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Homework 2

Math 336

#1 i $a_n = 4a_{n-1} - 6a_{n-2} + 4a_{n-3} - a_{n-4}$

$a_0 = 0 \quad a_1 = 1 \quad a_2 = 8 \quad a_3 = 27$
$0^3 \quad 1^3 \quad 2^3 \quad 3^3$

$a_4 = 4(27) - 6(8) + 4(1) - 0 = 64 \quad n^3$
$a_5 = 4(64) - 6(27) + 4(8) - 1 = 125 \quad n^3$
$a_6 = 4(125) - 6(64) + 4(27) - 8 = 216 \quad n^3$
$a_7 = 4(216) - 6(125) + 4(64) - 27 = 343 \quad n^3$
$a_8 = 4(343) - 6(216) + 4(125) - 64 = 512 \quad n^3$

All examples prove to be n^3

ii $a_n = n^3$

iii $n^3 = 4(n-1)^3 - 6(n-2)^3 + 4(n-3)^3 - (n-4)^3$

Expanded in Maple $\quad$ expand$(4(n-1)^3 - 6(n-2)^3 + 4(n-3)^3 - (n-4)^3) = n^3$

$n^3 = n^3$

#2 $\frac{dy}{dt} = \frac{y^3}{(t+1)}, \; y(0) = 1$ $\qquad \frac{dy}{y^3} = \frac{dt}{t+1} \xrightarrow{\text{integration}} \frac{-1}{2y^2} = \ln(t+1) + C_1$

$\frac{-1}{2y^2} = \ln(t+1) + C_1 \rightarrow \frac{-1}{2\ln(t+1)+C} = y^2 \longrightarrow y(t) = \sqrt{\frac{-1}{2\ln(t+1)+C}}$

$y(0) = \sqrt{\frac{-1}{2\ln(0+1)+C}} = 1$ $\qquad \sqrt{\frac{-1}{C}} = 1 \quad C = -1$

$\ln(0+1) \to 0$

$$\boxed{y(t) = \sqrt{\frac{-1}{2\ln(t+1)-1}}}$$

3 Solve $y''(t) - 3y'(t) + 2y(t) = 0 \quad y(0) = 2 \quad y'(0) = 3$

Char. equation $r^2 - 3r + 2 = 0$

$(r-2)(r-1) = 0$

$\lambda_{1,2} > 0$

$\lambda_1 = 1 \quad \lambda_2 = 2$

$y(t) = k_1 e^{t} + k_2 e^{2t} \qquad y'(t) = k_1 e^{t} + 2k_2 e^{2t}$

$y(0) = 2 = k_1 + k_2$ Line1 $\qquad$ Line2 - Line1

$y'(0) = 3 = k_1 + k_2 \cdot 2$ Line2 $\qquad 1 = k_2$ so $k_1 = 2 - 1 = 1$

$y(t) = e^{t} + e^{2t}$

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4 $\det\left(\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} - \lambda I\right) = \lambda^2 - 6\lambda + 9 + 16 = \lambda^2 - 6\lambda + 25 = \qquad \sqrt{-64} = 8i$

$\lambda = \frac{6 \pm \sqrt{36 - 4(25)}}{2} = 3 \pm \sqrt{-64} \cdot \frac{1}{2} = 3 \pm 4i$

$\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 3+4i \\ 3+4i \end{bmatrix}\begin{matrix} x_1 \\ y_1 \end{matrix} \qquad \begin{matrix} \cancel{3x_1} - 4y_1 = \cancel{3x_1} + 4ix_1 \\ 4x_1 + \cancel{3y_1} = \cancel{3y_1} + 4iy_1 \end{matrix} \qquad \begin{matrix} -4ix_1 - 4y_1 = 0 \\ 4x_1 - 4iy_1 = 0 \end{matrix}$

for $\lambda_1 = 3+4i$ $\qquad$ for $\lambda_2 = 3-4i$

$\vec{v}_1 = \begin{bmatrix} i \\ 1 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} -i \\ 1 \end{bmatrix}$