

HW2

2) $\frac{dy}{dt} = \frac{y^3}{(t+1)}, y(0)=1$

$$\frac{dy}{y^3} = \frac{dt}{(t+1)}$$

$$\frac{1}{2y^2} + C = \ln|t+1| \quad C=1$$

$$y^2 = \frac{1}{1-2\ln(t+1)}$$

$$y = \frac{1}{\sqrt{1-2\ln(t+1)}}$$

3) $y''(t) - 3y'(t) + 2y(t) = 0 \quad y(0)=2, y'(0)=3$

$$r^2 - 3r + 2 = 0$$

$$(r-2)(r-1) = 0$$

$$r_1 = 2, r_2 = 1$$

$$y(t) = C_1 e^{-3t} + C_2 e^t$$

$$y'(t) = -3C_1 e^{-3t} + C_2 e^t$$

$$r^2 - 3r + 2 = 0$$

$$(r-2)(r-1) = 0$$

$$r = 2, 1$$

$$y(t) = C_1 e^{2t} + C_2 e^t$$

$$y'(t) = 2C_1 e^{2t} + C_2 e^t$$

$$y(t) = e^{2t} + e^t$$

$$2 = C_1 + C_2 \quad C_2 = 2 + \frac{1}{4} = \frac{9}{4}$$

$$3 = -3C_1 + C_2 \quad C_2 = 3 + \frac{12}{4} - \frac{1}{4} = \frac{13}{4}$$

$$-1 = 4C_1 \quad C_1 = -\frac{1}{4}$$

$$C_2 = \frac{9}{4}$$

$$2 = C_1 + C_2 \quad C_1 = 1$$

$$3 = 2C_1 + C_2 \quad C_2 = 1$$

4) $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$

$$\begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix}$$

$$9 - 6\lambda + \lambda^2 + 16 = \lambda^2 - 6\lambda + 25$$

$$\lambda = \frac{6 \pm \sqrt{36 - 100}}{2} = \frac{6 \pm \sqrt{64}i}{2}$$

$$= 3 + 4i, 3 - 4i$$

$$\begin{bmatrix} -4i & -4 \\ 4 & 4i \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix} \begin{bmatrix} i & -i \end{bmatrix}$$