#Please do not post homework
#Julian Herman, 9/13/2021, Assignment 2

2) $\frac{dy}{dt} = \frac{y^3}{(t+1)}$, $y(0) = 1$

$$\int y^{-3}\,dy = \int \frac{1}{t+1}\,dt$$

$$\frac{-1}{2y^2} = \ln(t+1) + C$$

$$\frac{-1}{2(1)^2} = \ln(1) + C$$

$$\boxed{-\frac{1}{2} = C}$$

$$-\frac{1}{2y^2} = \ln(t+1) - \frac{1}{2}$$

$$\frac{-1}{2\left(\ln(t+1) - \frac{1}{2}\right)} = y^2$$

$$\frac{1}{-2\ln(t+1) + 1} = y^2$$

$$y = \pm\sqrt{\frac{1}{1 - 2\ln(t+1)}}$$

* The negative solution doesn't hold true for I.C. $y(0) = 1$

$$y(t) = \frac{1}{(1 - 2\ln(t+1))^{\frac{1}{2}}}$$

3) $y''(t) - 3y'(t) + 2y(t) = 0$, $y(0) = 2$, $y'(0) = 3$

$y = e^{rt}$

$r^2 e^{rt} - 3re^{rt} + 2e^{rt} = 0$

$e^{rt}(r^2 - 3r + 2) = 0$

$(r-2)(r-1) = 0$

$r = 2, 1$

$y(t) = C_1 e^{2t} + C_2 e^t$

$y'(t) = 2 \cdot C_1 \cdot e^{2t} + C_2 e^t$

$y(0) = C_1 + C_2 = 2 \Rightarrow C_1 = 2 - C_2$

$y'(0) = 2C_1 + C_2 = 3$

$2(2 - C_2) + C_2 = 3$

$4 - 2C_2 + C_2 = 3$

$\boxed{C_2 = 1} \Rightarrow \boxed{C_1 = 1}$

$\Rightarrow y(t) = e^{2t} + e^t$

4) $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$ $\quad \det \begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix} = 0$

$$(3-\lambda)^2 + 16 = 0$$

$$(3-\lambda)^2 = -16$$

$$3-\lambda = \pm 4i$$

$$\lambda = 3 \pm 4i$$

$$\lambda_1 = 3+4i \quad , \quad \lambda_2 = 3-4i$$

$\lambda_1 = 3+4i$: $\begin{bmatrix} 3-(3+4i) & -4 \\ 4 & 3-(3+4i) \end{bmatrix}$

$$\begin{bmatrix} -4i & -4 \\ 4 & -4i \end{bmatrix}$$

$$\begin{bmatrix} -i & -1 \\ 1 & -i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-ix_1 - x_2 = 0$$

$$x_1 - ix_2 = 0$$

$\boxed{x_2 = -ix_1}$

$\boxed{\text{Let } x_1 = 1: \quad x_2 = -i}$

$\Rightarrow$ one possible set of solutions

$\Rightarrow$ eigen vectors for $\lambda = 3+4i$

$c \cdot \begin{bmatrix} 1 \\ -i \end{bmatrix}$ for $c \in \mathbb{R}$

$\lambda_2 = 3-4i$: $\begin{bmatrix} 3-(3-4i) & -4 \\ 4 & 3-(3-4i) \end{bmatrix}$

$$\begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix}$$

$$\begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$y_1 i - y_2 = 0$ $\qquad \boxed{y_2 = y_1 \cdot i}$

$y_1 + y_2 i = 0$

Let $y_1 = 1$:

$y_2 = i$

$\Rightarrow$ eigenvectors for $\lambda_2 = 3-4i$:

$$c \cdot \begin{bmatrix} 1 \\ i \end{bmatrix} \text{ for } c \in \mathbb{R}$$

$\downarrow$ one possible set of solutions

multiplying by $-i$ gives another:

$$c \cdot \begin{bmatrix} -i \\ 1 \end{bmatrix} \text{ for } \lambda_2 = 3-4i$$

$$c \cdot \begin{bmatrix} i \\ 1 \end{bmatrix} \text{ for } \lambda_1 = 3+4i$$