Homework 20

Charles Griebell

DO NOT USE SIRS Demo to get the answer. SIRS demo in the print statement appears to have its own values of bata predetermined?

Note: For PRoblem 1, seeing all the numbers got really confusing and I was mixing them up in my head while reading. I am assuming that there interesting differences between the problems.

```
> read `C:/Users/cgrie/Dynam Models Bio/Homeworks/HW21/DMB.txt` ;
  read `C:/Users/cgrie/Dynam Models Bio/Homeworks/HW19/M19.txt` ;
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)

accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:

http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .

Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,

type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();

For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();

For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();

For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

Question 1:

Use each of the following procedures:

--- SIRS

---EquP

---SequP

---TimeSeries

---PhaseDiag

to
(i) compute equilibrium points
SIRS()
(ii)compute stable equilibrium points
(iii) plot

of the SIRS model that has a total of 1000 people at the start with 800 susceptible and 200 infected and none removed yet.

We will have our mesh size equal 0.01

```
> Help(SIRS);
  HelpCDM();
```

SIRS(s,i,beta,gamma,nu,N): The SIRS dynamical model with parameters beta,gamma, nu,N (see section 6.6 of Edelstein-Keshet), s is the number of Susceptibles, i is the number of infected, (the number of removed is given by N-s-i). N is the total population. Try:

SIRS(s,i,beta,gamma,nu,N);

The procedures giving the underlying transformations, followed by the list of variables used are:

ChemoStat, GeneNet, Lotka, RandNice, SIRS , SIRSdemo, Volterra, VolterraM **(2)**

```
> print(SIRS);
```

proc$(s, i, \mathrm{beta}, \mathrm{gamma}, \mathrm{nu}, N)$ **(3)**

$[-\mathrm{beta}*s*i+\mathrm{gamma}*(N-s-i), \mathrm{beta}*s*i-\mathrm{nu}*i]$

end proc

For

```
> print(SIRSdemo);
```

proc$(N, IN, \mathrm{gamma}, \mathrm{nu}, h, A)$ **(4)**

```
    local L, beta, i;
    print(`This is a numerical demonstration of the R0 phenomenon in the SIRS model using
    discretization with mesh size=`, h, `and letting it run until time t=`, A);
    print(`with population size`, N, `and fixed parameters nu=`, nu, `and gamma=`, gamma);
    print(`where we change beta from 0.2*nu/N to 4*nu/N `);
    print(`Recall that the epidemic will persist if beta exceeds nu/N, that in this case is`, nu/N);
    print(`We start with `, IN, `infected individuals, 0 removed and hence`, N - IN, susceptible);
    print(`We will show what happens once time is close to`, A);
    for i by 2 to 40 do
        beta := 1/10*i*nu/N;
        print(`beta is`, 1/10*i, `times the threshold value`);
        L := Dis2(SIRS(s, i, beta, gamma, nu, N), s, i, [N - IN, IN], h, A);
        print(`the long-term behavior is`);
        print([op(nops(L) - 3 ..nops(L), L)])
    end do
end proc
```

For

$\beta = 0.3 \cdot \frac{\nu}{1000}$, $\beta = 0.9 \cdot \frac{\nu}{1000}$, $\beta = 3.9 \cdot \frac{\nu}{1000}$

For each of the following choices

(i) $\nu = 2$, $\gamma = 7$

```
> t1b1 := SIRS(800,200,0.3*2/1000,7,2,1000);
  t1b2 := SIRS(800,200,0.9*2/1000,7,2,1000);
  t1b3 := SIRS(800,200,3.9*2/1000,7,2,1000);

  print(`If we used SIRSdemo`);

  t1b1 := SIRSdemo(1000,200,7,2,0.01,10);
  t1b2 := SIRSdemo(1000,200,7,2,0.01,10);
  t1b3 := SIRSdemo(1000,200,7,2,0.01,10);

  #Equilibrium Points
```

$$t1b1 := [-96.00000000, -304.0000000]$$

$$t1b2 := [-288.0000000, -112.0000000]$$

$$t1b3 := [-1248.000000, 848.000000]$$

If we used SIRSdemo

This is a numerical demonstration of the R0 phenomenon in the SIRS model using discretization with mesh size=, 0.01, *and letting it run until time t=*, 10

with population size, 1000, *and fixed parameters nu=*, 2, *and gamma=*, 7
*where we change beta from 0.2*nu/N to 4*nu/N*

Recall that the epidemic will persist if beta exceeds nu/N, that in this case is, $\frac{1}{500}$

We start with , 200, *infected individuals, 0 removed and hence*, 800, *susceptible*
We will show what happens once time is close to, 10

beta is, $\frac{1}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [998.9714573, 0.9819979429]], [9.99, [998.9714573, 0.9819979429]], [10.00, [998.9714573, 0.9819979429]], [10.01, [998.9714573, 0.9819979429]]]

beta is, $\frac{3}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [996.7436938, 2.957941386]], [9.99, [996.7436938, 2.957941386]], [10.00, [996.7436938, 2.957941386]], [10.01, [996.7436938, 2.957941386]]]

beta is, $\frac{1}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [994.2897923, 4.949714490]], [9.99, [994.2897923, 4.949714490]], [10.00, [994.2897923, 4.949714490]], [10.01, [994.2897923, 4.949714490]]]

beta is, $\frac{7}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [991.6117429, 6.957177951]], [9.99, [991.6117429, 6.957177951]], [10.00, [991.6117429, 6.957177951]], [10.01, [991.6117429, 6.957177951]]]

beta is, $\frac{9}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [988.7118377, 8.980171318]], [9.99, [988.7118377, 8.980171318]], [10.00, [988.7118377, 8.980171318]], [10.01, [988.7118377, 8.980171318]]]

beta is, $\frac{11}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [985.5926647, 11.01851342]], [9.99, [985.5926647, 11.01851342]], [10.00, [985.5926647, 11.01851342]], [10.01, [985.5926647, 11.01851342]]]

beta is, $\frac{13}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [982.2571008, 13.07200290]], [9.99, [982.2571008, 13.07200290]], [10.00, [982.2571008, 13.07200290]], [10.01, [982.2571008, 13.07200290]]]

beta is, $\frac{3}{2}$*, times the threshold value*

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[[9.98, [978.7083031, 15.14041874]], [9.99, [978.7083031, 15.14041874]], [10.00, [978.7083031, 15.14041874]], [10.01, [978.7083031, 15.14041874]]]

beta is, $\frac{17}{10}$*, times the threshold value*

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[[9.98, [974.9497004, 17.22352093]], [9.99, [974.9497004, 17.22352093]], [10.00, [974.9497004, 17.22352093]], [10.01, [974.9497004, 17.22352093]]]

beta is, $\frac{19}{10}$*, times the threshold value*

the long-term behavior is

[[9.98, [970.9849828, 19.32105116]], [9.99, [970.9849828, 19.32105116]], [10.00, [970.9849828, 19.32105116]], [10.01, [970.9849828, 19.32105116]]]

beta is, $\frac{21}{10}$*, times the threshold value*

the long-term behavior is

[[9.98, [966.8180914, 21.43273356]], [9.99, [966.8180914, 21.43273356]], [10.00, [966.8180914, 21.43273356]], [10.01, [966.8180914, 21.43273356]]]

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beta is, $\frac{5}{2}$*, times the threshold value*

the long-term behavior is

[[9.98, [957.8947362, 25.69736842]], [9.99, [957.8947362, 25.69736842]], [10.00, [957.8947362, 25.69736842]], [10.01, [957.8947362, 25.69736842]]]

beta is, $\frac{27}{10}$*, times the threshold value*

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[[9.98, [943.1050298, 32.19264787]], [9.99, [943.1050298, 32.19264787]], [10.00, [943.1050298, 32.19264787]], [10.01, [943.1050298, 32.19264787]]]

beta is, $\frac{33}{10}$, *times the threshold value*

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[[9.98, [937.8203878, 34.38257280]], [9.99, [937.8203878, 34.38257280]], [10.00, [937.8203878, 34.38257280]], [10.01, [937.8203878, 34.38257280]]]

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beta is, $\frac{39}{10}$, *times the threshold value*

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$t1b1 := ()$

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$$\textit{beta is, } \frac{39}{10}\textit{, times the threshold value}$$

the long-term behavior is

[[9.98, [920.9769715, 41.02161195]], [9.99, [920.9769715, 41.02161195]], [10.00, [920.9769715, 41.02161195]], [10.01, [920.9769715, 41.02161195]]]

$$t1b3 := (\ ) \quad \textbf{(5)}$$

```
> Help(SIRS);
  TimeSeries(SIRS(800,200,1,beta,N),[1,N],evalf([1,1]),0.01,10,1);
```

SIRS(s,i,beta,gamma,nu,N): The SIRS dynamical model with parameters beta,gamma, nu,N (see section 6.6 of Edelstein-Keshet), s is the number of

Susceptibles, i is the number of infected, (the number of removed is given by N-s-i). N is the total population. Try:

SIRS(s,i,beta,gamma,nu,N);

Error, invalid input: SIRS uses a 6th argument, N, which is missing
|C:/Users/cgrie/Dynam Models Bio/Homeworks/HW19/M19.txt:93|

(ii) $\nu = 3$, $\gamma = 6$

```
> t2b1 := SIRS(800,200,0.3*3/1000,3,6,1000);
  t2b2 := SIRS(800,200,0.9*3/1000,3,6,1000);
  t2b3 := SIRS(800,200,3.9*3/1000,3,6,1000);

  #Equilibrium Points
```

$$t2b1 := [-144.0000000, -1056.000000]$$
$$t2b2 := [-432.0000000, -768.0000000]$$
$$t2b3 := [-1872.000000, 672.000000] \quad \textbf{(6)}$$

(iii) $\nu = 4$, $\gamma = 1$

```
> t3b1 := SIRSdemo(800,200,0.3*4/1000,4,1,1000):
  t3b2 := SIRSdemo(800,200,0.9*4/1000,4,1,1000):
  t3b3 := SIRSdemo(800,200,3.9*4/1000,4,1,1000):

  #Equilibrium Points
```

This is a numerical demonstration of the R0 phenomenon in the SIRS model using discretization with mesh size=, 1, *and letting it run until time t=*, 1000 *with population size*, 800, *and fixed parameters nu*=, 4, *and gamma*=, 0.001200000000

*where we change beta from 0.2*nu/N to 4*nu/N*

Recall that the epidemic will persist if beta exceeds nu/N, that in this case is, $\frac{1}{200}$

We start with , 200, *infected individuals, 0 removed and hence*, 600, *susceptible*

We will show what happens once time is close to, 1000

beta is, $\frac{1}{10}$, *times the threshold value*

the long-term behavior is

[[998, [570.6007068, −2.714694026]], [999, [570.5894856, −2.714699647]], [1000, [570.5782835, −2.714705257]], [1001, [570.5671004, −2.714710858]]]

beta is, $\frac{3}{10}$, *times the threshold value*

the long-term behavior is

[[998, [169.2368529, −8.238396824]], [999, [169.2286028, −8.238434162]], [1000, [169.2203998, −8.238471287]], [1001, [169.2122435, −8.238508201]]]

beta is, $\frac{1}{2}$, *times the threshold value*

the long-term behavior is

[[998, [69.63560089, −14.12955489]], [999, [69.63559316, −14.12955499]], [1000, [69.63558553, −14.12955509]], [1001, [69.63557801, −14.12955518]]]

beta is, $\frac{7}{10}$, *times the threshold value*

the long-term behavior is

[[998, [37.02723754, −20.09283268]], [999, [37.02723754, −20.09283268]], [1000, [37.02723754, −20.09283268]], [1001, [37.02723754, −20.09283268]]]

beta is, $\frac{9}{10}$, *times the threshold value*

the long-term behavior is

[[998, [22.76259004, −26.07811510]], [999, [22.76259004, −26.07811510]], [1000, [22.76259004, −26.07811510]], [1001, [22.76259004, −26.07811510]]]

beta is, $\frac{11}{10}$, *times the threshold value*

the long-term behavior is

[[998, [15.34521888, −32.07161426]], [999, [15.34521888, −32.07161426]], [1000, [15.34521888, −32.07161426]], [1001, [15.34521888, −32.07161426]]]

beta is, $\frac{13}{10}$, *times the threshold value*

the long-term behavior is

[[998, [11.01983669, −38.06882380]], [999, [11.01983669, −38.06882380]], [1000, [11.01983669, −38.06882380]], [1001, [11.01983669, −38.06882380]]]

beta is, $\frac{3}{2}$, *times the threshold value*

the long-term behavior is

[[998, [8.284960426, −44.06794195]], [999, [8.284960426, −44.06794195]], [1000, [8.284960426, −44.06794195]], [1001, [8.284960426, −44.06794195]]]

beta is, $\frac{17}{10}$, *times the threshold value*

the long-term behavior is

[[998, [6.448867539, −50.06813864]], [999, [6.448867539, −50.06813864]], [1000, [6.448867539, −50.06813864]], [1001, [6.448867539, −50.06813864]]]

beta is, $\frac{19}{10}$, *times the threshold value*

the long-term behavior is

[[998, [5.157952671, −56.06898954]], [999, [5.157952671, −56.06898954]], [1000, [5.157952671, −56.06898954]], [1001, [5.157952671, −56.06898954]]]

beta is, $\frac{21}{10}$, *times the threshold value*

the long-term behavior is

[[998, [4.216508797, −62.07025981]], [999, [4.216508797, −62.07025981]], [1000, [4.216508797, −62.07025981]], [1001, [4.216508797, −62.07025981]]]

beta is, $\frac{23}{10}$, *times the threshold value*

the long-term behavior is

[[998, [3.509220927, −68.07181106]], [999, [3.509220927, −68.07181106]], [1000, [3.509220927, −68.07181106]], [1001, [3.509220927, −68.07181106]]]

beta is, $\frac{5}{2}$, *times the threshold value*

the long-term behavior is

[[998, [2.964615876, −74.07355754]], [999, [2.964615876, −74.07355754]], [1000, [2.964615876, −74.07355754]], [1001, [2.964615876, −74.07355754]]]

beta is, $\frac{27}{10}$, *times the threshold value*

the long-term behavior is

[[998, [2.536505333, −80.07544381]], [999, [2.536505333, −80.07544381]], [1000, [2.536505333, −80.07544381]], [1001, [2.536505333, −80.07544381]]]

beta is, $\frac{29}{10}$, *times the threshold value*

the long-term behavior is

[[998, [2.193976761, −86.07743277]], [999, [2.193976761, −86.07743277]], [1000, [2.193976761, −86.07743277]], [1001, [2.193976761, −86.07743277]]]

beta is, $\frac{31}{10}$, *times the threshold value*

the long-term behavior is

[[998, [1.915715176, −92.07949886]], [999, [1.915715176, −92.07949886]], [1000, [1.915715176, −92.07949886]], [1001, [1.915715176, −92.07949886]]]

beta is, $\frac{33}{10}$, *times the threshold value*

the long-term behavior is

[[998, [1.686641012, −98.08162397]], [999, [1.686641012, −98.08162397]], [1000, [1.686641012, −98.08162397]], [1001, [1.686641012, −98.08162397]]]

beta is, $\frac{7}{2}$, *times the threshold value*

the long-term behavior is

[[998, [1.495844876, −104.0837950]], [999, [1.495844876, −104.0837950]], [1000, [1.495844876, −104.0837950]], [1001, [1.495844876, −104.0837950]]]

beta is, $\frac{37}{10}$, *times the threshold value*

the long-term behavior is

[[998, [1.335277818, −110.0860023]], [999, [1.335277818, −110.0860023]], [1000, [1.335277818, −110.0860023]], [1001, [1.335277818, −110.0860023]]]

beta is, $\frac{39}{10}$, *times the threshold value*

the long-term behavior is

[[998, [1.198897204, −116.0882387]], [999, [1.198897204, −116.0882387]], [1000, [1.198897204, −116.0882387]], [1001, [1.198897204, −116.0882387]]]

This is a numerical demonstration of the R0 phenomenon in the SIRS model using discretization with mesh size=, 1, *and letting it run until time t=*, 1000

with population size, 800, *and fixed parameters nu=*, 4, *and gamma=*, 0.003600000000

*where we change beta from 0.2*nu/N to 4*nu/N*

Recall that the epidemic will persist if beta exceeds nu/N, that in this case is, $\frac{1}{200}$

We start with , 200, *infected individuals, 0 removed and hence*, 600, *susceptible*

We will show what happens once time is close to, 1000

beta is, $\frac{1}{10}$, *times the threshold value*

the long-term behavior is

[[998, [699.8712665, −2.650067845]], [999, [699.8781943, −2.650064367]], [1000, [699.8850937, −2.650060903]], [1001, [699.8919648, −2.650057453]]]

beta is, $\frac{3}{10}$*, times the threshold value*

the long-term behavior is

[[998, [354.2961877, −7.405664437]], [999, [354.2955886, −7.405667155]], [1000, [354.2949943, −7.405669851]], [1001, [354.2944048, −7.405672526]]]

beta is, $\frac{1}{2}$*, times the threshold value*

the long-term behavior is

[[998, [177.7640147, −12.77794981]], [999, [177.7640141, −12.77794982]], [1000, [177.7640135, −12.77794982]], [1001, [177.7640129, −12.77794983]]]

beta is, $\frac{7}{10}$*, times the threshold value*

the long-term behavior is

[[998, [101.5943080, −18.51093945]], [999, [101.5943080, −18.51093945]], [1000, [101.5943080, −18.51093945]], [1001, [101.5943080, −18.51093945]]]

beta is, $\frac{9}{10}$*, times the threshold value*

the long-term behavior is

[[998, [64.57142868, −24.38485714]], [999, [64.57142868, −24.38485714]], [1000, [64.57142868, −24.38485714]], [1001, [64.57142868, −24.38485714]]]

beta is, $\frac{11}{10}$*, times the threshold value*

the long-term behavior is

[[998, [44.31201255, −30.31912324]], [999, [44.31201255, −30.31912324]], [1000, [44.31201255, −30.31912324]], [1001, [44.31201255, −30.31912324]]]

beta is, $\frac{13}{10}$*, times the threshold value*

the long-term behavior is

[[998, [32.15891038, −36.28257207]], [999, [32.15891038, −36.28257207]], [1000, [32.15891038, −36.28257207]], [1001, [32.15891038, −36.28257207]]]

beta is, $\frac{3}{2}$*, times the threshold value*

the long-term behavior is

[[998, [24.34108531, −42.26162790]], [999, [24.34108531, −42.26162790]], [1000, [24.34108531, −42.26162790]], [1001, [24.34108531, −42.26162790]]]

beta is, $\frac{17}{10}$*, times the threshold value*

the long-term behavior is

[[998, [19.03308578, −48.24971910]], [999, [19.03308578, −48.24971910]], [1000, [19.03308578, −48.24971910]], [1001, [19.03308578, −48.24971910]]]

$$\textit{beta is, } \frac{19}{10}\textit{, times the threshold value}$$

the long-term behavior is

[[998, [15.27213473, −54.24337968]], [999, [15.27213473, −54.24337968]], [1000, [15.27213473, −54.24337968]], [1001, [15.27213473, −54.24337968]]]

$$\textit{beta is, } \frac{21}{10}\textit{, times the threshold value}$$

the long-term behavior is

[[998, [12.51405624, −60.24065060]], [999, [12.51405624, −60.24065060]], [1000, [12.51405624, −60.24065060]], [1001, [12.51405624, −60.24065060]]]

$$\textit{beta is, } \frac{23}{10}\textit{, times the threshold value}$$

the long-term behavior is

[[998, [10.43342038, −66.24036031]], [999, [10.43342038, −66.24036031]], [1000, [10.43342038, −66.24036031]], [1001, [10.43342038, −66.24036031]]]

$$\textit{beta is, } \frac{5}{2}\textit{, times the threshold value}$$

the long-term behavior is

[[998, [8.826320786, −72.24177475]], [999, [8.826320786, −72.24177475]], [1000, [8.826320786, −72.24177475]], [1001, [8.826320786, −72.24177475]]]

$$\textit{beta is, } \frac{27}{10}\textit{, times the threshold value}$$

the long-term behavior is

[[998, [7.559902201, −78.24441565]], [999, [7.559902201, −78.24441565]], [1000, [7.559902201, −78.24441565]], [1001, [7.559902201, −78.24441565]]]

$$\textit{beta is, } \frac{29}{10}\textit{, times the threshold value}$$

the long-term behavior is

[[998, [6.544682858, −84.24796086]], [999, [6.544682858, −84.24796086]], [1000, [6.544682858, −84.24796086]], [1001, [6.544682858, −84.24796086]]]

$$\textit{beta is, } \frac{31}{10}\textit{, times the threshold value}$$

the long-term behavior is

[[998, [5.718653171, −90.25218715]], [999, [5.718653171, −90.25218715]], [1000, [5.718653171, −90.25218715]], [1001, [5.718653171, −90.25218715]]]

beta is, $\frac{33}{10}$*, times the threshold value*

the long-term behavior is

[[998, [5.037766831, −96.25693596]], [999, [5.037766831, −96.25693596]], [1000, [5.037766831, −96.25693596]], [1001, [5.037766831, −96.25693596]]]

beta is, $\frac{7}{2}$*, times the threshold value*

the long-term behavior is

[[998, [4.470053563, −102.2620922]], [999, [4.470053563, −102.2620922]], [1000, [4.470053563, −102.2620922]], [1001, [4.470053563, −102.2620922]]]

beta is, $\frac{37}{10}$*, times the threshold value*

the long-term behavior is

[[998, [3.991861648, −108.2675707]], [999, [3.991861648, −108.2675707]], [1000, [3.991861648, −108.2675707]], [1001, [3.991861648, −108.2675707]]]

beta is, $\frac{39}{10}$*, times the threshold value*

the long-term behavior is

[[998, [3.585394582, −114.2733074]], [999, [3.585394582, −114.2733074]], [1000, [3.585394582, −114.2733074]], [1001, [3.585394582, −114.2733074]]]

This is a numerical demonstration of the R0 phenomenon in the SIRS model using discretization with mesh size=, 1, *and letting it run until time t=*, 1000

with population size, 800, *and fixed parameters nu=*, 4, *and gamma=*, 0.01560000000

*where we change beta from 0.2*nu/N to 4*nu/N*

Recall that the epidemic will persist if beta exceeds nu/N, that in this case is, $\frac{1}{200}$

We start with , 200, *infected individuals, 0 removed and hence*, 600, *susceptible*

We will show what happens once time is close to, 1000

beta is, $\frac{1}{10}$*, times the threshold value*

the long-term behavior is

[[998, [774.1863191, −2.612906841]], [999, [774.1863194, −2.612906840]], [1000, [774.1863197, −2.612906840]], [1001, [774.1863200, −2.612906840]]]

beta is, $\frac{3}{10}$*, times the threshold value*

the long-term behavior is

[[998, [618.5671617, −6.216447772]], [999, [618.5671617, −6.216447772]], [1000, [618.5671617, −6.216447772]], [1001, [618.5671617, −6.216447772]]]

beta is, $\frac{1}{2}$, *times the threshold value*

the long-term behavior is

$[[998, [441.3523149, -9.483096064]], [999, [441.3523149, -9.483096064]], [1000, [441.3523149, -9.483096064]], [1001, [441.3523149, -9.483096064]]]$

beta is, $\frac{7}{10}$, *times the threshold value*

the long-term behavior is

$[[998, [308.4987543, -13.44178052]], [999, [308.4987543, -13.44178052]], [1000, [308.4987543, -13.44178052]], [1001, [308.4987543, -13.44178052]]]$

beta is, $\frac{9}{10}$, *times the threshold value*

the long-term behavior is

$[[998, [219.9572201, -18.09173259]], [999, [219.9572201, -18.09173259]], [1000, [219.9572201, -18.09173259]], [1001, [219.9572201, -18.09173259]]]$

beta is, $\frac{11}{10}$, *times the threshold value*

the long-term behavior is

$[[998, [161.7398166, -23.21474110]], [999, [161.7398166, -23.21474110]], [1000, [161.7398166, -23.21474110]], [1001, [161.7398166, -23.21474110]]]$

beta is, $\frac{13}{10}$, *times the threshold value*

the long-term behavior is

$[[998, [122.6493511, -28.63612983]], [999, [122.6493511, -28.63612983]], [1000, [122.6493511, -28.63612983]], [1001, [122.6493511, -28.63612983]]]$

beta is, $\frac{3}{2}$, *times the threshold value*

the long-term behavior is

$[[998, [95.59718973, -34.24531616]], [999, [95.59718973, -34.24531616]], [1000, [95.59718973, -34.24531616]], [1001, [95.59718973, -34.24531616]]]$

beta is, $\frac{17}{10}$, *times the threshold value*

the long-term behavior is

$[[998, [76.29481577, -39.97539912]], [999, [76.29481577, -39.97539912]], [1000, [76.29481577, -39.97539912]], [1001, [76.29481577, -39.97539912]]]$

beta is, $\frac{19}{10}$, *times the threshold value*

the long-term behavior is

$[[998, [62.12952577, -45.78562060]], [999, [62.12952577, -45.78562060]], [1000,$

[62.12952577, −45.78562060]], [1001, [62.12952577, −45.78562060]]]

beta is, $\frac{21}{10}$*, times the threshold value*

the long-term behavior is

[[998, [51.47141044, −51.65055400]], [999, [51.47141044, −51.65055400]], [1000, [51.47141044, −51.65055400]], [1001, [51.47141044, −51.65055400]]]

beta is, $\frac{23}{10}$*, times the threshold value*

the long-term behavior is

[[998, [43.27454482, −57.55388290]], [999, [43.27454482, −57.55388290]], [1000, [43.27454482, −57.55388290]], [1001, [43.27454482, −57.55388290]]]

beta is, $\frac{5}{2}$*, times the threshold value*

the long-term behavior is

[[998, [36.84852180, −63.48483694]], [999, [36.84852180, −63.48483694]], [1000, [36.84852180, −63.48483694]], [1001, [36.84852180, −63.48483694]]]

beta is, $\frac{27}{10}$*, times the threshold value*

the long-term behavior is

[[998, [31.72533545, −69.43611523]], [999, [31.72533545, −69.43611523]], [1000, [31.72533545, −69.43611523]], [1001, [31.72533545, −69.43611523]]]

beta is, $\frac{29}{10}$*, times the threshold value*

the long-term behavior is

[[998, [27.57991287, −75.40264664]], [999, [27.57991287, −75.40264664]], [1000, [27.57991287, −75.40264664]], [1001, [27.57991287, −75.40264664]]]

beta is, $\frac{31}{10}$*, times the threshold value*

the long-term behavior is

[[998, [24.18141504, −81.38083007]], [999, [24.18141504, −81.38083007]], [1000, [24.18141504, −81.38083007]], [1001, [24.18141504, −81.38083007]]]

beta is, $\frac{33}{10}$*, times the threshold value*

the long-term behavior is

[[998, [21.36261382, −87.36805678]], [999, [21.36261382, −87.36805678]], [1000, [21.36261382, −87.36805678]], [1001, [21.36261382, −87.36805678]]]

beta is, $\frac{7}{2}$*, times the threshold value*

the long-term behavior is

[[998, [19.00015921, −93.36240248]], [999, [19.00015921, −93.36240248]], [1000, [19.00015921, −93.36240248]], [1001, [19.00015921, −93.36240248]]]

beta is, $\frac{37}{10}$, *times the threshold value*

the long-term behavior is

[[998, [17.00157121, −99.36242451]], [999, [17.00157121, −99.36242451]], [1000, [17.00157121, −99.36242451]], [1001, [17.00157121, −99.36242451]]]

beta is, $\frac{39}{10}$, *times the threshold value*

the long-term behavior is

[[998, [15.29648241, −105.3670251]], [999, [15.29648241, −105.3670251]], [1000, [15.29648241, −105.3670251]], [1001, [15.29648241, −105.3670251]]] **(7)**

(iv) $\nu = 7$, $\gamma = 10$

```
> t4b1 := SIRS(800,200,0.3*7/1000,7,10,1000);
  t4b2 := SIRS(800,200,0.9*7/1000,7,10,1000);
  t4b3 := SIRS(800,200,3.9*7/1000,7,10,1000);

  #Equilibrium Points
```

t4b1 := [−336.0000000, −1664.000000]

t4b2 := [−1008.000000, −992.000000]

t4b3 := [−4368.000000, 2368.000000] **(8)**

The Time Series

```
> TimeSeries(t
```

The

PROBLEM 2

do NOT WORK on any random functions until after completing the other problems. The answers will not make sense, because the problems change

d

PROBLEM 3

Carefully read, and understand, the Maple code for Orbk, and use it to find, numerically, the stable equilibrium point of the difference equation

$$x_{n+1} = \frac{3 + x_{n-1} + x_{n-2} + x_{n-3}}{1 + x_n + x_{n-2}}$$

First, Convert to canonical form

$$REC := x_n = \frac{3 + x_{n-2} + x_{n-3} + x_{n-4}}{1 + x_{n-1} + x_{n-3}}$$

$$REC := x_n = \frac{3 + x_{n\text{-}2} + x_{n\text{-}3} + x_{n\text{-}4}}{1 + x_{n\text{-}1} + x_{n\text{-}3}} \tag{9}$$

To fin the

$$REC := x_n = \frac{3 + x_{n\text{-}2} + x_{n\text{-}3} + x_{n\text{-}4}}{1 + x_{n\text{-}1} + x_{n\text{-}3}} \tag{10}$$

```
> #This is put in maple language. It is very very very likely
  correct after referring to the OrbK documentation
  genTerm := (3+z[2]+z[3]+z[4])/(1+z[1]+z[3]);
```

$$genTerm := \frac{3 + z_2 + z_3 + z_4}{1 + z_1 + z_3} \tag{11}$$

QUESTION: Do I need to move the x_n from the LHS to the RHS, or do ignore and only put the RHS into OrbK

Remember, for OrbK there cannot be any equals sign in the

```
> #Help(Orbk);
```

print stuff

```
> help(rand);
```

```
> RNG := rand(-10.0..10.0);
  RNG();
```

$$RNG := (\ ) \mapsto RandomTools{:-}Generate(float('range' = -10.0\,..10.0, 'method' = 'uniform'))$$

$$-3.23567403 \tag{12}$$

```
> Norm();
```

$$Norm(\ ) \tag{13}$$

x

```
> #INITIAL CONDITIONS
  #print(Arbitrary);
```

```
#INI1 := [RNG(),RNG(),RNG(),RNG()];
#Orbk(4,z,genTerm,evalf(INI1),1000,1010);
#print(`All Positive`);
#all positive
#INI2 := [abs(RNG()),abs(RNG()),abs(RNG()),abs(RNG())];
#Orbk(4,z,genTerm,evalf(INI2),1000,1010);
#print(`All negative`);
#INI3 := [-1*abs(RNG()),-1*abs(RNG()),-1*abs(RNG()),-1*abs(RNG())
];
#Orbk(4,z,genTerm,evalf(INI3),1000,1010);
#print(`close to zero`);
#INI4 := [-0.01*abs(RNG()),-0.02*abs(RNG()),0.03*abs(RNG()),0.08*
abs(RNG())];
Orbk(4,z,genTerm,evalf(INI4),1000,1010);

print(`alleged equilibrium`);
Orbk(4,z,genTerm,evalf([0,0,0,-3]),1000,1010);
```

bad input

FAIL

alleged equilibrium

$$[-4.576515660, 0.1552753666, -4.576515657, 0.1552753665, -4.576515658, 0.1552753663, -4.576515661, 0.1552753665, -4.576515664, 0.1552753667, -4.576515661] \quad (14)$$

We will try to find the fixed points:

This is when $f'(x) = x$

```
> Help(EquP);
```

EquP(F,x): Given a transformation F in the list of variables finds all the Equilibrium points of the continuous-time dynamical system x'(t)=F(x(t))

*EquP([5/2*x*(1-x)],[x]);*

EquP([y(1-x-y),x*(3-2*x-y)],[x,y]);* **(15)**

```
> EquP(genTerm,[z[1],z[2],z[3],z[4]]);
```

bad input

FAIL **(16)**

We will try to find our fixed points and possible stable fixed points.

```
> Help1();
  Help(JAC);
```

The SUPPORTING procedures are

IsContStable, IsDisStable, JAC, RandNice, ToSys

JAC(F,x): The Jacobian Matrix (given as a list of lists) of the transformation F in the list of variables x. Try:

JAC([x+y,x^2+y^2],[x,y]); **(17)**

If we want to look at stability more closely, we can take the jacobian, substitute numerical values of the equilibrium point for x and y, and see what happens

FIRST, convert 4th order difference equation into a first-order system

```
> Help(ToSys);
```

ToSys(k,z,f): converts the kth order difference equation x(n)=f(x[n-1],x[n-2],...x[n-k]) to a first-order system

x1(n)=F(x1(n-1),x2(n-1), ...,xk(n-1)), it gives the unerlying transormation, followed by the set of variables

Try:

ToSys(2,z,z[1]+z[2]); **(18)**

```
> firstOrder := ToSys(4,z,genTerm);
```

$$firstOrder := \left[\frac{3 + z_2 + z_3 + z_4}{1 + z_1 + z_3}, z_1, z_2, z_3\right], [z_1, z_2, z_3, z_4] \quad \textbf{(19)}$$

This is enough information to find the equilibrium points

```
> EqPts := evalf(EquP(firstOrder));
```

$$EqPts := \{[0., 0., 0., -3.]\} \quad \textbf{(20)}$$

```
> EqPts[1][1];
```

$$0. \quad \textbf{(21)}$$

Then, find the jacobian matrix of the first order system to determine stability

```
> jm := JAC(firstOrder);
```

$$jm := \left[\left[-\frac{3+z_2+z_3+z_4}{(1+z_1+z_3)^2}, \frac{1}{1+z_1+z_3}, \frac{-2+z_1-z_2-z_4}{(1+z_1+z_3)^2}, \frac{1}{1+z_1+z_3}\right], [1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0]\right] \quad \textbf{(22)}$$

Then USe the subs commant to replace the values of z[1] , z[2], z[3], z[4] with numerical values.

```
> j1 := subs({z[1]= EqPts[1][1],z[2]=EqPts[1][2],z[3]=EqPts[1][3],z[4]=EqPts[1][4]},jm[1]);
```

$$j1 := [-0., 1., 1.000000000, 1.] \quad \textbf{(23)}$$

Putting this back into matrix form we get

```
> J := Matrix([j1,evalf(jm[2]),evalf(jm[3]),evalf(jm[4])]);
```

$$J := \begin{bmatrix} -0. & 1. & 1.000000000 & 1. \\ 1. & 0. & 0. & 0. \\ 0. & 1. & 0. & 0. \\ 0. & 0. & 1. & 0. \end{bmatrix} \quad \textbf{(24)}$$

Now we need determine if the fixed point is stable by finding the eigenvalues

```
> evalf(abs(Eigenvalues(J)));
```

$$\begin{bmatrix} 1.46557123187677 \\ 0.999999999999999 \\ 0.826031357654187 \\ 0.826031357654187 \end{bmatrix} \quad \textbf{(25)}$$

Here, we see that [0. , 0. , 0. , -3.] is not a stable fixed point because at least one of the real parts of the eigenvalues is greater than 1

I dont think theres any equilibrium points because

Made more progress above   i just kept getting bugs below

```
> #Try the SFPe command
  Help11();
  #Dose SFPe need fully algebraic? is SFP the remedy to this issue
  SFPe(genTerm,[z1,z2,z[3],z[4]]);
```

SFPe(f,x), Orbk(k,z,f,INI,K1,K2)

Error, (in Engine:-Dispatch) badly formed input to solve: not fully algebraic |C:/Users/cgrie/Dynam Models Bio/Homeworks/HW19/M19.txt:278|

```
> Help9();
```

Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x) **(26)**

```
> print(SFP);
```

proc(f, x) **(27)**

local $L, i, f1, pt, Ls$;

$L := FP(f, x)$;

$Ls := [\ ]$;

$f1 := diff(f, x)$;

for i **to** $nops(L)$ **do**

$pt := L[i]$; **if** $\mathrm{abs}(subs(x = pt, f1)) < 1$ **then** $Ls := [op(Ls), pt]$ **end if**

end do;

Ls

end proc

```
> SFP(FirstOrder[1],[z[1],z[2],z[3],z[4]]);
```

Error, (in FP) invalid input: solve expects its 1st argument, eqs, to be of type {`and`, `not`, `or`, algebraic, relation(algebraic), ({list, set})({`and`, `not`, `or`, algebraic, relation(algebraic)})}, but received FirstOrder[1] = [z[1], z[2], z[3], z[4]] |C:/Users/cgrie/Dynam Models Bio/Homeworks/HW19/M19.txt:359|

```
> #DISREGARD the
```