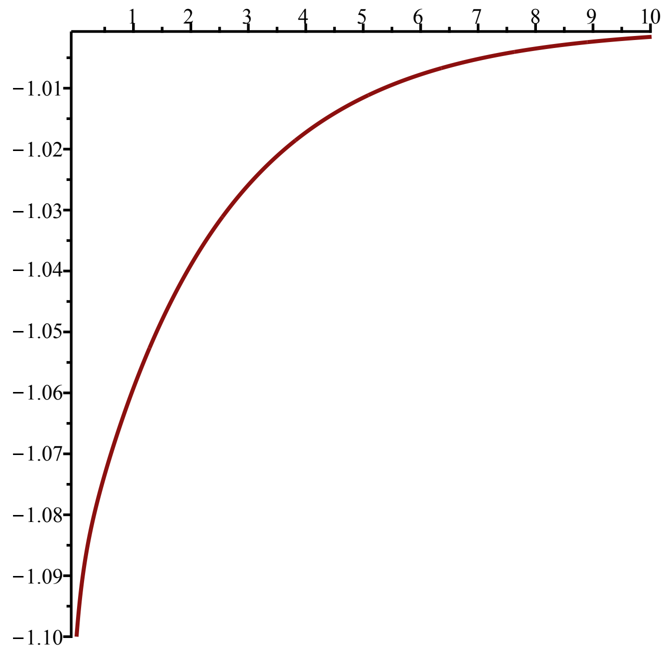
> $SEquP(F, [x, y])$

$$\{[3., -1.]\} \quad \textbf{(48)}$$

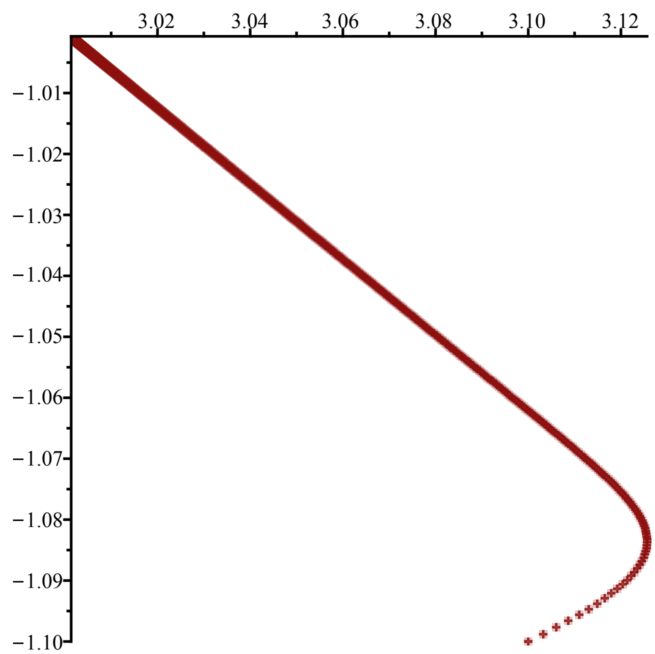
> $TimeSeries(F, [x, y], [3.1, -1.1], 0.01, 10, 1)$



> $TimeSeries(F, [x, y], [3.1, -1.1], 0.01, 10, 2)$



> $PhaseDiag(F, [x, y], [3.1, -1.1], 0.01, 10)$



>

>

> $Help(Orbk)$

Orbk(k,z,f,INI,K1,K2): Given a positive integer k, a letter (symbol), z, an expression f of z[1], ..., z[k] (representing a multi-variable function of the variables z[1],...,z[k]
a vector INI representing the initial values [x[1],..., x[k]], and (in applications) positive integres K1 and K2, outputs the
values of the sequence starting at n=K1 and ending at n=K2. of the sequence satisfying the difference equation

x[n]=f(x[n-1],x[n-2],..., x[n-k+1]):

This is a generalization to higher-order difference equation of procedure Orb(f,x,x0,K1,K2). For example, try:

*Orbk(1,z,5/2*z[1]*(1-z[1]),[0.5],1000,1010);*

To get the Fibonacci sequence, type:

Orbk(2,z,z[1] +z[2],[1,1],1000,1010);

To get the part of the orbit between n=1000 and n=1010, of the 3rd order recurrence given in Eq. (4) of the Ladas-Amleh paper

https://sites.math.rutgers.edu/~zeilberg/Bio21/AmlehLadas.pdf

with initial conditions x(0)=1, x(1)=3, x(2)=5, Type:

Orbk(3,z,z[2]/(z[2] +z[3]),[1.,3.,5.],1000,1010);

To get the part of the orbit between n=1000 and n=1010, of the 3rd order recurrence given in Eq. (5) of the Ladas-Amleh paper

with initial conditions x(0)=1, x(1)=3, x(2)=5, Type:

Orbk(3,z,(z[1] +z[3])/z[2],[1.,3.,5.],1000,1010);

To get the part of the orbit between n=1000 and n=1010, of the 3rd order recurrence given in Eq. (6) of the Ladas-Amleh paper

with initial conditions x(0)=1, x(1)=3, x(2)=5, Type:

Orbk(3,z,(1 +z[3])/z[1],[1.,3.,5.],1000,1010);

To get the part of the orbit between n=1000 and n=1010, of the 3rd order recurrence given in Eq. (7) of the Ladas-Amleh paper

with initial conditions x(0)=1, x(1)=3, x(2)=5, Type:

Orbk(3,z,(1 +z[1])/(z[2] +z[3]),[1.,3.,5.],1000,1010); **(49)**

```
> print(Orbk)
proc(k, z, f, INI, K1, K2)                                              (50)
    local L, i, newguy;
    L := INI;
    if not (type(k, integer) and type(z, symbol) and type(INI, list) and nops(INI) = k and
    type(K1, integer) and type(K2, integer) and 0 < K1 and K1 < K2) then
        print(bad input); RETURN(FAIL)
    end if;
    while nops(L) < K2 do
        newguy := subs({seq(z[i] = L[-i], i = 1 ..k)}, f); L := [op(L), newguy]
    end do;
```

$[op(K1..K2, L)]$

end proc

>

>

> #3)

> $x(n+1) := \frac{3+x(n-1)+x(n-2)+x(n-3)}{1+x(n)+x(n-2)}$

$$x(n+1) := \frac{3+x(n-1)+x(n-2)+x(n-3)}{1+x(n)+x(n-2)} \quad \textbf{(51)}$$

> $x(n) := subs(n=n-1, x(n+1))$

$$x(n) := \frac{3+x(n-2)+x(n-3)+x(n-4)}{1+x(n-1)+x(n-3)} \quad \textbf{(52)}$$

> $F := \frac{3+z[2]+z[3]+z[4]}{1+z[1]+z[3]}$

$$F := \frac{3+z_2+z_3+z_4}{1+z_1+z_3} \quad \textbf{(53)}$$

> $Orbk(4, z, F, [9.1, 0.2, 1.2, 0.9], 10000, 10010)$

[0.8518413801, 5.473832241, 0.8518413801, 5.473832241, 0.8518413801, 5.473832241, 0.8518413801, 5.473832241, 0.8518413801, 5.473832241, 0.8518413801] **(54)**

> $Orbk(4, z, F, [11.1, 0.2, 1.2, 0.9], 10000, 10010)$

[0.8082760883, 6.176729621, 0.8082760883, 6.176729621, 0.8082760883, 6.176729621, 0.8082760883, 6.176729621, 0.8082760883, 6.176729621, 0.8082760883] **(55)**

> $Orbk(4, z, F, [0.1, 0.2, 0.2, 0.1], 10000, 10010)$

[1.106519003, 3.385317674, 1.106519003, 3.385317674, 1.106519003, 3.385317674, 1.106519003, 3.385317674, 1.106519003, 3.385317674, 1.106519003] **(56)**

> $Orbk(4, z, F, [0.1, 0.2, 0.9, 0.1], 10000, 10010)$

[0.9864991099, 4.097128884, 0.9864991099, 4.097128884, 0.9864991099, 4.097128884, 0.9864991099, 4.097128884, 0.9864991099, 4.097128884, 0.9864991099] **(57)**

> $Orbk(4, z, F, [0.01, 0.02, 0.01, 0.01], 10000, 10010)$

[1.023849787, 3.840652307, 1.023849787, 3.840652307, 1.023849787, 3.840652307, 1.023849787, 3.840652307, 1.023849787, 3.840652307, 1.023849787] **(58)**

> $Orbk(4, z, F, [0.1, 0.3, 0.2, 0.4], 10000, 10010)$

[1.363810701, 2.525906831, 1.363810701, 2.525906831, 1.363810701, 2.525906831, 1.363810701, 2.525906831, 1.363810701, 2.525906831, 1.363810701] **(59)**

> $Orbk(4, z, F, [1.0, 1.0, 1.0, 1.0], 10000, 10010)$

[1.567364551, 2.139552296, 1.567364551, 2.139552296, 1.567364551, 2.139552296, 1.567364551, 2.139552296, 1.567364551, 2.139552296, 1.567364551] **(60)**

> $Orbk(4, z, F, [2.0, 2.0, 2.0, 2.0], 10000, 10010)$

[1.866943730, 1.780228265, 1.866943730, 1.780228265, 1.866943730, 1.780228265, **(61)**

1.866943730, 1.780228265, 1.866943730, 1.780228265, 1.866943730]

> $Orbk(4, z, F, [2.1, 2.1, 2.1, 2.1], 10000, 10010)$

[1.890626777, 1.758425358, 1.890626777, 1.758425358, 1.890626777, 1.758425358, 1.890626777, 1.758425358, 1.890626777, 1.758425358, 1.890626777] **(62)**

> $Orbk(4, z, F, [1.9, 1.9, 1.9, 1.9], 10000, 10010)$

[1.842416563, 1.803619196, 1.842416563, 1.803619196, 1.842416563, 1.803619196, 1.842416563, 1.803619196, 1.842416563, 1.803619196, 1.842416563] **(63)**

> $Orbk(4, z, F, [1.85, 1.85, 1.85, 1.85], 10000, 10010)$

[1.829812874, 1.815974626, 1.829812874, 1.815974626, 1.829812874, 1.815974626, 1.829812874, 1.815974626, 1.829812874, 1.815974626, 1.829812874] **(64)**

> $Orbk(4, z, F, [1.8, 1.8, 1.8, 1.8], 10000, 10010)$

[1.816968812, 1.828808992, 1.816968812, 1.828808992, 1.816968812, 1.828808992, 1.816968812, 1.828808992, 1.816968812, 1.828808992, 1.816968812] **(65)**

> $Orbk(4, z, F, [1.81, 1.81, 1.81, 1.81], 10000, 10010)$

[1.819557373, 1.826202283, 1.819557373, 1.826202283, 1.819557373, 1.826202283, 1.819557373, 1.826202283, 1.819557373, 1.826202283, 1.819557373] **(66)**

> $Orbk(4, z, F, [1.81, 1.81, 1.81, 1.81], 10000, 10010)$

> **for** i **by** 0.001 **from** 1.8 **to** 1.85 **do** $print(Orbk(4, z, F, [i, i, i, i], 1000, 1010))$ **od**:

[1.816968812, 1.828808992, 1.816968812, 1.828808992, 1.816968812, 1.828808992, 1.816968812, 1.828808992, 1.816968812, 1.828808992, 1.816968812]

[1.817228119, 1.828547406, 1.817228119, 1.828547406, 1.817228119, 1.828547406, 1.817228119, 1.828547406, 1.817228119, 1.828547406, 1.817228119]

[1.817487328, 1.828286021, 1.817487328, 1.828286021, 1.817487328, 1.828286021, 1.817487328, 1.828286021, 1.817487328, 1.828286021, 1.817487328]

[1.817746432, 1.828024844, 1.817746432, 1.828024844, 1.817746432, 1.828024844, 1.817746432, 1.828024844, 1.817746432, 1.828024844, 1.817746432]

[1.818005439, 1.827763868, 1.818005439, 1.827763868, 1.818005439, 1.827763868, 1.818005439, 1.827763868, 1.818005439, 1.827763868, 1.818005439]

[1.818264345, 1.827503097, 1.818264345, 1.827503097, 1.818264345, 1.827503097, 1.818264345, 1.827503097, 1.818264345, 1.827503097, 1.818264345]

[1.818523149, 1.827242530, 1.818523149, 1.827242530, 1.818523149, 1.827242530, 1.818523149, 1.827242530, 1.818523149, 1.827242530, 1.818523149]

[1.818781854, 1.826982165, 1.818781854, 1.826982165, 1.818781854, 1.826982165, 1.818781854, 1.826982165, 1.818781854, 1.826982165, 1.818781854]

[1.819040462, 1.826722000, 1.819040462, 1.826722000, 1.819040462, 1.826722000, 1.819040462, 1.826722000, 1.819040462, 1.826722000, 1.819040462]

[1.819298967, 1.826462041, 1.819298967, 1.826462041, 1.819298967, 1.826462041, 1.819298967, 1.826462041, 1.819298967, 1.826462041, 1.819298967]

[1.819557373, 1.826202283, 1.819557373, 1.826202283, 1.819557373, 1.826202283,

1.819557373, 1.826202283, 1.819557373, 1.826202283, 1.819557373]
[1.819815681, 1.825942725, 1.819815681, 1.825942725, 1.819815681, 1.825942725,
1.819815681, 1.825942725, 1.819815681, 1.825942725, 1.819815681]
[1.820073887, 1.825683371, 1.820073887, 1.825683371, 1.820073887, 1.825683371,
1.820073887, 1.825683371, 1.820073887, 1.825683371, 1.820073887]
[1.820331995, 1.825424217, 1.820331995, 1.825424217, 1.820331995, 1.825424217,
1.820331995, 1.825424217, 1.820331995, 1.825424217, 1.820331995]
[1.820590003, 1.825165264, 1.820590003, 1.825165264, 1.820590003, 1.825165264,
1.820590003, 1.825165264, 1.820590003, 1.825165264, 1.820590003]
[1.820847911, 1.824906513, 1.820847911, 1.824906513, 1.820847911, 1.824906513,
1.820847911, 1.824906513, 1.820847911, 1.824906513, 1.820847911]
[1.821105722, 1.824647960, 1.821105722, 1.824647960, 1.821105722, 1.824647960,
1.821105722, 1.824647960, 1.821105722, 1.824647960, 1.821105722]
[1.821363433, 1.824389609, 1.821363433, 1.824389609, 1.821363433, 1.824389609,
1.821363433, 1.824389609, 1.821363433, 1.824389609, 1.821363433]
[1.821621043, 1.824131459, 1.821621043, 1.824131459, 1.821621043, 1.824131459,
1.821621043, 1.824131459, 1.821621043, 1.824131459, 1.821621043]
[1.821878557, 1.823873506, 1.821878557, 1.823873506, 1.821878557, 1.823873506,
1.821878557, 1.823873506, 1.821878557, 1.823873506, 1.821878557]
[1.822135971, 1.823615754, 1.822135971, 1.823615754, 1.822135971, 1.823615754,
1.822135971, 1.823615754, 1.822135971, 1.823615754, 1.822135971]
[1.822393287, 1.823358200, 1.822393287, 1.823358200, 1.822393287, 1.823358200,
1.822393287, 1.823358200, 1.822393287, 1.823358200, 1.822393287]
[1.822650503, 1.823100846, 1.822650503, 1.823100846, 1.822650503, 1.823100846,
1.822650503, 1.823100846, 1.822650503, 1.823100846, 1.822650503]
[***1.822907621, 1.822843691, 1.822907621, 1.822843691, 1.822907621, 1.822843691,***
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> *#It appears that there is an equilibrium point somewhere near 1.822...*

> *Help*(*ToSys*)

ToSys(k,z,f): converts the kth order difference equation x(n)=f(x[n-1],x[n-2],...x[n-k]) to a first-order system
x1(n)=F(x1(n-1),x2(n-1), ...,xk(n-1)), it gives the unerlying transormation, followed by the set of variables
Try:
ToSys(2,z,z[1]+z[2]); **(68)**

> *ToSys*(4, *z*, *F*)

$$\left[\frac{3+z_2+z_3+z_4}{1+z_1+z_3}, z_1, z_2, z_3\right], [z_1, z_2, z_3, z_4] \quad \textbf{(69)}$$

> *Help*(*SFP*)

SFP(F,x): Given a transformation F in the list of variables finds all the STABLE fixed point of the transformation x->F(x), i.e. the set of solutions of
the system {x[1]=F[1], ..., x[k]=F[k]} that are stable. Try:
*SFP([5/2*x*(1-x)],[x]);*
*SFP([(1+x+y)/(2+3*x+y), (3+x+2*y)/(5+x+3*y)],[x,y]);* **(70)**

> $SFP\left(\left[\frac{3+z_2+z_3+z_4}{1+z_1+z_3}, z_1, z_2, z_3\right], [z_1, z_2, z_3, z_4]\right)$

{[1.822875656, 1.822875656, 1.822875656, 1.822875656]} **(71)**

> *Orbk*(4, *z*, *F*, [1.822875656, 1.822875656, 1.822875656, 1.822875656], 1000, 1010)

[1.822875656, 1.822875655, 1.822875656, 1.822875655, 1.822875656, 1.822875655, 1.822875656, 1.822875655, 1.822875656, 1.822875655, 1.822875656] **(72)**

> *#EQ point* : 1.822875656

> *#IS IT STABLE THOUGH?*

> *Orbk*(4, *z*, *F*, [1.822875656 + 0.1, 1.822875656 + 0.1, 1.822875656 + 0.1, 1.822875656 + 0.1], 1000, 1010)

[1.848105624, 1.798117869, 1.848105624, 1.798117869, 1.848105624, 1.798117869, 1.848105624, 1.798117869, 1.848105624, 1.798117869, 1.848105624] **(73)**

> {*op*(%)}

{1.798117869, 1.848105624} **(74)**

> *#I would argue that it is UNSTABLE because a relatively small change (+0.1) caused it to oscillate between two values (period of 2):* {1.798117869, 1.848105624}