```
> # OK to post homework
  # Max Mekhanikov, September 5th 2021, Assignment #1

  #  Question 1
>
> F := proc(n, p1, p2, p3, c0, c1, c2) option remember :
  if n = 0 then
  c0 :
  elif n = 1 then
  c1 :
  elif n = 2 then
  c2 :
  else
  expand(p1·F(n − 1, p1, p2, p3, c0, c1, c2) + p2·F(n − 2, p1, p2, p3, c0, c1, c2) + p3·F(n
      − 3, p1, p2, p3, c0, c1, c2));
  fi:
  end:

  seq(F(i, p1, p2, p3, c0, c1, c2), i = 4);
```

$$c0\,p1\,p3 + c1\,p1\,p2 + c2\,p1^2 + p3\,c1 + p2\,c2 \quad (1)$$

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>
```

 # In terms of the given initial condition vatiables (p1,p2,p3,c0,c1,c2) the expected number of females to be born at time n = 4 is shown above. This same sequence can be used to show the expected value numerically but would require given values for the inital conditions. As shown below, all of the variables are set to 1 (p1=p2=p3=c0=c1=c2=1) and the code shows the expected numeric values from n=1 up to n=4.

```
>  seq(F(i, 1, 1, 1, 1, 1, 1), i = 1 ..4);
```

$$1, 1, 3, 5 \quad (2)$$

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> # Question 2
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 # The same code from question 1 can be reused to output the number of females born at time n when provided an input value for n. For instance, at n = 10 the only change required is to alter the " i = " value in the last line as shown.

```
>
>  seq(F(i, p1, p2, p3, c0, c1, c2), i = 10);
```

$$c0\,p1^7\,p3 + c1\,p1^7\,p2 + c2\,p1^8 + 6\,c0\,p1^5\,p2\,p3 + c1\,p1^6\,p3 + 6\,c1\,p1^5\,p2^2 + 7\,c2\,p1^6\,p2 + 5\,c0\,p1^4\,p3^2 + 10\,c0\,p1^3\,p2^2\,p3 + 10\,c1\,p1^4\,p2\,p3 + 10\,c1\,p1^3\,p2^3 + 6\,c2\,p1^5\,p3 + 15\,c2\,p1^4\,p2^2 + 12\,c0\,p1^2\,p2\,p3^2 + 4\,c0\,p1\,p2^3\,p3 + 4\,c1\,p1^3\,p3^2 + 18\,c1\,p1^2\,p2^2\,p3 + 4\,c1\,p1\,p2^4 + 20\,c2\,p1^3\,p2\,p3 + 10\,c2\,p1^2\,p2^3 + 3\,c0\,p1\,p3^3 + 3\,c0\,p2^2\,p3^2 + 9\,c1\,p1\,p2\,p3^2 + 4\,c1\,p2^3\,p3 + 6\,c2\,p1^2\,p3^2 + 12\,c2\,p1\,p2^2\,p3 + c2\,p2^4 + c1\,p3^3 + 3\,c2\,p2\,p3^2 \quad (3)$$

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>
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 # This can be achieved for any value of n simply by changing the last line to the desired

duscrete ime value. Additionally, just as in the first question, we can also alter this code to show the numeric value when changing the initial conditions to numbers such as 1, rather than leaving in variable form.

> $seq(F(i, 1, 1, 1, 1, 1, 1), i = 10);$

$$193 \quad (4)$$

> *# This output shows us that when p1=p2=p3=c0=c1=c2=1, we can expect 193 females to be born at n=10.*

> *#* ***Question*** **3**

> *# c0 = c1 = c2 = 1, n = 1000*

> *# Assuming p1 = p2 = p3, a p value of one third would lead to a stable population at time n = 1000, a p value of one fourth would cause the population to go extinct at n=1000, and a p value of one half would cause population explosion.*

> $seq\left(F\left(i, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 1, 1, 1\right), i = 1000\right);$

$$1 \quad (5)$$

> *# Above : p1=p2=p3=*$\frac{1}{3}$*, causing stable population*

> $seq\left(evalf\left(F\left(i, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 1, 1, 1\right)\right), i = 1000\right);$

$$1.095771968\ 10^{-61} \quad (6)$$

> *# Above : p1=p2=p3=*$\frac{1}{4}$*, causing population extinction*

> $seq\left(evalf\left(F\left(i, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1, 1, 1\right)\right), i = 1000\right);$

$$1.276032978\ 10^{91} \quad (7)$$

> *# Above : p1=p2=p3=*$\frac{1}{2}$*, causing population explosion*