1. A sequence is defined in terms of the recurrence, and initial conditions

$a_n = 4a_{n-1} - 6a_{n-2} + 4a_{n-3} - a_{n-4}$, $a_0=0$, $a_1=1$. $a_2=8$, . $a_3=27$,

(i) Find a_n for $1 \le n \le 8$. (ii) Can you guess an explicit formula for a_n in terms of n? (iii) Can you prove it?

You are welcome to use Maple for the computations. You can write a short Maple code for a(n) using "option remember".

1. $a_n = 4a_{n-1} - 6a_{n-2} + 4a_{n-3} - a_{n-4}$

$a_0 = 0 \quad a_1 = 1 \quad a_2 = 8 \quad a_3 = 27$

i. $a_4 = 4(27) - 6(8) + 4(1) - 0 = 64$

$a_5 = 4(64) - 6(27) + 4(8) - 1 = 125$

$a_6 = 4(125) - 6(64) + 4(27) - 8 = 216$

$a_7 = 4(216) - 6(125) + 4(64) - 27 = 343$

ii. $a_n = n^3$

iii. $4(n-1)^3 - 6(n-2)^3 + 4(n-3)^3 - (n-4)^3 = n^3$

2. $\frac{dy}{dt} = \frac{y^3}{t+1} \quad y(0) = 1$

$\frac{dy}{y^3} = \frac{dt}{t+1} \rightarrow -\frac{1}{2y^2} = \ln(t+1) + C \rightarrow -\frac{1}{2} = \ln(1) + C$

$\rightarrow C = -\frac{1}{2}$

$-\frac{1}{2y^2} = \ln(t+1) - \frac{1}{2}$

$-2y^2 = \frac{1}{\ln(t+1) - \frac{1}{2}}$

$y = \sqrt{\frac{1}{-2\ln(t+1) + 1}}$

2. Solve the following differential equation, subject to the given initial condition dy y3

dt=(t+1) , y(0)=1 . Do it both by hand and using Maple (as we did in Lecture 2).

3. Solve the following differential equation, subject to the given initial conditions y′′(t)−3y′(t)+2y(t) = 0 , y(0) = 2 , y′(0) = 3 .

Do it both by hand and using Maple (as we did in Lecture 2)

dsolve({(D^2)(y(t))-3*Dy(t)+2*y(t)=0, y(0)=2, Dy(0)=3}, y(t))

y(t)=e^(t)+e^(2*t)

3. $y(t) = e^{rt}$

$y'(t) = re^{rt}$

$y''(t) = r^2e^{rt}$

$r^2e^{rt} - 3re^{rt} + 2e^{rt} = 0$

$e^{rt}(r^2 - 3r + 2) = 0$

$(r-1)(x-2) = 0$

$r = 1 \quad r = 2$

$y(t) = C_1e^{t} + C_2e^{2t}$

$y(0) = 2 = C_1 + C_2$

$y'(t) = tC_1e^{t} + 2C_2e^{2t}$

$y'(0) = 3 = 2C_2$

$C_2 = \frac{3}{2}$

$2 = C_1 + C_2$

$2 = C_1 + \frac{3}{2}$

$C1 = \frac{1}{2}$

4. Find all the eigenvalues and corresponding eigenvectors of the matrix

3	-4
4	3

Do it both by hand and using Maple (as we did in Lecture 2).

with(LinearAlgebra)

evalf(Eigenvalues(matrix([[3, -4][4,3]])))

Matrix(1,2,[[3+4*I, 3-4*I]])

evalf(Eigenvects(matrix([[3, -4], [4,3]])))

Vector[column](2,[3+4*I, 3-4*I]), Matrix(2,2,[[I,-I], [1., 1.]])

4. $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$

$$\det \begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix} = (3-\lambda)(3-\lambda) - (-4)(4) = \lambda^2 - 6\lambda + 25$$

$\lambda = 3-4i \qquad \lambda = 3+4i$

$\lambda = 3-4i$

$$\begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix} = \begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix}$$

$\lambda = 3+4i$

$$\begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix} = \begin{bmatrix} -4i & -4 \\ 4 & -4i \end{bmatrix}$$