# Dynamic Modeling HW2

1) (ii) $a_n = n^3$

(iii) ① Check initial conditions

$*\ (0)^3 = 0$ $\quad *\ (3)^3 = 27$ $\quad *\ (6)^3 = 216$

$*\ (1)^3 = 1$ $\quad *\ (4)^3 = 64$ $\quad *\ (7)^3 = 343$

$*\ (2)^3 = 8$ $\quad *\ (5)^3 = 125$ $\quad *\ (8)^3 = 512$

② Prove using algebra sequence

$$n^3 = 4(n-1)^3 - 6(n-2)^3 + 4(n-3)^3 - (n-4)^3$$

$$n^3 = 4(n^3 - 3n^2 + 3n - 1) - 6(n^3 - 6n^2 + 12n - 8) + 4(n^3 - 9n^2 + 27n - 27) - (n^3 - 12n^2 + 48n - 64)$$

$$n^3 = \boxed{4n^3} - 12n^2 + \cancel{12n} - \cancel{4} - \boxed{6n^3} + 36n^2 - \cancel{72n} + \cancel{48} + \boxed{4n^3} - 36n^2 + \cancel{108n} - \cancel{108} \boxed{-n^3} + 12n^2 - \cancel{48n} + \cancel{64}$$

$$n^3 = n^3 + 0n^2 + 0n + 0$$

$$n^3 = n^3 \checkmark$$

3) $y''(t) - 3y'(t) + 2y(t) = 0$ ; $y(0) = 2$, $y'(0) = 3$

General Soln : $y(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$

$\lambda^2 - 3\lambda + 2 = 0$

$(\lambda - 1)(\lambda - 2)$

$$\boxed{y(t) = e^t + e^{2t}}$$

$y(t) = c_1 e^t + c_2 e^{2t}$ $\qquad (2 = c_1 + c_2) \cdot -1$

$y'(t) = c_1 e^t + 2c_2 e^{2t}$ $\qquad + \ 3 = c_1 + 2c_2$

$1 = c_2$

$c_1 + 1 = 2$

$c_1 = 1$

2) $\frac{dy}{dt} = \frac{y^3}{(t+1)}$, $y(0) = 1$

$\int \frac{dy}{y^3} = \int \frac{dt}{(t+1)}$

$-\frac{1}{2y^2} = \ln|t+1| + c$

$-\frac{1}{2} = \ln(1) + c$

$-\frac{1}{2} = 0 + c$

$c = -\frac{1}{2}$

$-\frac{1}{2y^2} = \ln|t+1| - \frac{1}{2}$

$\cancel{2} \cdot -\frac{1}{\cancel{2}y^2} = \frac{2\ln|t+1| - 1}{2} \cdot 2$

$-1 \cdot -\frac{1}{y^2} = (2\ln|t+1| - 1) \cdot -1$

$\frac{1}{y^2} = 1 - 2\ln|t+1|$

$\sqrt{y^2} = \sqrt{\frac{1}{1-2\ln|t+1|}}$

$\boxed{y = \frac{1}{\sqrt{1-2\ln|t+1|}}}$

4) $\begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix} \Rightarrow (3-\lambda)^2 + 16$

$\lambda^2 - 6\lambda + 9 + 16$

$\lambda^2 - 6\lambda + 25$

$\lambda = \frac{6 \pm \sqrt{36 - 4(1)(25)}}{2} = \frac{6 \pm \sqrt{-64}}{2} = \frac{6 \pm 8i}{2} = 3 \pm 4i$

$\boxed{\lambda = 3 \pm 4i}$

$\lambda = 3+4i$: $\begin{bmatrix} 3-3-4i & -4 \\ 4 & 3-3-4i \end{bmatrix} = \begin{bmatrix} -4i & -4 \\ 4 & -4i \end{bmatrix} \xrightarrow[\frac{1}{4}r_2 \to r_2]{-\frac{1}{4}r_1 \to r_1} \begin{bmatrix} i & 1 \\ 1 & -i \end{bmatrix} \xrightarrow{ir_1 + r_2 \to r_2} \begin{bmatrix} i & 1 \\ 0 & 0 \end{bmatrix}$

$x_2 = -ix_1$ $\boxed{\begin{bmatrix} 1 \\ -i \end{bmatrix} \Rightarrow \lambda = 3+4i}$

$\lambda = 3-4i$

$\begin{bmatrix} 3-3+4i & -4 \\ 4 & 3-3+4i \end{bmatrix} = \begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix} \xrightarrow[\frac{1}{4}r_2 \to r_2]{\frac{1}{4}r_1 \to r_1} \begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_2} \begin{bmatrix} 1 & i \\ i & -1 \end{bmatrix} \xrightarrow{-ir_1 + r_2 \to r_2} \begin{bmatrix} 1 & i \\ 0 & 0 \end{bmatrix}$

$x_1 = -ix_2$ $\boxed{\begin{bmatrix} -i \\ 1 \end{bmatrix} \Rightarrow \lambda = 3-4i}$

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> #OK to post homework
  #Nikita John, September 13th, Assignment 2
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> #1(i): Computation of reccurence equation
  a := proc(n) option remember
  if n = 0 then
  0 :
  elif n = 1 then
  1 :
  elif n = 2 then
  8 :
  elif n = 3 then
  27 :
  else
  expand(4·a(n − 1) − 6·a(n − 2) + 4·a(n − 3) − a(n − 4)) :
  fi:
  end:
```

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> seq(a(i), i = 1 ..8)
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$$1, 8, 27, 64, 125, 216, 343, 512 \tag{1}$$

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> #2: Solving 1st order DE with Maple
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$$dsolve\left(\left\{D(y)(t) = \frac{y(t)^3}{t+1}, y(0) = 1\right\}, y(t)\right);$$

$$y(t) = \frac{1}{\sqrt{1 - 2\ln(t+1)}} \tag{2}$$

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> #3: Solving 2nd order DE with Maple
  dsolve({D(D(y))(t) - 3·D(y)(t) + 2·y(t) = 0, y(0) = 2, D(y)(0) = 3}, y(t));
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$$y(t) = e^{2t} + e^{t} \tag{3}$$

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> #4: Finding the eigenvalues and eigenvectors of a matrix in Maple
  with(LinearAlgebra);
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[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, **(4)** *QRDecomposition*, *RandomMatrix*, *RandomVector*, *Rank*, *RationalCanonicalForm*, *ReducedRowEchelonForm*, *Row*, *RowDimension*, *RowOperation*, *RowSpace*, *ScalarMatrix*, *ScalarMultiply*, *ScalarVector*, *SchurForm*, *SingularValues*, *SmithForm*, *SplitForm*, *StronglyConnectedBlocks*, *SubMatrix*, *SubVector*, *SumBasis*, *SylvesterMatrix*, *SylvesterSolve*, *ToeplitzMatrix*, *Trace*, *Transpose*, *TridiagonalForm*, *UnitVector*, *VandermondeMatrix*, *VectorAdd*, *VectorAngle*, *VectorMatrixMultiply*, *VectorNorm*, *VectorScalarMultiply*, *ZeroMatrix*, *ZeroVector*, *Zip*]

```
> A := Matrix([[3, -4], [4, 3]]);
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$$A := \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} \tag{5}$$

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> evalf(Eigenvalues(A));
```

$$\begin{bmatrix} 3. + 4.\,\mathrm{I} \\ 3. - 4.\,\mathrm{I} \end{bmatrix} \tag{6}$$

```
> evalf(Eigenvectors(A));
```

$$\begin{bmatrix} 3. + 4.\,\mathrm{I} \\ 3. - 4.\,\mathrm{I} \end{bmatrix}, \begin{bmatrix} \mathrm{I} & -\mathrm{I} \\ 1. & 1. \end{bmatrix} \tag{7}$$

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>
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