```
> read `/Users/Deven/Desktop/Fall 2021/Dynamic Models of Biology/DMB.txt`
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

```
> # Deven Singh
  # Assignment 20
  #OK TO POST
> # Question 1: When Beta = 0.3
> # (i)
> Help(SIRS);
```

SIRS(s,i,beta,gamma,nu,N): The SIRS dynamical model with parameters beta,gamma, nu,N (see section 6.6 of Edelstein-Keshet), s is the number of
Susceptibles, i is the number of infected, (the number of removed is given by N-s-i). N is the total population. Try:

SIRS(s,i,beta,gamma,nu,N); **(2)**

> $N := 1000:$
> $\mathrm{nu} := 2:$
> $F := SIRS\left(s, i, \frac{0.3 \cdot \mathrm{nu}}{N}, 5, \mathrm{nu}, N\right);$

$$F := [-0.0006000000000\, s\, i + 5000 - 5\, s - 5\, i, 0.0006000000000\, s\, i - 2\, i] \quad \textbf{(3)}$$

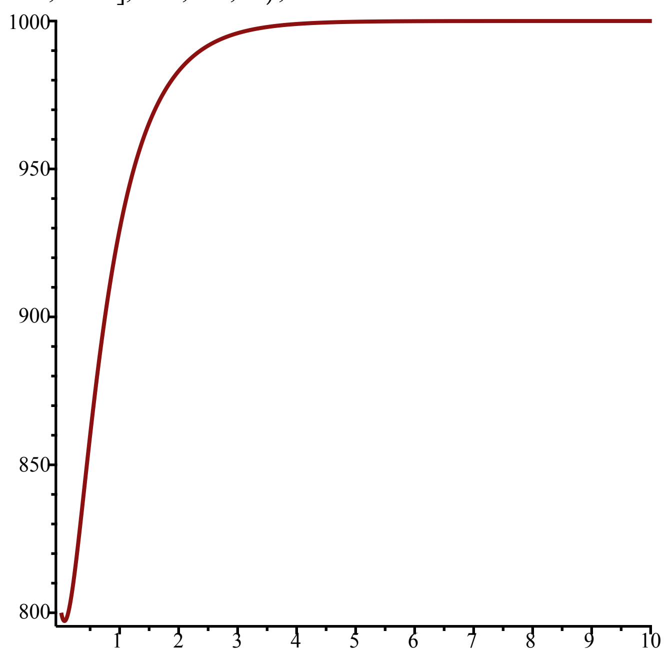
> $EquP(F, [s, i]);$

$$\{[1000., 0.], [3333.333333, -1666.666667]\} \quad \textbf{(4)}$$

> $SEquP(F, [s, i]);$

$$\{[1000., 0.]\} \quad \textbf{(5)}$$

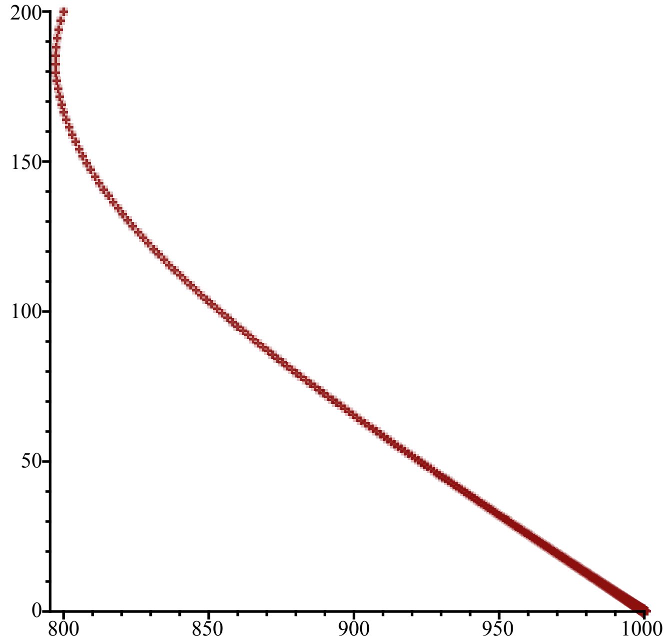
> $TimeSeries(F, [s, i], [800, 200], .01, 10, 1);$



> $Help(PhaseDiag);$

PhaseDiag(F,x,pt,h,A): Inputs a transformation F in the list of variables x (of length 2), i.e. a mapping from R^2 to R^2 gives the
The phase diagram of the solution with initial condition x(0)=pt
dx/dt=F[1](x(t)) by a discrete time dynamical system with step-size h from t=0 to t=A
Try:
PhaseDiag([x(1-y),y*(1-x)],[x,y],[0.5,0.5], 0.01, 10);* **(6)**

> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10);$



> # (ii)

> $N := 1000:$
$nu := 3:$
$F := SIRS\left(s, i, \frac{0.3 \cdot nu}{N}, 6, nu, N\right);$

$$F := [-0.0009000000000\, s\, i + 6000 - 6\, s - 6\, i,\ 0.0009000000000\, s\, i - 3\, i] \quad \textbf{(7)}$$

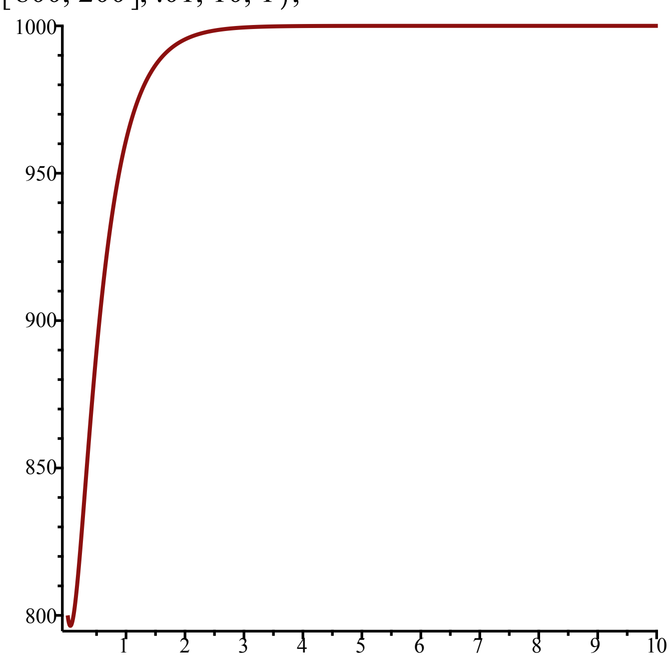
> $EquP(F, [s, i]);$

$$\{[1000., 0.], [3333.333333, -1555.555556]\} \quad \textbf{(8)}$$

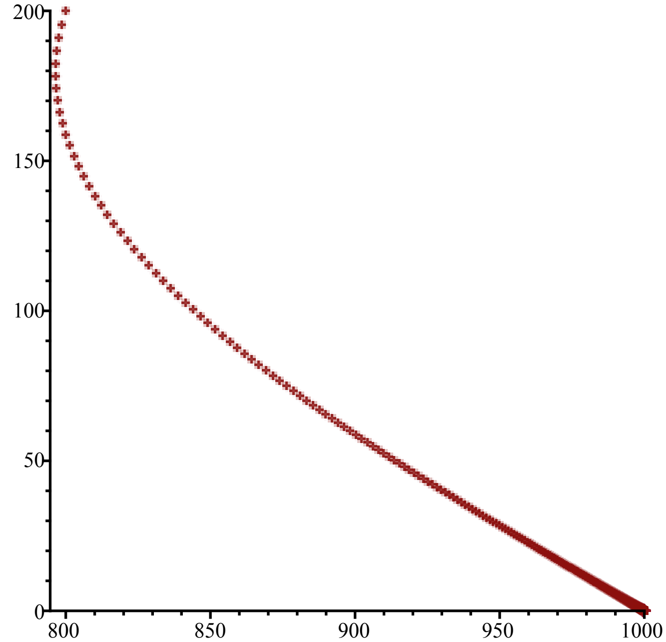
> $SEquP(F, [s, i]);$

$$\{[1000., 0.]\} \quad \textbf{(9)}$$

> $TimeSeries(F, [s, i], [800, 200], .01, 10, 1);$



> *PhaseDiag*(*F*, [*s*, *i*], [800, 200], 0.01, 10);



> #(iii)

> $N := 1000:$
nu := 4 :
$F := SIRS\left(s, i, \frac{0.3 \cdot \text{nu}}{N}, 1, \text{nu}, N\right);$

$$F := [-0.001200000000\, s\, i + 1000 - s - i,\ 0.001200000000\, s\, i - 4\, i] \quad \textbf{(10)}$$

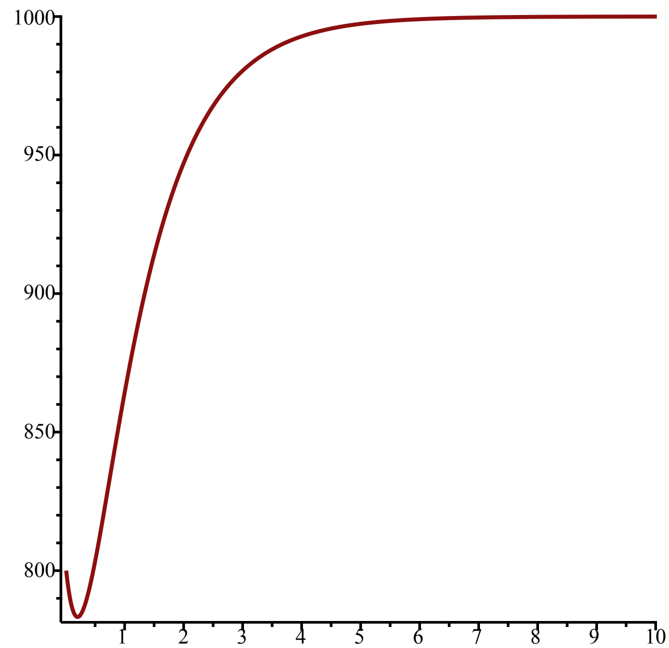
> *EquP*(*F*, [*s*, *i*]);

$$\{[1000., 0.], [3333.333333, -466.6666667]\} \quad \textbf{(11)}$$

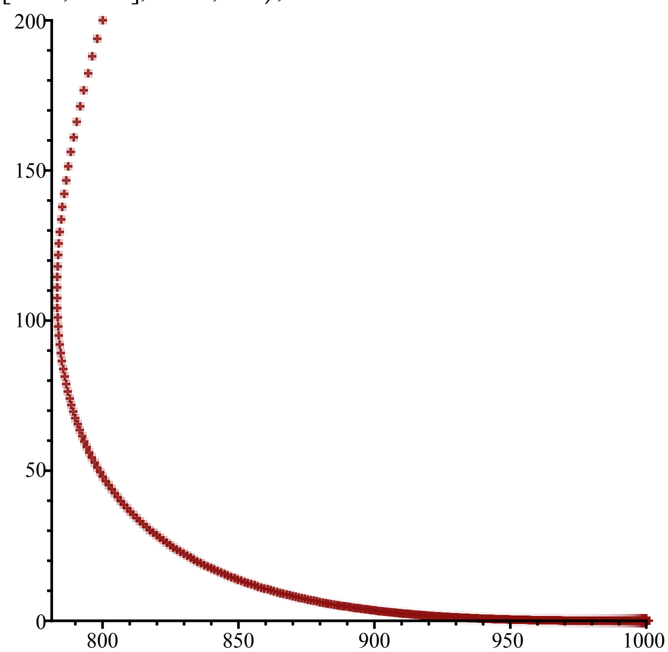
> *SEquP*(*F*, [*s*, *i*]);

$$\{[1000., 0.]\} \quad \textbf{(12)}$$

> *TimeSeries*(*F*, [*s*, *i*], [800, 200], .01, 10, 1);




```
> PhaseDiag(F, [s, i], [800, 200], 0.01, 10);
```




```
> #(iv)
> N := 1000 :
  nu := 7 :
```

$$F := SIRS\left(s, i, \frac{0.3 \cdot \text{nu}}{N}, 10, \text{nu}, N\right);$$

$$F := [-0.002100000000\, s\, i + 10000 - 10\, s - 10\, i, 0.002100000000\, s\, i - 7\, i] \quad \textbf{(13)}$$

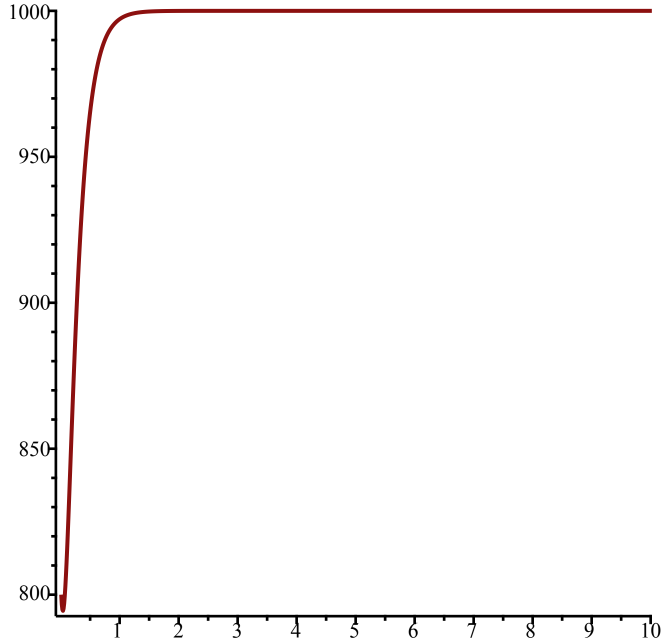
```
> EquP(F, [s, i]);
```

$$\{[1000., 0.], [3333.333333, -1372.549020]\} \quad \textbf{(14)}$$

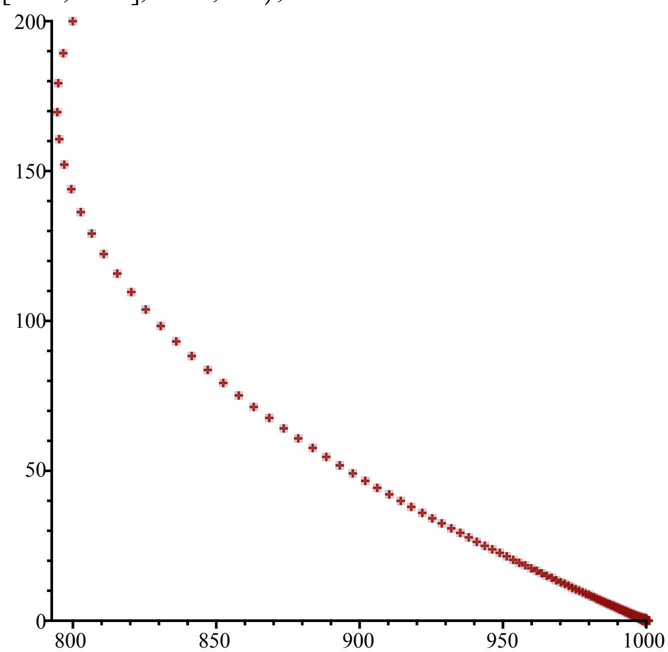
```
> SEquP(F, [s, i]);
```

$$\{[1000., 0.]\} \quad \textbf{(15)}$$

> $TimeSeries(F, [s, i], [800, 200], .01, 10, 1);$



> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10);$



> *#Question 1: When Beta = 0.9*

> *#(i)*

> $N := 1000:$
$nu := 2:$
$F := SIRS\left(s, i, \frac{0.9 \cdot nu}{N}, 5, nu, N\right);$

$$F := [-0.001800000000\, s\, i + 5000 - 5\, s - 5\, i, 0.001800000000\, s\, i - 2\, i] \quad \textbf{(16)}$$

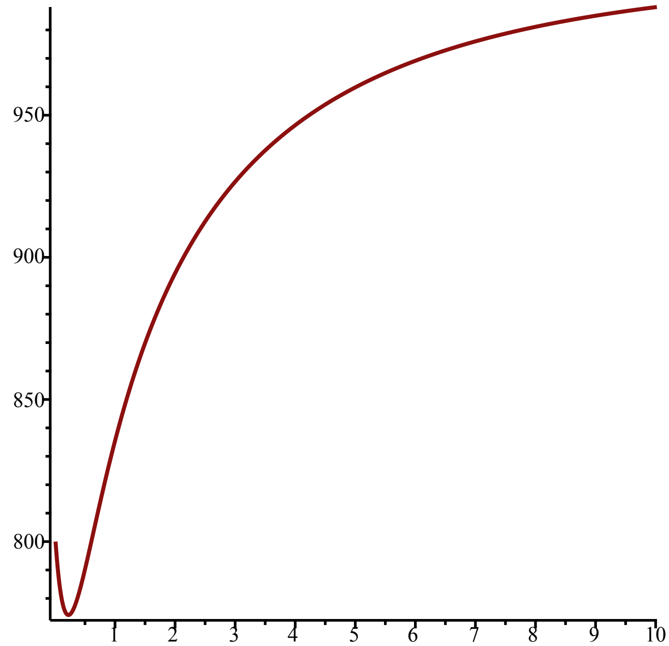
> $EquP(F, [s, i]);$

$$\{[1000., 0.], [1111.111111, -79.36507937]\} \quad \textbf{(17)}$$

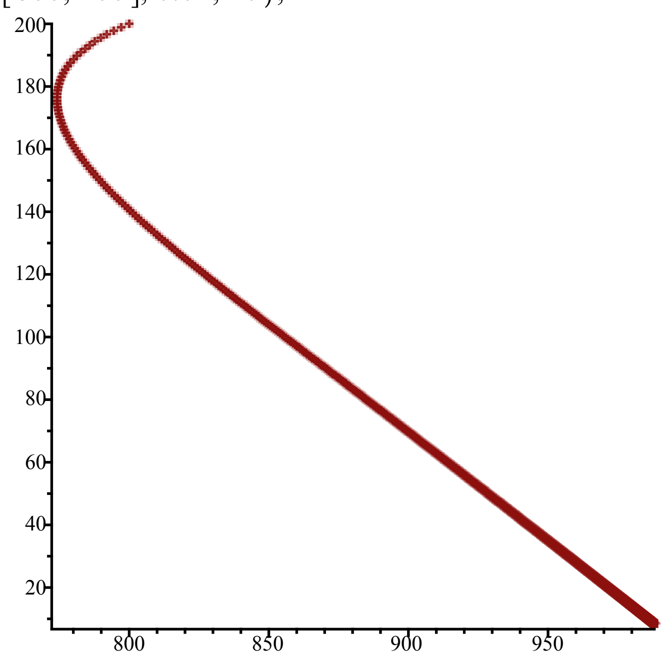
> $SEquP(F, [s, i]);$

$$\{[1000., 0.]\}\tag{18}$$

> $TimeSeries(F, [s, i], [800, 200], .01, 10, 1);$



> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10);$



> #(ii)

> $N := 1000:$

$\mathrm{nu} := 3:$

$F := SIRS\left(s, i, \frac{0.9 \cdot \mathrm{nu}}{N}, 6, \mathrm{nu}, N\right);$

$$F := [-0.002700000000\, s\, i + 6000 - 6\, s - 6\, i, 0.002700000000\, s\, i - 3\, i]\tag{19}$$

> $EquP(F, [s, i]);$

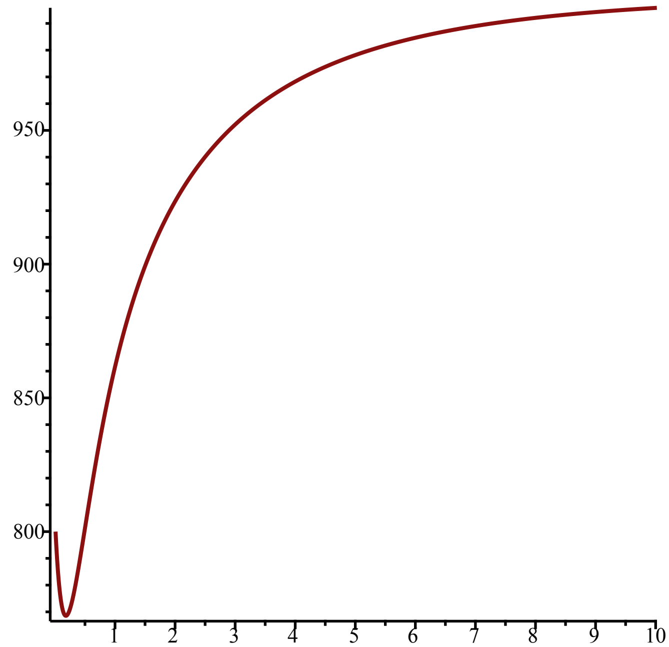
(20)

$\{[1000., 0.], [1111.111111, -74.07407407]\}$ **(20)**

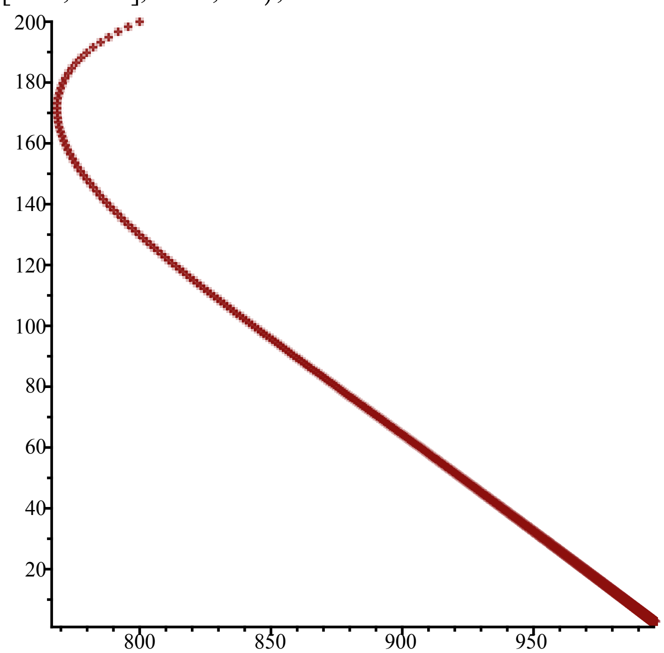
> $SEquP(F, [s, i]);$

$\{[1000., 0.]\}$ **(21)**

> $TimeSeries(F, [s, i], [800, 200], .01, 10, 1);$



> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10);$



> #(iii)

> $N := 1000 :$
$\mathrm{nu} := 4 :$
$F := SIRS\left(s, i, \frac{0.9 \cdot \mathrm{nu}}{N}, 1, \mathrm{nu}, N\right);$

$F := [-0.003600000000\, s\, i + 1000 - s - i, 0.003600000000\, s\, i - 4\, i]$ **(22)**

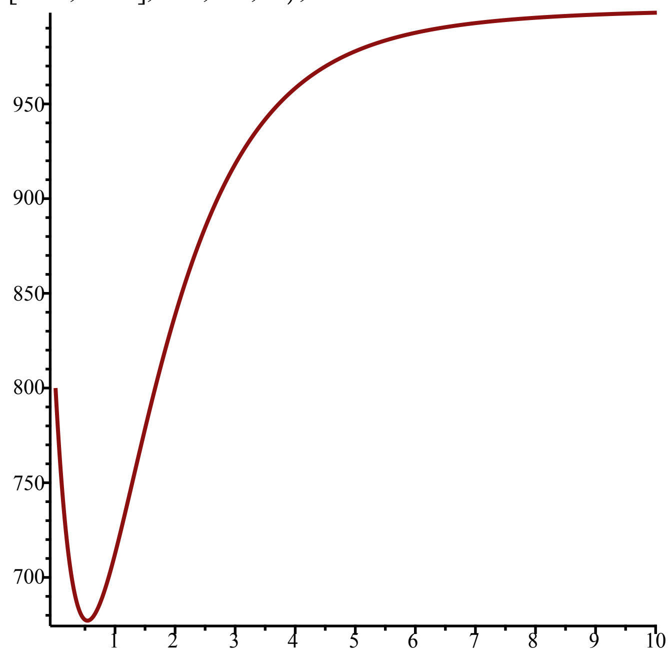
```
> EquP(F, [s, i]);
```

{[1000., 0.], [1111.111111, −22.22222222]} **(23)**

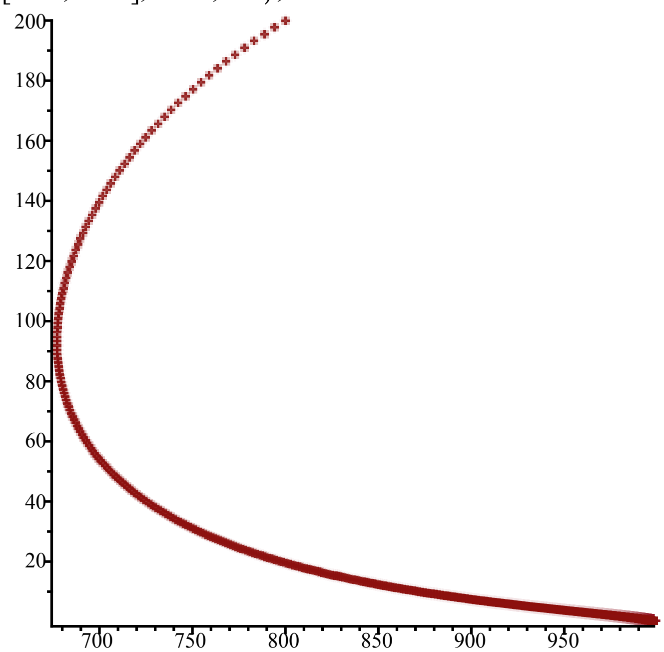
```
> SEquP(F, [s, i]);
```

{[1000., 0.]} **(24)**

```
> TimeSeries(F, [s, i], [800, 200], .01, 10, 1);
```




```
> PhaseDiag(F, [s, i], [800, 200], 0.01, 10);
```




```
> #(iv)
> N := 1000 :
  nu := 7 :
```

$F := SIRS\left(s, i, \frac{0.9 \cdot \mathrm{nu}}{N}, 10, \mathrm{nu}, N\right);$

(25)

$$F := [-0.006300000000\, s\, i + 10000 - 10\, s - 10\, i,\ 0.006300000000\, s\, i - 7\, i] \quad (25)$$

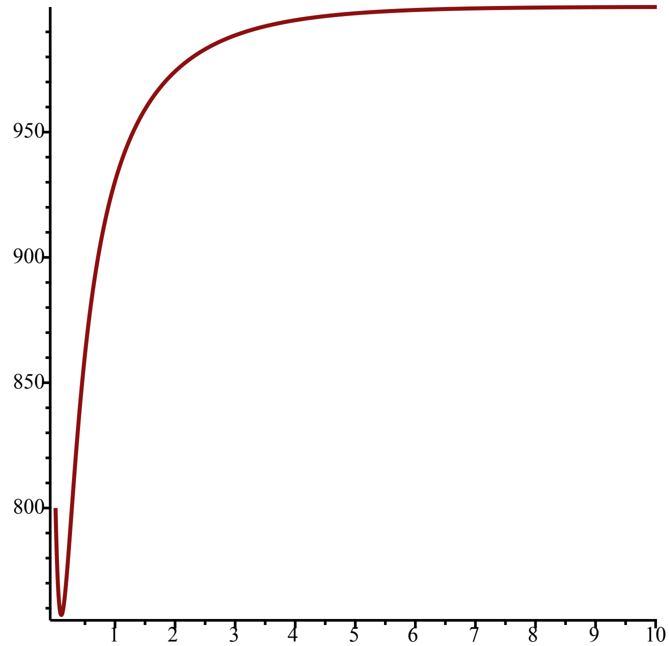
```
> EquP(F, [s, i]);
```

$$\{[1000., 0.], [1111.111111, -65.35947712]\} \quad (26)$$

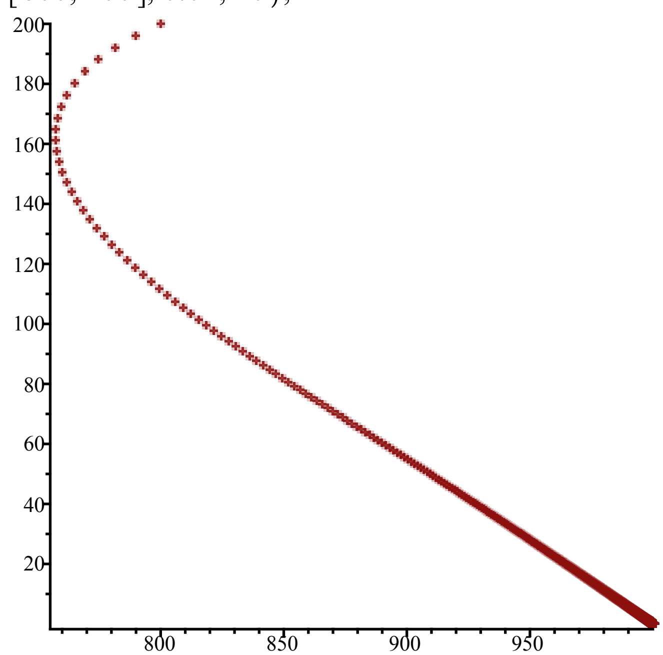
```
> SEquP(F, [s, i]);
```

$$\{[1000., 0.]\} \quad (27)$$

```
> TimeSeries(F, [s, i], [800, 200], .01, 10, 1);
```




```
> PhaseDiag(F, [s, i], [800, 200], 0.01, 10);
```




```
> #Question 1: When Beta = 3.9
> #(i)
> N := 1000 :
  nu := 2 :
```

$$F := SIRS\left(s, i, \frac{3.9 \cdot \mathrm{nu}}{N}, 5, \mathrm{nu}, N\right);$$

$$F := [-0.007800000000\, s\, i + 5000 - 5\, s - 5\, i, 0.007800000000\, s\, i - 2\, i] \quad \textbf{(28)}$$

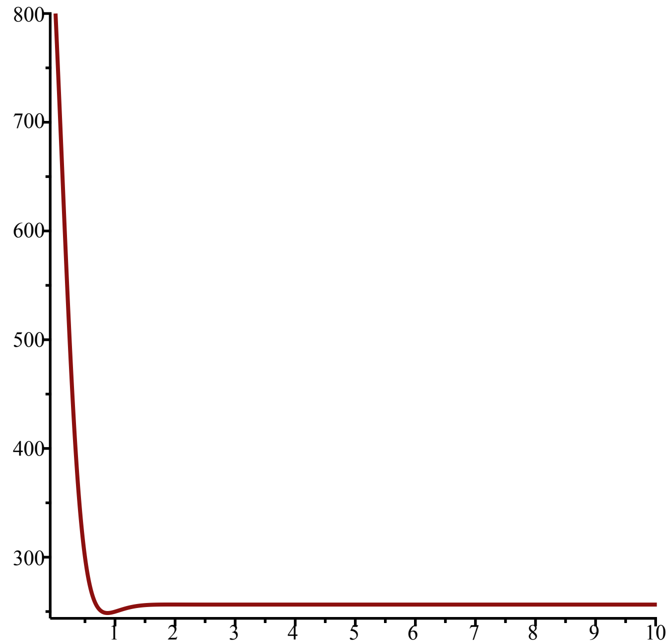
> $EquP(F, [s, i]);$

$$\{[256.4102564, 531.1355311], [1000., 0.]\} \quad \textbf{(29)}$$

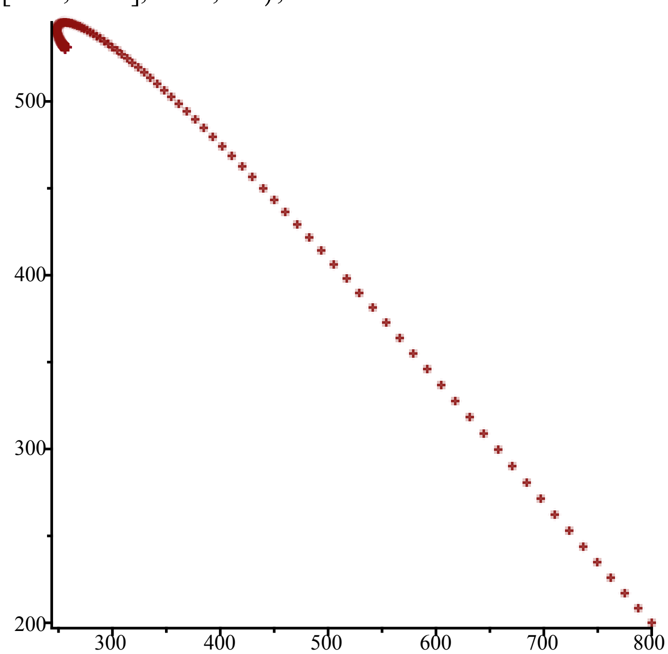
> $SEquP(F, [s, i]);$

$$\{[256.4102564, 531.1355311]\} \quad \textbf{(30)}$$

> $TimeSeries(F, [s, i], [800, 200], .01, 10, 1);$



> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10);$



> #(ii)

> $N := 1000:$

nu := 3 :

$F := SIRS\left(s, i, \frac{3.9 \cdot \text{nu}}{N}, 6, \text{nu}, N\right);$

$$F := [-0.01170000000\, s\, i + 6000 - 6\, s - 6\, i, 0.01170000000\, s\, i - 3\, i] \quad \textbf{(31)}$$

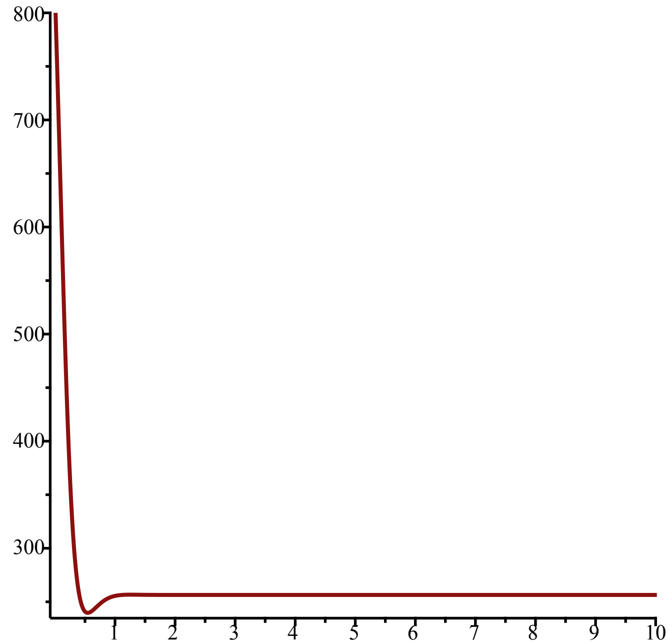
> $EquP(F, [s, i]);$

$$\{[256.4102564, 495.7264957], [1000., 0.]\} \quad \textbf{(32)}$$

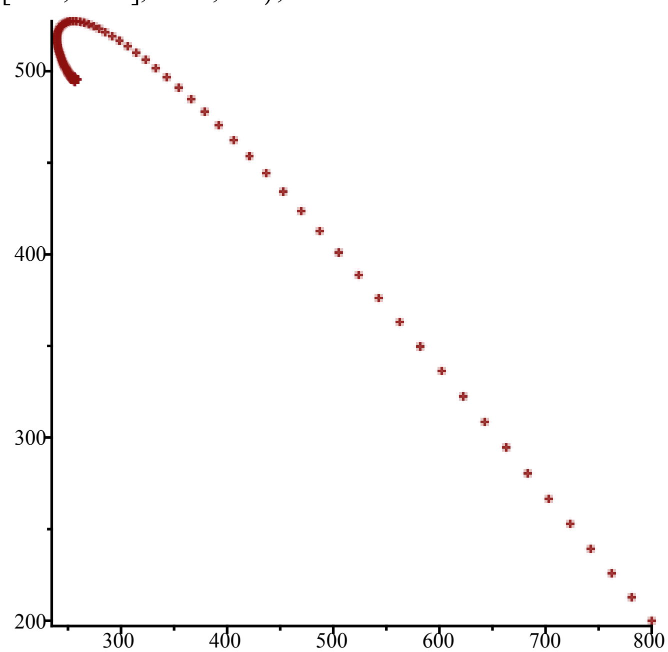
> $SEquP(F, [s, i]);$

$$\{[256.4102564, 495.7264957]\} \quad \textbf{(33)}$$

> $TimeSeries(F, [s, i], [800, 200], .01, 10, 1);$



> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10);$



> #(iii)

> $N := 1000:$
$nu := 4:$
$F := SIRS\left(s, i, \frac{3.9 \cdot nu}{N}, 1, nu, N\right);$

$$F := [-0.01560000000\, s\, i + 1000 - s - i, 0.01560000000\, s\, i - 4\, i] \quad \textbf{(34)}$$

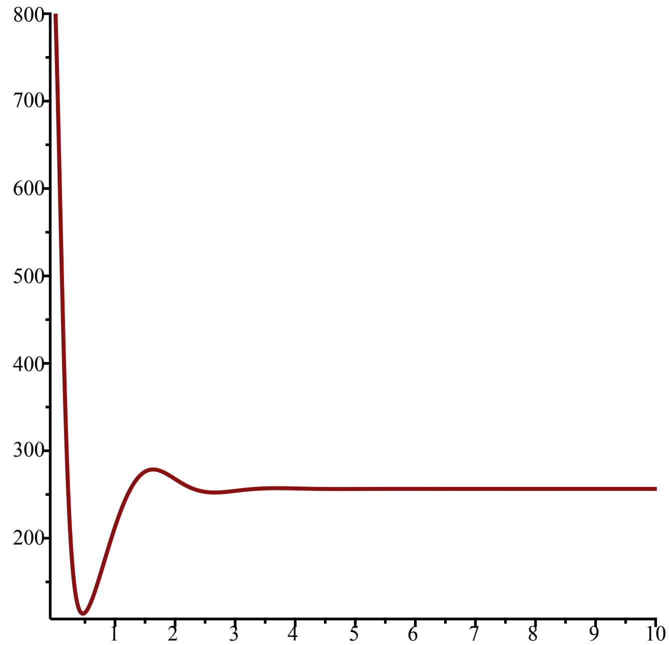
> $EquP(F, [s, i]);$

$$\{[256.4102564, 148.7179487], [1000., 0.]\} \quad \textbf{(35)}$$

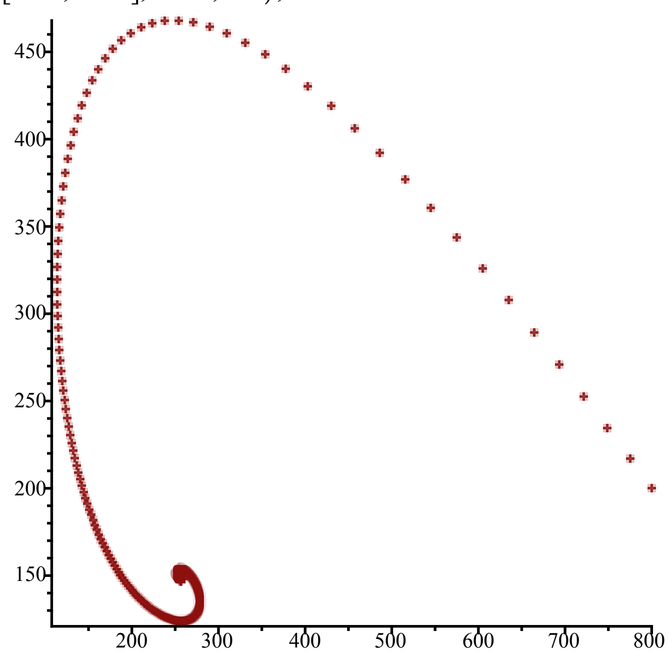
> $SEquP(F, [s, i]);$

$$\{[256.4102564, 148.7179487]\} \quad \textbf{(36)}$$

> $TimeSeries(F, [s, i], [800, 200], .01, 10, 1);$



> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10);$



> #(iv)

> $N := 1000:$
> $\text{nu} := 7:$
> $F := SIRS\left(s, i, \frac{3.9\cdot\text{nu}}{N}, 10, \text{nu}, N\right);$

$$F := [-0.02730000000\, s\, i + 10000 - 10\, s - 10\, i, 0.02730000000\, s\, i - 7\, i] \quad \textbf{(37)}$$

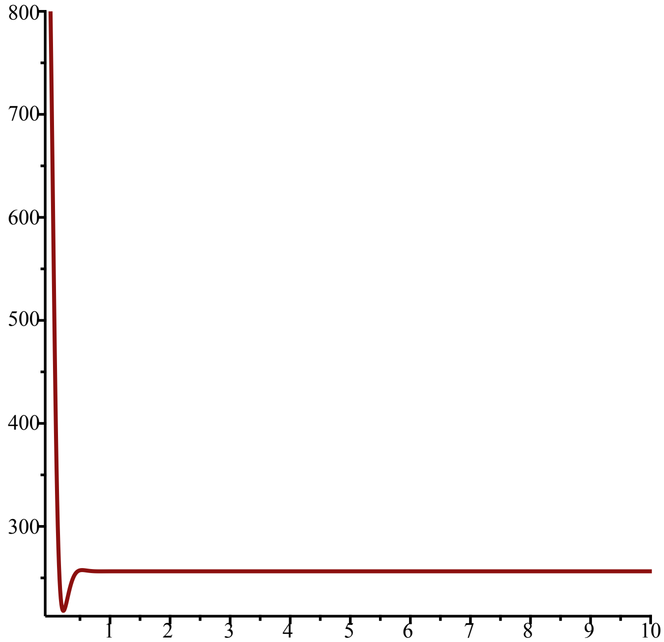
> $EquP(F, [s, i]);$

$$\{[256.4102564, 437.4057315], [1000., 0.]\} \quad \textbf{(38)}$$

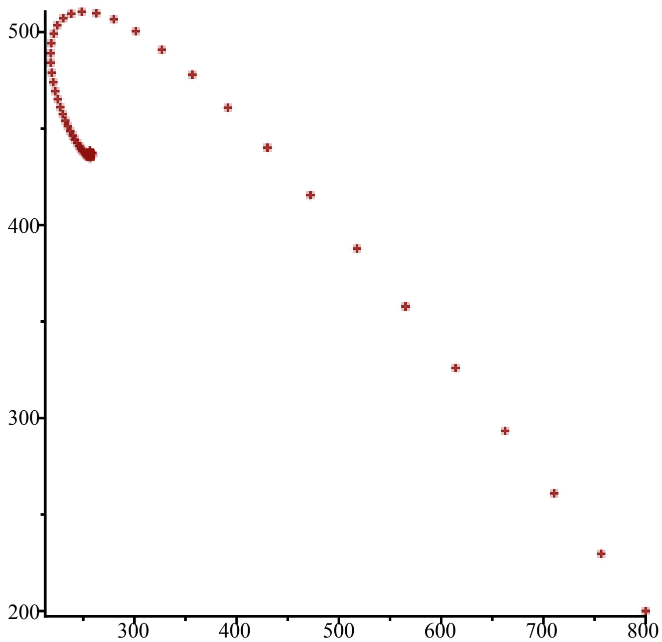
> $SEquP(F, [s, i]);$

$$\{[256.4102564, 437.4057315]\} \quad \textbf{(39)}$$

> $TimeSeries(F, [s, i], [800, 200], .01, 10, 1);$



> $PhaseDiag(F, [s, i], [800, 200], 0.01, 10);$



> #Question 2

> $F1 := RandNice([x, y], 3);$

$$F1 := [(3 - 3x - y)(2 - x - y), (2 - 2x - 3y)(3 - 2x - 2y)] \quad \textbf{(40)}$$

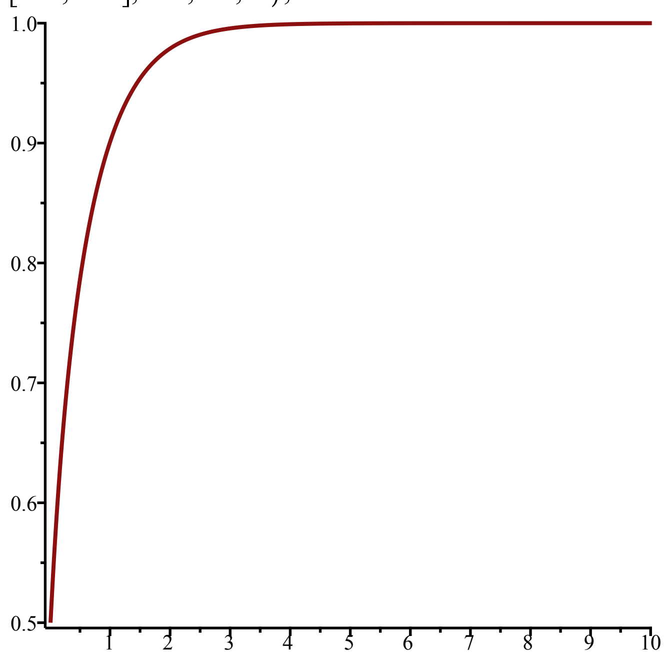
> $EquP(F1, [x, y]);$

$$\left\{[1, 0], [4, -2], \left[\frac{3}{4}, \frac{3}{4}\right]\right\} \quad \textbf{(41)}$$

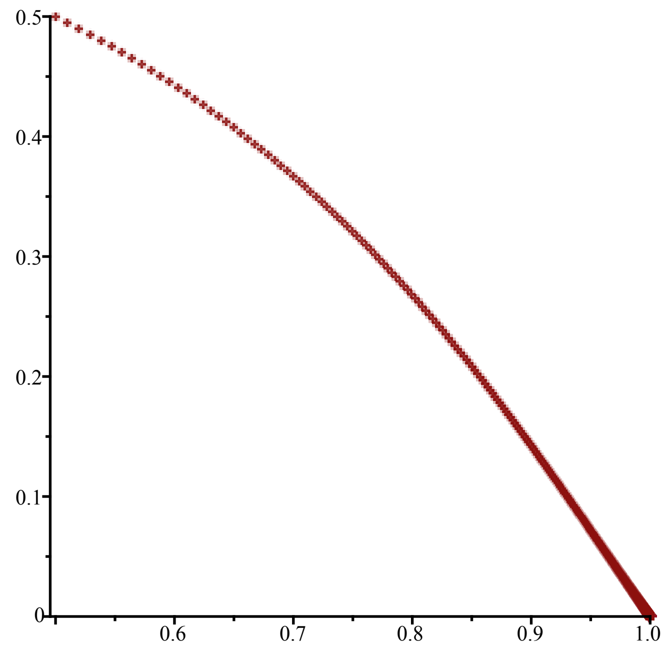
> $SEquP(F1, [x, y]);$

$$\{[1., 0.]\} \quad \textbf{(42)}$$

> $TimeSeries(F1, [x, y], [0.5, 0.5], .01, 10, 1);$



> $PhaseDiag(F1, [x, y], [0.5, 0.5], 0.01, 10);$



> $F2 := RandNice([x, y], 3);$

$$F2 := [(1-2x-y)(2-3x-3y), (3-2x-y)(3-3x-3y)] \quad \textbf{(43)}$$

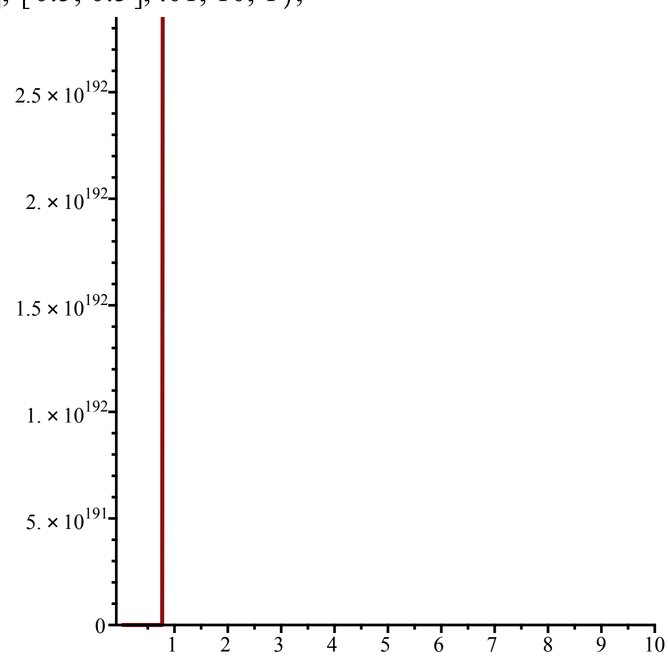
> $EquP(F2, [x, y]);$

$$\left\{[0, 1], \left[\frac{7}{3}, -\frac{5}{3}\right]\right\} \quad \textbf{(44)}$$

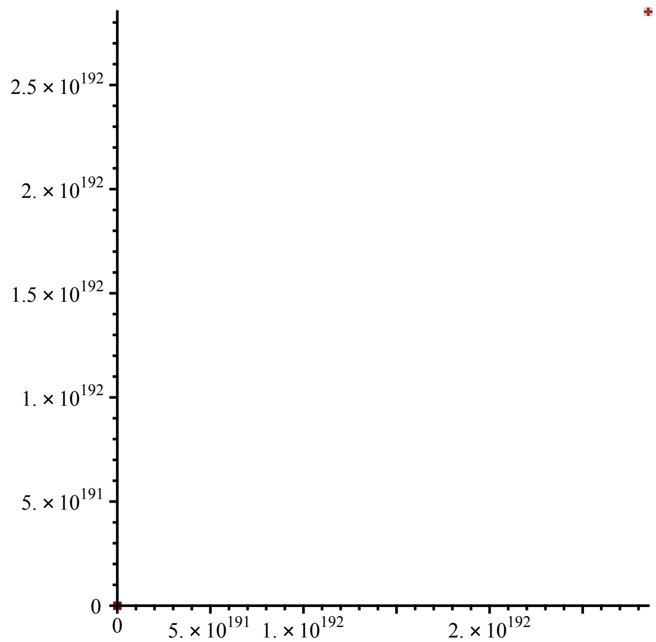
> $SEquP(F2, [x, y]);$

$$\varnothing \quad \textbf{(45)}$$

> $TimeSeries(F2, [x, y], [0.5, 0.5], .01, 10, 1);$



> $PhaseDiag(F2, [x, y], [0.5, 0.5], 0.01, 10);$



> $F3 := RandNice([x, y], 3);$

$$F3 := [(1 - 3x - 3y)(2 - 3x - y), (1 - 3x - y)(1 - 2x - 2y)] \quad \textbf{(46)}$$

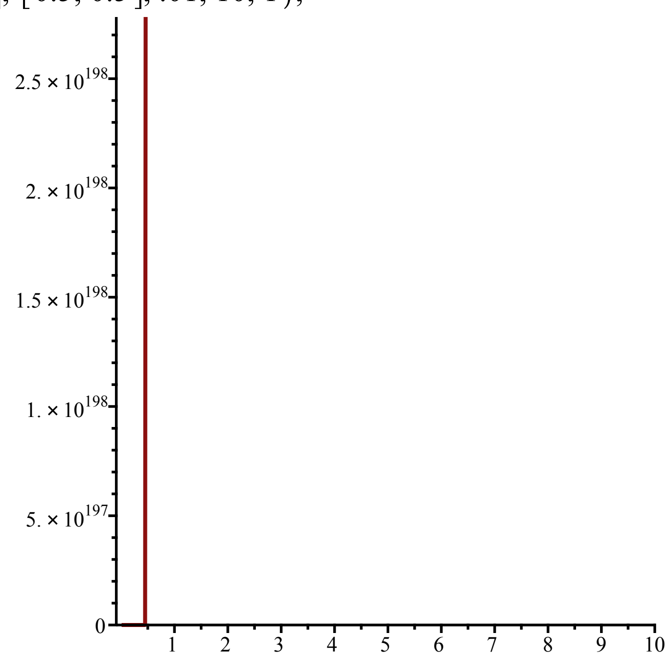
> $EquP(F3, [x, y]);$

$$\left\{\left[\frac{1}{3}, 0\right], \left[\frac{3}{4}, -\frac{1}{4}\right]\right\} \quad \textbf{(47)}$$

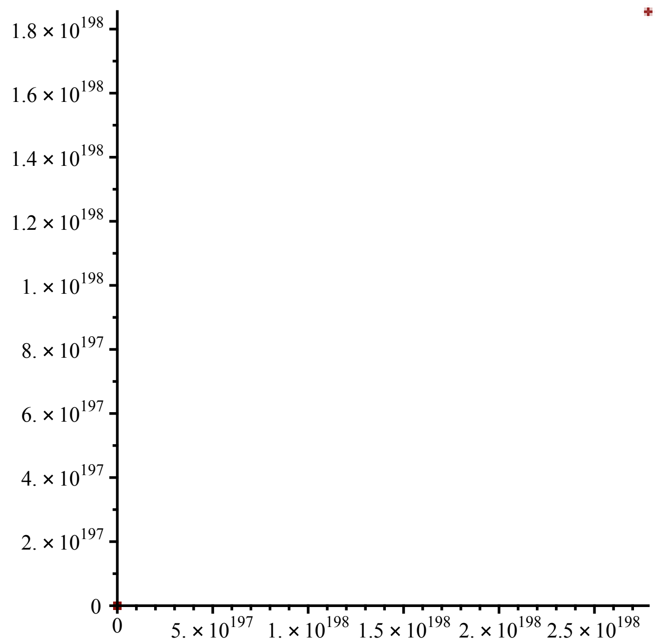
> $SEquP(F3, [x, y]);$

$$\emptyset \quad \textbf{(48)}$$

> $TimeSeries(F3, [x, y], [0.5, 0.5], .01, 10, 1);$



> $PhaseDiag(F3, [x, y], [0.5, 0.5], 0.01, 10);$



> #Question 3

> $\# \dfrac{x(n) = 3 + x(n-2) + x(n-3) + x(n-4)}{(1 + x(n-1) + x(n-3))}$

> # Orbk takes too long to compute past K2=40. I am trying to figure out a solution to this problem

> $evalf\left(Orbk\left(4, z, \dfrac{(3 + z[3] + z[2] + z[1])}{(1 + z[4] + z[2])}, [1, 4, 19, 32], 30, 40\right)\right);$

[1.835435444, 1.819261539, 1.822007470, 1.826194343, 1.818049841, 1.822480262, 1.824702558, 1.820999184, 1.823957273, 1.823989772, 1.821803818] **(49)**

> $E := ToSys\left(4, z, \dfrac{(3 + z[3] + z[2] + z[1])}{(1 + z[4] + z[2])}\right);$

$$E := \left[\frac{3 + z_3 + z_2 + z_1}{1 + z_4 + z_2}, z_1, z_2, z_3\right], [z_1, z_2, z_3, z_4] \quad \textbf{(50)}$$

> $SFP(E);$

{[1.822875656, 1.822875656, 1.822875656, 1.822875656]} **(51)**

>