

Houdai Bittini, 9/8/21, HW2

1) a)  $a_n | n \leq 8$   $a_0 = 0$   $a_1 = 1$   $a_2 = 8$   $a_3 = 21$

$$a_4 = 4(12) - 6(8) + 4(1) - 0 = 64$$

$$a_5 = 4(64) - 6(27) + 4(8) - 1 = 125$$

$$a_6 = 6^3 = 216$$

$$a_7 = 7^3 = 343$$

$$a_8 = 8^3 = 512 = 4(343) - 6(216) + 4(125) - 64 = 512 \checkmark$$

$$a_n = n^3 \quad a_0 = 0^3 = 0 \checkmark \quad a_1 = 1^3 = 1 \checkmark \quad a_2 = 2^3 = 8 \checkmark \quad a_3 = 27 = 3^3 \checkmark$$

$$n^3 = 4(n-1)^3 - 6(n-2)^3 + 4(n-3)^3 - (n-4)^3 = n^3 \checkmark$$

2)  $\frac{dy}{dt} = \frac{y^3}{(t+1)}$   $y(0) = 1$   $\int \frac{dy}{y^3} = \int \frac{dt}{(t+1)}$   $-\frac{1}{2y^2} = \ln(t+1) + C$   $y(0) = 1$   
 $-\frac{1}{2} = \ln(1) + C$   $C = -\frac{1}{2}$   
 $-\frac{1}{2y^2} = \ln(t+1) - \frac{1}{2}$   $y = \sqrt{\frac{1}{2(\ln(t+1) + \frac{1}{2})}}$

3)  $y''(t) - 3y'(t) + 2y(t) = 0$   $y(0) = 2$   $y'(0) = 3$   $y(t) = e^{rt}$   
 $r^2 e^{rt} - 3r e^{rt} + 2e^{rt} = 0$   $r^2 - 3r + 2 = 0$   $(r-2)(r-1) = 0$   $r = 1, 2$   
 $y(t) = c_1 e^t + c_2 e^{2t}$   $2 = c_1 + c_2$   $c_2 = 1$   
 $y'(t) = c_1 e^t + 2c_2 e^{2t}$   $3 = c_1 + 2c_2$   $c_1 = 1$   
 $y(t) = e^t + e^{2t}$

4)  $\begin{bmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{bmatrix}$   $(3-\lambda)^2 + 16$   $9 - 6\lambda + \lambda^2 + 16$   $\frac{6 \pm \sqrt{36 - 4(15)}}{2} = 3 \pm 4i$

$$\lambda = 3 + 4i \rightarrow \begin{bmatrix} -4i & -4 \\ 4 & -4i \end{bmatrix} \rightarrow \begin{bmatrix} i & -1 \\ -1 & i \end{bmatrix} \rightarrow \begin{bmatrix} i & -1 \\ 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -i \\ 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} i \\ 1 \end{bmatrix}$$

$$\lambda = 3 - 4i \rightarrow \begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix} \rightarrow \begin{bmatrix} i & -1 \\ -1 & i \end{bmatrix} \rightarrow \begin{bmatrix} 1 & i \\ 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -i \\ 1 \end{bmatrix}$$