> # *Max Mekhanikov - Homework 2*

>

> # *Question 1*

>

```
> a := proc(n) option remember:
if n = 0 then
 0:
elif n = 1 then
 1:
elif n = 2 then
 8:
 elif n = 3 then
 27:
else
 4·a(n − 1) − 6·a(n − 2) + 4·a(n − 3) − a(n − 4);
fi:
end:

seq(a(i), i = 1 ..8);
```

$$1, 8, 27, 64, 125, 216, 343, 512 \quad (1)$$

> # After analyzing the results for the first eight terms of the sequence, it becomes clear that the explicit formula in terms of n is **n^3.** We can prove this by manually computing 1<n<8 and obtaining the same results.

>

> $1^3; 2^3; 3^3; 4^3; 5^3; 6^3; 7^3; 8^3;$

$$1$$
$$8$$
$$27$$
$$64$$
$$125$$
$$216$$
$$343$$
$$512 \quad (2)$$

> # *Question 2 — Work by hand attached below*

>

> $dsolve\left(\left\{D(y)(t) = \frac{y(t)^3}{t+1}, y(0) = 1\right\}, y(t)\right);$

$$y(t) = \frac{1}{\sqrt{1 - 2\ln(t+1)}} \quad (3)$$

> # ***Question* 3 - *Work* by *hand attached below***

>

> *dsolve*( {D(D(*y*))(*t*) − 3·D(*y*)(*t*) + 2·*y*(*t*) = 0, *y*(0) = 2, D(*y*)(0) = 3}, *y*(*t*));

$$y(t) = e^{t} + e^{2t} \tag{4}$$

> # ***Question* 4 - *Work* by *hand attached below***

> *with*(*LinearAlgebra*) : *Matrix*([[3, -4], [4, 3]]);

$$\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} \tag{5}$$

> *Eigenvalues*(*Matrix*([[3, -4], [4, 3]]));

$$\begin{bmatrix} 3 + 4\,\mathrm{I} \\ 3 - 4\,\mathrm{I} \end{bmatrix} \tag{6}$$

> *Eigenvectors*(*Matrix*([[3, -4], [4, 3]]));

$$\begin{bmatrix} 3 + 4\,\mathrm{I} \\ 3 - 4\,\mathrm{I} \end{bmatrix}, \begin{bmatrix} \mathrm{I} & -\mathrm{I} \\ 1 & 1 \end{bmatrix} \tag{7}$$

Question 2

$$dy/dt = y^3/t+1 \quad , y(0)=1$$

$$\int \frac{dy}{y^3} = \int \frac{dt}{t+1}$$

$$-\frac{1}{2y^2} + C_1 = \ln(|t+1|) + C_2$$

$$1 = -2y^2(\ln(t+1) + C_3)$$

$$\frac{-1}{2(\ln(t+1)+C_3)} = y^2$$

$$y = -1/\sqrt{2(\ln(t+1)+c_3)}$$

$y(0)=1$

$$1 = -1/\sqrt{2(\ln(1)+c_3)}$$

$$1 = -1/2(0+C_3)$$

$$1 = -1/2C_3 \rightarrow 2C_3 = -1 \ , \ c_3 = -1/2$$

$$y(t) = 1/\sqrt{1-2\ln(t+1)}$$

## Question 3

$$y''(t) - 3y'(t) + 2y(t) = 0, \quad y(0) = 2, \quad y'(0) = 3$$

$$(e^{\gamma t})'' - 3(e^{\gamma t})' + 2e^{\gamma t} = 0$$

$$\gamma^2 e^{\gamma t} - 3\gamma e^{\gamma t} + 2e^{\gamma t} = 0$$

$$e^{\gamma t}(\gamma^2 - 3\gamma + 2) = 0$$

$$\hookrightarrow \gamma = 2, 1 \qquad \gamma_1 \neq \gamma_2$$

$$y(t) = c_1 e^{2t} + c_2 e^{t}, \quad y(0) = 2, \quad y'(0) = 3$$

$$2 = c_1 + c_2$$

$$y'(t) = 2c_1 e^{2t} + c_2 e^{t}$$

$$3 = 2c_1 + c_2$$

$$3 = 2c_1 + (2 - c_1)$$

$$c_1 = 1$$

$$c_2 = 1$$

$$y(t) = e^{2t} + e^{t}$$

# Question 4

$$A = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} \qquad \det(A - \lambda I) = 0$$

$$\begin{vmatrix} 3-\lambda & -4 \\ 4 & 3-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)^2 + 16 = 0$$

$$9 - 6\lambda + \lambda^2 + 16 = 0$$

$$\lambda^2 - 6\lambda + 25 = 0$$

$$\lambda = \frac{6 \pm \sqrt{36 - 100}}{2} = \frac{6 \pm \sqrt{-64}}{2}$$

$$\lambda = 3 \pm 4i$$

1) $\lambda = 3 + 4i$ $\quad \begin{bmatrix} -4i & -4 \\ 4 & -4i \end{bmatrix} \rightarrow \begin{bmatrix} -4i & -4 \\ 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -i & -1 \\ 0 & 0 \end{bmatrix}$

$\vec{v}_1 = \begin{pmatrix} i \\ 1 \end{pmatrix}$

2) $\lambda = 3 - 4i$ $\quad \begin{bmatrix} 4i & -4 \\ 4 & 4i \end{bmatrix} \rightarrow \begin{bmatrix} 4i & -4 \\ 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & i \\ 0 & 0 \end{bmatrix}$

$\vec{v}_1 = \begin{pmatrix} -i \\ 1 \end{pmatrix}$