4. Find all the eigenvalues ...

Dynamic Models Bio Homework 2

1. A sequence is defined in terms of the recurrence, and its initial conditions

$a_n = 4a_{n-1} - 6a_{n-2} + 4a_{n-3} - a_{n-4}$,

$$\begin{cases} a_0 = 0 \\ a_1 = 1 \\ a_2 = 8 \end{cases}$$
$a_3 = 27$

1 7 19 / 6 6 ... / 2 8

my guess is $\boxed{a_n = n^3}$

~~A~~ all 3rd Degree differences are 6

(i) Find a_n for $1 \le n \le 8$

$a_4 = 4(27) - 6(8) + 4(1) - 0$
$= 108 - 48 + 4$
$= 60 + 4$
$= 64$

We just

$a_5 = 4(64) - 6(27) + 4(8) - 1$
$= 256 - 162 + 32 - 1$
$= 256 - 130 - 1$
$= 126 - 1$
$= 125$

OH I was Allowed to use maple on this.

Do maple code

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We will use Induction to prove

$a_n = 4a_{n-1} - 6a_{n-2} + 4a_{n-3} - a_{n-4}$

given $a_0 = 0$, $a_1 = 1$, $a_2 = 8$, $a_3 = 27$

can have its general term a_n rewritten as:

$a_n = n^3$.

Proof. We can use the information from the base step

$a_0 = 0$, $a_1 = 1$, $a_2 = 8$, $a_3 = 27$, $a_4 = 64$

as our scaffold.

NO! We have to use recurrence trick

let $a_n = 1$ $a_{n-1} = \frac{1}{r}$ $a_{n-2} = \frac{1}{r^2}$ $a_{n-3} = \frac{1}{r^3}$

$a_{n-4} = \frac{1}{r^4}$

Thus, we have $1 - \frac{4}{r} + \frac{6}{r^2} - \frac{4}{r^3} + \frac{1}{r^4} = 0$

which multiplied by r^4 gives us $r^4 - 4r^3 + 6r^2 - 4r + 1 = 0$

which, when factored is $(r-1)^4$

We have to ~~make~~ ~~a~~ pretty large system

Dynamic Models Bio Homework 2

3. Solve $y''(t) - 3y'(t) + 2y(t) = 0$

$y(0) = 2 \quad y'(0) = 3.$

Guess $y(t) = e^{rt}$.

Therefore $y''(t) = r^2 e^{rt}$

$y'(t) = re^{rt}$

Therefore

$y''(t) - 3y'(t) + 2y(t) = 0$

is

$r^2 e^{rt} - 3re^{rt} + 2e^{rt} = 0$

And factoring out "e^{rt}", we have

$e^{rt}(r^2 - 3r + 2) = 0$

which implies the roots of the characteristic equation are given by

$(r-2)(r-1) \Rightarrow r = 1 \;\&\; r = 2$

Now,

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Since $r=1$ and $r=2$,

our general solution with undetermined coefficients is:

$$y(t) = c_1 e^{t} + c_2 e^{2t},$$

And with Initial Conditions

$y(0)=2 \quad y'(0)=3$

We also must have

$$y'(t) = c_1 e^{t} + 2c_2 e^{2t}$$

To obtain the system

$$\begin{cases} y(0)=2 = c_1 e^{0} + c_2 e^{0} \\ \quad = c_1 + c_2 \\ y'(0)=3 = c_1 e^{0} + 2c_2 e^{0} \end{cases}$$

Thus, $c_1 = c_2 = 1$

$\Rightarrow$ general solution is

$$y(t) = e^{t} + e^{2t}.$$

Dynam Models Bio Homework 2

4. Find all the eigenvalues and corresponding eigenvectors of

$$\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$$

Eigenvalues: $\det\begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix} = 0$

$$(3-\lambda)(3-\lambda) - ((-4)(4)) = 0$$

$$9 - 3\lambda - 3\lambda + 16 + \lambda^2 = 0$$

$$25 - 6\lambda + \lambda^2 = 0$$

$$\lambda = \frac{\cancel{+}6 \pm \sqrt{(-6)^2 - 100}}{2} = \frac{6 \pm \sqrt{-64}}{2}$$

Eigenvectors: $\lambda = 3 \pm 4i$

To find eigenvector we will solve

1: $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = 3 + 4i \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$

$$\begin{cases} 3x_0 - 4y_0 = (3+4i)(x_0) \\ 4x_0 + 3y_0 = (3+4i)(y_0) \end{cases}$$

which is rewritten as

$$\begin{cases} -4y_0 - 4i x_0 = 0 \\ 4x_0 - 4i y_0 = 0 \end{cases}$$

Dynamic Models Bio Homework 2

Knowing

$$\begin{cases} -4y_0 - 4ix_0 = 0 \\ 4x_0 - 4iy_0 = 0 \end{cases}$$

Multiply both sides of top equation by i to get

$$\begin{cases} 4x_0 - 4iy_0 = 0 \\ 4x_0 - 4iy_0 = 0 \end{cases}$$

This redundant system already yields our two eigenvectors, so we dont need to use an additional eigenvalue to get our other eigenvector.

To get our eigenvector, all we need to do is find x_0 and y_0 such that

$$4x_0 - 4iy_0 = 0 .$$

way Let $y_0 = i$ to get rid of the imaginary term. therefore, $4x_0 + 4 = 0$, therefore, $x_0 = -1$

Therefore, our eigenvector is

$$\begin{pmatrix} -1 \\ i \end{pmatrix}$$