HW2 RichaNayak

1) i) Using maple

ii) Looking at the values from $1 \le n \le 8$ we can guess $a(n) = n^3$ to be the explicit form.

iii) To prove $a(n) = n^3$

we can use maple command to prove

$$n^3 = 4(n-1)^3 - 6(n-2)^3 + 4(n-3)^3 - (n-4)^3$$

code - Expand $(4(n-1)^3 - 6(n-2)^3 + 4(n-3)^3 - (n-4)^3)$

2) $\dfrac{dy}{dt} = \dfrac{y^3}{t+1}$ $\quad y(0) = 1$

$y^{-2}$

$$\int \frac{1}{y^3}\,dy = \int \frac{1}{t+1}\,dt$$

$$-\frac{y^{-2}}{2} = \ln(t+1) + C$$

$$-\frac{(1)^{-2}}{2} = \ln(0+1) + C$$

$$C = -\frac{1}{2}$$

$$-\frac{y^{-2}}{2} = \ln(t+1) - \frac{1}{2}$$

$$\frac{1}{y^2} = 1 - 2\ln(t+1)$$

$$y^2 = \frac{1}{1 - 2\ln(t+1)}$$

$$y = \frac{1}{\sqrt{1 - 2\ln(t+1)}}$$

3) $y''(t) - 3y'(t) + 2y(t) = 0$, $y(0) = 2$, $y'(0) = 3$

$r^2 - 3r + 2 = 0$

$(r-1)(r-2) = 0$

$r = 1, 2$

$y(t) = C_1e^t + C_2e^{2t}$

$y'(t) = C_1e^t + 2C_2e^{2t}$

$y(0) = 2$ ∴ $2 = C_1 + C_2$

$y'(0) = 3$ ∴ $3 = C_1 + 2C_2$

$-1 = -C_2$

$C_2 = 1$ $C_1 = 1$

$y(t) = e^t + e^{2t}$

4) $A = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$

$\begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix} = (3-\lambda)^2 - (-16)$

$= 9 - 6\lambda + \lambda^2 + 16$

$= \lambda^2 - 6\lambda + 25$

Eigenvalues roots are $3-4i$, $3+4i$

Eigenvector

$\lambda = 3+4i$

$$\begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix} = \begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -i \\ 1 \end{bmatrix}$$

$\lambda = 3 + 4i$

$$\begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix} = \begin{bmatrix} -4i & -4 \\ 4 & -4i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} i \\ 1 \end{bmatrix}$$

#Richa Nayak
#Homework - 2 (maple part)
#Dynamic Models in Biology - Dr. Z

#1. $a(n) = 4a(n-1) - 6a(n-2) + 4a(n-3) - a(n-4)$, $a(0) = 0$, $a(1) = 1$, $a(2) = 8$, $a(3) = 27$

#a) find an for 1<= n <=8

```
a := proc(n) option remember: if n = 0 then 0 elif n = 1 then 1 elif n = 2 then 8 elif n = 3 then 27 :
 else 4·a(n − 1) − 6·a(n − 2) + 4·a(n − 3) − a(n − 4) :fi:end:
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seq(a(i), i = 1 ..8)
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$$1, 8, 27, 64, 125, 216, 343, 512 \tag{1}$$

#b) & c) done by hand

#2. solving the differential equation using maple

$$dsolve\left(\left\{\mathrm{D}(y)(t) = \frac{y(t)^3}{t+1}, y(0) = 1\right\}, y(t)\right)$$

$$y(t) = \frac{1}{\sqrt{1 - 2\ln(t+1)}} \tag{2}$$

#3. solving differential equation using maple

$$dsolve(\{\mathrm{D}(\mathrm{D}(y))(t) - 3\cdot\mathrm{D}(y)(t) + 2\cdot y(t) = 0, y(0) = 2, \mathrm{D}(y)(0) = 3\}, y(t))$$

$$y(t) = \mathrm{e}^{2t} + \mathrm{e}^{t} \tag{3}$$

#4. Finding the eigenvalues and eigenvectors using maple

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with(LinearAlgebra) :
A := Matrix([[3,-4], [4, 3]])
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$$A := \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} \tag{4}$$

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evalf(Eigenvalues(A))
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$$\begin{bmatrix} 3. + 4.\,\mathrm{I} \\ 3. - 4.\,\mathrm{I} \end{bmatrix} \tag{5}$$

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evalf(Eigenvectors(A))[2]
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$$\begin{bmatrix} \mathrm{I} & -\mathrm{I} \\ 1. & 1. \end{bmatrix} \tag{6}$$

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$expand\left(4(n-1)^3 - 6(n-2)^3 + 4(n-3)^3 - (n-4)^3\right)$

$$n^3 \qquad \textbf{(1)}$$