

HW-2 On top of Homework Jeton Hida, 9/13/21 Assignment 2

2. $\frac{dy}{dt} = \frac{y^3}{(t+1)}$, $y(0) = 1$

$$\int \frac{1}{y^3} dy = \int \frac{1}{t+1} dt$$

$$-\frac{1}{2y^2} = \ln|t+1| + C$$

$$y^2 = \frac{-2\ln|t+1| + C}{1}$$

$$y = \sqrt{-2\ln|t+1| + C}$$

$$y(0) = \sqrt{-2\ln|1| + C} = 1 \Rightarrow C = 1$$

$$y(t) = \sqrt{1 - 2\ln|t+1|}$$

3. $y'' - 3y' + 2y = 0$, $y(0) = 2$, $y'(0) = 3$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$(\lambda - 2)(\lambda - 1) = 0$$

$$\lambda = 2, \lambda = 1$$

$$y(t) = c_1 e^{2t} + c_2 e^t$$

$$y'(t) = 2c_1 e^{2t} + c_2 e^t$$

$$y(0) = c_1 + c_2 = 2$$

$$y'(0) = 2c_1 + c_2 = 3$$

$$-c_1 = -1$$

$$c_1 = 1$$

$$c_2 = 1$$

$$y(t) = e^{2t} + e^t$$

4. $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$

$$\det(A - \lambda I) = (3 - \lambda)^2 + 16 = 0$$

$$0 = (3 - \lambda)^2 + 16$$

$$(3 - \lambda) = \pm 4i$$

$$\lambda = 3 + 4i$$

$$\lambda = 3 - 4i$$

$$\lambda_1 = 3 - 4i$$

$$\begin{bmatrix} -4i & -4 \\ 4 & -4i \end{bmatrix} v_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad v_1 = \begin{bmatrix} -i \\ 1 \end{bmatrix}$$

$$\lambda_2 = 3 + 4i$$

$$\begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix} v_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} i \\ 1 \end{bmatrix}$$