

Hw 2

$$1) i) a_4 = 4(27) - 6(8) + 4(1) + 0$$

$$a_4 = 64$$

$$ii) a_5 = 4(64) - 6(27) + 4(8) - 1$$

$$a_5 = 125$$

$$a_6 = 216$$

$$a_7 = 343$$

$$a_8 = 512$$

$$ii) a_n = 4a_{n-1} - 6a_{n-2} + 4a_{n-3} - a_{n-4} = n^3$$

$$iii) n^3 = 4(n-1)^3 - 6(n-2)^3 + 4(n-3)^3 - (n-4)$$

$$2) \frac{dy}{dt} = \frac{y^3}{(t+1)}, \quad y(0) = 1$$

$$\int \frac{1}{y^3} dy = \int \frac{1}{(t+1)} dt$$

$$\frac{-1}{2y^2} = \ln(t+1) + C$$

$$y^2 = \frac{-1}{2 \ln(t+1) + C}$$

$$y = \frac{1}{\sqrt{2 \ln(t+1) + C}}$$

$$1 = \frac{1}{\sqrt{2(0) + C}}$$

$$1 = \sqrt{\frac{-1}{C}} \quad C = -1$$

$$y = \frac{1}{\sqrt{2 \ln(t+1) - 1}}$$

Hw 2

$$3) \quad y''(t) - 3y'(t) + 2y(t) = 0, \quad y(0) = 2 \\ y'(0) = 3$$

$$(e^{\lambda t})'' - (3e^{\lambda t})' + 2e^{\lambda t} = 0$$

$$e^{\lambda t} (\lambda^2 - 3\lambda + 2) = 0$$

$$\lambda = 2, 1$$

$$y = C_1 e^{2t} + C_2 e^t$$

$$y'(t) = 2C_1 e^{2t} + C_2 e^t$$

$$2 = C_1 + C_2$$

$$C_1 = 1, C_2 = 1$$

$$3 = 2C_1 + C_2$$

$$y = e^{2t} + e^t$$

$$4) \begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\lambda^2 - 6\lambda + 25 = 0$$

$$\lambda = 3 + 4i, 3 - 4i$$

$$\lambda_1 = 3 - 4i, \begin{pmatrix} 4i - 4 \\ 4 & 4i \end{pmatrix}, v_1 = \begin{pmatrix} -i \\ 1 \end{pmatrix}$$

$$\lambda_2 = 3 + 4i, \begin{pmatrix} -4i - 4 \\ 4 & -4i \end{pmatrix}, v_2 = \begin{pmatrix} i \\ 1 \end{pmatrix}$$