1. $a_n = 4a_{n-1} - 6a_{n-2} + 4a_{n-3} - a_{n-4}$ $\quad a_0 = 0 \quad a_1 = 1$
$a_2 = 8 \quad a_3 = 27$

i.
$a_4 = 4(27) - 6(8) + 4(1) - 0 = 64$

$a_5 = 4(64) - 6(27) + 4(8) - 1 = 125$

$a_6 = 4(125) - 6(64) + 4(27) - 8 = 216$

$a_7 = 4(216) - 6(125) + 4(64) - 27 = 343$

$a_8 = 4(343) - 6(216) + 4(125) - 64 = 576$

ii. $a_n = n^3$

iii. $a_n = 4(n-1)^3 - 6(n-2)^3 + 4(n-3)^3 - (n-4)^3$
$= 4n^3 - 12n^2 + 12n - 1 - 6n^3 + 36n^2 - 72n + 48 + 4n^3 - 36n^2 + 108n$
$- 108 + n^3 - 12n^2 + 48n - 64$
$= n^3$

2. $\dfrac{dy}{dt} = \dfrac{y^3}{t+1} \qquad y(0) = 1$

$\dfrac{dy}{y^3} = \dfrac{dt}{t+1}$

$-\dfrac{1}{2y^2} = \ln(t+1) + c$

$-\dfrac{1}{2} = \ln(1) + c$

$c = -\dfrac{1}{2}$

$-\dfrac{1}{2y^2} = \ln(t+1) - \frac{1}{2}$

$-2y^2 = \dfrac{1}{\ln(t+1) - \frac{1}{2}}$

$y = \sqrt{\dfrac{1}{-2\ln(t+1) + 1}}$

3. $y''(t) - 3y'(t) + 2y(t) = 0 \qquad y(0) = 2 \quad y'(0) = 3$

$y(t) = e^{rt} \qquad y'(t) = re^{rt} \qquad y''(t) = r^2e^{rt}$

$r^2e^{rt} - 3re^{rt} + 2e^{rt} = 0$

$r^2 - 3r + 2 = 0$

$(r-2)(r-1) = 0$

$r = 2, 1$

$y(t) = c_1e^{2t} + c_2e^{t} \qquad y'(t) = 2c_1e^{2t} + c_2e^{t}$

$2 = c_1 + c_2 \qquad 1 = c_1$

$3 = 2c_1 + c_2 \qquad 1 = c_2$

$y(t) = e^{2t} + e^{t}$

4. $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} \qquad \begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix}$

$0 = (3-\lambda)^2 + 16$

$0 = 9 - 6\lambda + \lambda^2 + 16$

$0 = \lambda^2 - 6\lambda + 25$

$\lambda = \frac{6 \pm \sqrt{36-100}}{2}$

$\lambda = \frac{6 \pm \sqrt{-64}}{2}$

$\lambda = 3 \pm 4i$

3. dsolve({(D^2)(y)(t) - 3*D(y)(t) + 2*y(t) = 0, y(0) = 2, D(y)(0) = 3}, y(t))

    y(t) = e^(t) + e^(2*t)

4. with(LinearAlgebra):

   evalf(Eigenvalues(matrix([[3, -4], [4, 3]])))

    Matrix(1, 2, [[3. + 4.*I, 3. - 4.*I]])

   evalf(Eigenvects(matrix([[3, -4], [4, 3]])))

    Vector[column](2, [3. + 4.*I, 3. - 4.*I]), Matrix(2, 2, [[I, -I], [1., 1.]])