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> #OK to post homework
> #Anusha Nagar, 9.13.2021, Assignment 2
>
> #Problem 1
> #Given: a(n) = 4 a(n − 1) − 6 a(n − 2) + 4 a(n − 3) − a(n − 4), a(0) = 0,
      a(1) = 1, a(2) = 8, a(3) = 27
> #(i) Find a(n) for 1 ≤ n ≤ 8
> a :=proc(n) option remember :
  if n = 0 then
   0 :
  elif n = 1 then
   1 :
  elif n = 2 then
   8 :
  elif n = 3 then
   27 :
  else
   4 * a(n-1)-6 * a(n-2) + 4 * a(n-3) − a(n − 4) :
  fi:
  end:

  seq(a(i), i = 1 ..8);
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$$1, 8, 27, 64, 125, 216, 343, 512 \quad (1)$$

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> #(ii) The explicit formula seems to be a(n)=n^3
> #(iii) Proof is below
> seq(a(n) − n^3, n = 0 ..50);
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$$0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \quad (2)$$

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> #As we see above, the solution converges.
> #We can also check another way. We first see that initial
    conditions are satisfied. a(0)=0^3, a(1) = 1^3, a(2) = 2^3, a(3) = 3^3
    . We then check if n^3 satisfies the recurrnce
> expand(4 * (n-1)^3 − 6 * (n-2)^3 + 4 * (n-3)^3 − (n − 4)^3);
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$$n^3 \quad (3)$$

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> #We see that the recurrence is satisfied
>
> #Problem 2
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> #BY hand: we do separation of variables. This leads to $dy \cdot y^{-3}$ = dt $\cdot (t+1)^{-1}$. Integrating both sides, you find $-1 \cdot (2\ y^2)^{-1} + C$ = ln(t + 1)

> #Initial condition is y(0)=1. When we plug thisin, we get -0.5+C = 0, so C = 0.5. Our final equation is $-1 \cdot (2y^2)^{-1} + 0.5$ = ln(t + 1). In explicit terms, this becomes y = $\mathrm{sqrt}\left(\frac{1}{2\left(-\ln(t+1)+\frac{1}{2}\right)}\right)$

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> #With maple:
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> dsolve({D(y)(t) = y(t)^3/(t+1), y(0) = 1}, y(t));
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$$y(t) = \frac{1}{\sqrt{1 - 2\ln(t+1)}} \quad (4)$$

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> #Problem 3
> #By HAND: r^2 - 3 r + 2 = 0. Factor to get·(r - 2)(r - 1),
    and roots are 1 and 2. So here, the solution ends up as y = c1
    ·e^t + c2·e^(2 t). Using our initial conditions, we find that 2 = c1
    + c2, and 3 = c1 + 2 c2. We see that c1 = 1and c2 = 1 here
    . Final equation is y(t) = e^t + e^(2 t)
> #By MAPLE
> dsolve({D(D(y))(t) - 3 D(y)(t) + 2 y(t) = 0, y(0) = 2, D(y)(0) = 3},
    y(t));
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$$y(t) = e^{2t} + e^{t} \quad (5)$$

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> #Problem 4
> #BY HAND: The characteristic matrix becomes [3-lam  -4; 4  3-lam]
    . Taking the determinant of this = 0, our characteristic
    equation becomes (3-lam)(3-lam)-(4)(-4) = 0. Expanding, we get
    lam^2 - 6 lam + 25 = 0. Roots here become  3 +-4 i. Plugging in 3
    + 4 i for our lamd in our matrix, we get our matrix to be [
    -4 i  -4;  4  -4 i]. Our eigenvector here becomes [i; 1]. For 3
    - 4 i, we take the conjugate vector·(I think that's the name
    for it) to be [-i; 1], as we can easily
    do this when our eigenvalues are complex.
> #By maple:
> A := Matrix([[3,-4], [4, 3]]);
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$$A := \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} \quad (6)$$

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> with(LinearAlgebra);
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[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, (7) *MatrixNorm*, *MatrixPower*, *MatrixScalarMultiply*, *MatrixVectorMultiply*, *MinimalPolynomial*, *Minor*, *Modular*, *Multiply*, *NoUserValue*, *Norm*, *Normalize*, *NullSpace*, *OuterProductMatrix*, *Permanent*, *Pivot*, *PopovForm*, *ProjectionMatrix*, *QRDecomposition*, *RandomMatrix*, *RandomVector*, *Rank*, *RationalCanonicalForm*, *ReducedRowEchelonForm*, *Row*, *RowDimension*, *RowOperation*, *RowSpace*, *ScalarMatrix*, *ScalarMultiply*, *ScalarVector*, *SchurForm*, *SingularValues*, *SmithForm*, *SplitForm*, *StronglyConnectedBlocks*, *SubMatrix*, *SubVector*, *SumBasis*, *SylvesterMatrix*, *SylvesterSolve*, *ToeplitzMatrix*, *Trace*, *Transpose*, *TridiagonalForm*, *UnitVector*, *VandermondeMatrix*, *VectorAdd*, *VectorAngle*, *VectorMatrixMultiply*, *VectorNorm*, *VectorScalarMultiply*, *ZeroMatrix*, *ZeroVector*, *Zip*]

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> Eigenvalues(A);
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$$\begin{bmatrix} 3+4\,\mathrm{I} \\ 3-4\,\mathrm{I} \end{bmatrix} \tag{8}$$

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> evalf(Eigenvectors(A));
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$$\begin{bmatrix} 3.+4.\,\mathrm{I} \\ 3.-4.\,\mathrm{I} \end{bmatrix}, \begin{bmatrix} \mathrm{I} & -\mathrm{I} \\ 1. & 1. \end{bmatrix} \tag{9}$$

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