Dynamical Models in Biology HW 2

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1.

Using Maple,

$a_1 = 1$ $a_5 = 125$

$a_2 = 8$ $a_6 = 216$

$a_3 = 27$ $a_7 = 343$

$a_4 = 64$ $a_8 = 512$

$a_n = n^3$

Prof:

$$4(n-1)^3 - 6(n-2)^3 + 4(n-3)^3 - (n-4)^3$$

$$= 4(n^3 - 3n^2 + 3n - 1) - 6(n^3 - 6n^2 + 12n - 8)$$
$$+ 4(n^3 - 9n^2 + 27n - 27) - (n^3 - 12n^2 + 48n - 64)$$

$$= n^3$$

2. By hand:

$$\int \frac{1}{y^3}\,dy = \int \frac{1}{t+1}\,dt$$

$$-\frac{1}{2y^2} = \ln|t+1| + C$$

$$\frac{-1}{2} = C$$

$$\frac{-1}{2y^2} = \ln(t+1) - \frac{1}{2}$$

$$1 = 1 - 2\ln(t+1)\,y^2$$

$$y(t) = \sqrt{\frac{1}{1-2\ln(t+1)}}$$

In Maple:

dsolve({y(0) = 1, D(y)(t) = y(t)^3/(t + 1)}, y(t));

=> y(t) = 1/sqrt(1 - 2*ln(t + 1))

3. By hand:

$$y''(t) - 3y'(t) + 2y(t) = 0$$

$$r^2 - 3r + 2 = 0$$

$$r = 1, 2$$

$$y = C_1 e^t + C_2 e^{2t}$$

$$2 = C_1 + C_2$$

$$y' = c_1 e^t + 2c_2 e^{2t}$$

$$3 = c_1 + 2c_2$$

$$3 = 2 - c_2 + 2c_2 = 2 + c_2$$

$$c_2 = 1, \quad c_1 = 1$$

$$\boxed{y(t) = e^t + e^{2t}}$$

In Maple:

dsolve({D(D(y))(t) - 3*D(y)(t) + 2*y(t) = 0, y(0) = 2, D(y)(0) = 3}, y(t))

=> y(t) = exp(2*t) + exp(t)

4. By hand:

$$\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$$

$$\det \begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix} = (3-\lambda)^2 + 16$$

$$= 9 - 6\lambda + \lambda^2 + 16$$

$$= \lambda^2 - 6\lambda + 25$$

$$\lambda = \frac{6 \pm \sqrt{36-100}}{2} = \boxed{3 \pm 4i}$$

$\lambda = 3+4i$

$$\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} (3+4i)v_1 \\ (3+4i)v_2 \end{pmatrix}$$

$$3v_1 - 4v_2 = (3+4i)v_1$$

$$4v_1 + 3v_2 = (3+4i)v_2$$

$$4v_1 = 4iv_2$$

$$V = \begin{pmatrix} i \\ 1 \end{pmatrix}$$

$\lambda = 3-4i$

$$\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} (3-4i)v_1 \\ (3-4i)v_2 \end{pmatrix}$$

$$3v_1 - 4v_2 = (3-4i)v_1$$

$$4v_1 + 3v_2 = (3-4i)v_2$$

$$4v_1 = -4iv_2$$

$$v_1 = -iv_2$$

$$V = \begin{pmatrix} -i \\ 1 \end{pmatrix}$$

In Maple:

Eigenvalues(Matrix([[3, -4], [4, 3]]))

=> [[3+4i], [3-4i]]

Eigenvectors(Matrix([[3, -4], [4, 3]]))

=> [[3+4i], [3-4i]], [[I, -I], [1, 1]]