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#Problem 1
# The recursion is  $R(0) = c_0, R(1) = c_1, R(2) = c_2, R(3) = p_1 \cdot R(2) + p_2 \cdot R(1) + p_3 \cdot R(0), \dots, R(n) =$ 
#  $p_1 \cdot R(n-1) + p_2 \cdot R(n-2) + p_3 \cdot R(n-3)$ 
#  $R(4) = p_1 \cdot R(3) + p_2 \cdot R(2) + p_3 \cdot R(1) = p_1 \cdot (p_1 \cdot c_2 + p_2 \cdot c_1 + p_3 \cdot c_0) + p_2 \cdot c_2 + p_3 \cdot c_1$ 
# Problem 2

F := proc(p1, p2, p3, c0, c1, c2, n) option remember:
if n = 0 then
    c0 :
elif n = 1 then
    c1 :
elif n = 2 then
    c2 :
else
    expand(p1 \cdot F(p1, p2, p3, c0, c1, c2, n - 1) + p2 \cdot F(p1, p2, p3, c0, c1, c2, n - 2) + p3 \cdot F(p1, p2,
        p3, c0, c1, c2, n - 3))
fi:
end:

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$$seq(F(1, 1, 1, 1, 1, 1, 1, i), i=1..10); \quad 1, 1, 3, 5, 9, 17, 31, 57, 105, 193 \quad (1)$$

$$F(1, 1, 1, 1, 1, 1, 4) \quad \textcolor{blue}{5} \quad (2)$$

#Problem 3

(i) extinction

$$\text{evalf}\left(\text{seq}\left(F\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 1, 1, 1, i\right), i=1..10\right)\right);$$

$$1., 1., 0.7500000000, 0.6875000000, 0.6093750000, 0.5117187500, 0.4521484375, \\0.3933105469, 0.3392944336, 0.2961883545 \quad (3)$$

$$\begin{aligned} & \text{evalf}\left(\text{seq}\left(F\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 1, 1, 1, i\right), i=990..1000\right)\right); \\ & 4.468254626 \cdot 10^{-61}, 3.882363014 \cdot 10^{-61}, 3.373295354 \cdot 10^{-61}, 2.930978249 \cdot 10^{-61}, \\ & 2.546659154 \cdot 10^{-61}, 2.212733189 \cdot 10^{-61}, 1.922592648 \cdot 10^{-61}, 1.670496248 \cdot 10^{-61}, \\ & 1.451455521 \cdot 10^{-61}, 1.261136104 \cdot 10^{-61}, 1.095771968 \cdot 10^{-61} \end{aligned} \quad (4)$$

(ii) stable population

$$\text{evalf}\left(\text{seq}\left(F\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 1, 1, 1, i\right), i=990..1000\right)\right);$$

$$1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1. \quad (6)$$

(iii) population explosion

$$\text{evalf}\left(\text{seq}\left(F\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1, 1, 1, i\right), i=1..10\right)\right);$$

$$1., 1., 1.500000000, 1.750000000, 2.125000000, 2.687500000, 3.281250000, 4.046875000, \\5.007812500, 6.167968750 \quad (7)$$

$$\begin{aligned} & \operatorname{evalf}\left(\operatorname{seq}\left(F\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1, 1, 1, i\right), i=990..1000\right)\right); \\ & 1.561649092 \cdot 10^{90}, 1.926687579 \cdot 10^{90}, 2.377054517 \cdot 10^{90}, 2.932695594 \cdot 10^{90}, \\ & 3.618218845 \cdot 10^{90}, 4.463984478 \cdot 10^{90}, 5.507449458 \cdot 10^{90}, 6.794826391 \cdot 10^{90}, \\ & 8.383130163 \cdot 10^{90}, 1.034270301 \cdot 10^{91}, 1.276032978 \cdot 10^{91} \end{aligned} \tag{8}$$