

> # Max Mekhanikov – HW 19

= # Question 1

>

```
Dis2 :=proc(F,x,y,pt,h,A) local L,i:  
L := Orb2([x + h * F[1],y + h * F[2]],x,y,pt,0,trunc(A/h)):  
L := [seq([i*h,[L[i][1],L[i][2]]],i=1..nops(L))]:  
end:
```

```
Orb2 :=proc(F,x,y,pt0,K1,K2) local pt,L,i:  
pt := pt0:
```

```
for i from 1 to K1-1 do  
pt := subs({x=pt[1],y=pt[2]},F):  
od:
```

```
L := []:  
for i from K1 to K2 do  
L := [op(L),pt]:  
pt := normal(subs({x=pt[1],y=pt[2]},F)):  
od:  
L:  
end:
```

```
SIRS :=proc(s,i,beta,gamma,nu,N) : [-beta*s*i + gamma*(N-s-i), beta*s*i - nu*i]:  
end:
```

#Dis2(F,x,y,pt,h,A): The approximate orbit of the Dynamical system approximating the 2D for  
the autonomous continuous dynamical process

#dx/dt=F[1](x(t),y(t))  
#dy/dt=F[2](x(t),y(t)), x(0)=pt[1], y(0)=pt[2] with mesh size h from t=0 to t=A  
Dis2 :=proc(F,x,y,pt,h,A) local L,i :

```
L := Orb2([x + h * F[1],y + h * F[2]],x,y,pt,0,trunc(A/h)):  
L := [seq([i*h,[L[i][1],L[i][2]]],i=1..nops(L))]:  
end:
```

```
SIRSDemo :=proc(N,IN,gamma,nu,h,A) local L,beta,i:  
print(`This is a numerical demonstration of the R0 phenomenon in the SIRS model using  
discretization with mesh size=`,h, `and letting it run until time t=`,A):  
print(`with population size`,N, `and fixed parameters nu=`,nu, `and gamma=`,gamma):  
print(`where we change beta from 0.2*nu/N to 4*nu/N `):
```

```

print(`Recall that the epidemic will persist if beta exceeds nu/N, that in this case is `, nu/N) :
print(`We start with `, IN, `infected individuals, 0 removed and hence`, N-IN, `susceptible`) :
print(`We will show what happens once time is close to`, A) :
for i from 1 by 2 to 40 do
  beta := i/10 * (nu/N) :
  print(`beta is `, i/10, `times the threshold value`) :
  L := Dis2(SIRS(s, i, beta, gamma, nu, N), s, i, [N-IN, IN], h, A) :
  print(`the long-term behavior is`) :
  print([op(nops(L)-3 ..nops(L), L)]) :
od:
end:

```

> SIRSdemo(1000, 200, 3, 1, 0.01, 10)

This is a numerical demonstration of the R0 phenomenon in the SIRS model using discretization with mesh size=, 0.01, and letting it run until time t=, 10

with population size, 1000, and fixed parameters nu=, 1, and gamma=, 3

where we change beta from  $0.2 \cdot \text{nu}/N$  to  $4 \cdot \text{nu}/N$

Recall that the epidemic will persist if beta exceeds nu/N, that in this case is,  $\frac{1}{1000}$

We start with , 200, infected individuals, 0 removed and hence, 800, susceptible

We will show what happens once time is close to, 10

beta is,  $\frac{1}{10}$ , times the threshold value

the long-term behavior is

[[9.98, [998.9666995, 0.9909989667]], [9.99, [998.9666995, 0.9909989667]], [10.00, [998.9666995, 0.9909989667]], [10.01, [998.9666995, 0.9909989667]]]

beta is,  $\frac{3}{10}$ , times the threshold value

the long-term behavior is

[[9.98, [996.7009881, 2.978970309]], [9.99, [996.7009881, 2.978970309]], [10.00, [996.7009881, 2.978970309]], [10.01, [996.7009881, 2.978970309]]]

beta is,  $\frac{1}{2}$ , times the threshold value

the long-term behavior is

[[9.98, [994.1715221, 4.974854288]], [9.99, [994.1715221, 4.974854288]], [10.00, [994.1715221, 4.974854288]], [10.01, [994.1715221, 4.974854288]]]

beta is,  $\frac{7}{10}$ , times the threshold value

the long-term behavior is

[[9.98, [991.3807432, 6.978577656]], [9.99, [991.3807432, 6.978577656]], [10.00, [991.3807432, 6.978577656]], [10.01, [991.3807432, 6.978577656]]]

*beta is,  $\frac{9}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [988.3315033, 8.990054852]], [9.99, [988.3315033, 8.990054852]], [10.00, [988.3315033, 8.990054852]], [10.01, [988.3315033, 8.990054852]]]$

*beta is,  $\frac{11}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [985.0270559, 11.00918827]], [9.99, [985.0270559, 11.00918827]], [10.00, [985.0270559, 11.00918827]], [10.01, [985.0270559, 11.00918827]]]$

*beta is,  $\frac{13}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [981.4710448, 13.03586861]], [9.99, [981.4710448, 13.03586861]], [10.00, [981.4710448, 13.03586861]], [10.01, [981.4710448, 13.03586861]]]$

*beta is,  $\frac{3}{2}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [977.6674922, 15.06997519]], [9.99, [977.6674922, 15.06997519]], [10.00, [977.6674922, 15.06997519]], [10.01, [977.6674922, 15.06997519]]]$

*beta is,  $\frac{17}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [973.6207848, 17.11137641]], [9.99, [973.6207848, 17.11137641]], [10.00, [973.6207848, 17.11137641]], [10.01, [973.6207848, 17.11137641]]]$

*beta is,  $\frac{19}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [969.3356593, 19.15993017]], [9.99, [969.3356593, 19.15993017]], [10.00, [969.3356593, 19.15993017]], [10.01, [969.3356593, 19.15993017]]]$

*beta is,  $\frac{21}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [964.8171858, 21.21548438]], [9.99, [964.8171858, 21.21548438]], [10.00, [964.8171858, 21.21548438]], [10.01, [964.8171858, 21.21548438]]]$

*beta is,  $\frac{23}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [960.0707508, 23.27787743]], [9.99, [960.0707508, 23.27787743]], [10.00,$

[960.0707508, 23.27787743]], [10.01, [960.0707508, 23.27787743]]]

*beta is,  $\frac{5}{2}$ , times the threshold value*

*the long-term behavior is*

[[9.98, [955.1020392, 25.34693877]], [9.99, [955.1020392, 25.34693877]], [10.00, [955.1020392, 25.34693877]], [10.01, [955.1020392, 25.34693877]]]

*beta is,  $\frac{27}{10}$ , times the threshold value*

*the long-term behavior is*

[[9.98, [949.9170149, 27.42248950]], [9.99, [949.9170149, 27.42248950]], [10.00, [949.9170149, 27.42248950]], [10.01, [949.9170149, 27.42248950]]]

*beta is,  $\frac{29}{10}$ , times the threshold value*

*the long-term behavior is*

[[9.98, [944.5219011, 29.50434292]], [9.99, [944.5219011, 29.50434292]], [10.00, [944.5219011, 29.50434292]], [10.01, [944.5219011, 29.50434292]]]

*beta is,  $\frac{31}{10}$ , times the threshold value*

*the long-term behavior is*

[[9.98, [938.9231598, 31.59230516]], [9.99, [938.9231598, 31.59230516]], [10.00, [938.9231598, 31.59230516]], [10.01, [938.9231598, 31.59230516]]]

*beta is,  $\frac{33}{10}$ , times the threshold value*

*the long-term behavior is*

[[9.98, [933.1274712, 33.68617582]], [9.99, [933.1274712, 33.68617582]], [10.00, [933.1274712, 33.68617582]], [10.01, [933.1274712, 33.68617582]]]

*beta is,  $\frac{7}{2}$ , times the threshold value*

*the long-term behavior is*

[[9.98, [927.1417118, 35.78574860]], [9.99, [927.1417118, 35.78574860]], [10.00, [927.1417118, 35.78574860]], [10.01, [927.1417118, 35.78574860]]]

*beta is,  $\frac{37}{10}$ , times the threshold value*

*the long-term behavior is*

[[9.98, [920.9729335, 37.89081195]], [9.99, [920.9729335, 37.89081195]], [10.00, [920.9729335, 37.89081195]], [10.01, [920.9729335, 37.89081195]]]

*beta is,  $\frac{39}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [914.6283415, 40.00114971]], [9.99, [914.6283415, 40.00114971]], [10.00, [914.6283415, 40.00114971]], [10.01, [914.6283415, 40.00114971]]]$  (1)

>  $SIRSdemo(1000, 200, 3, 2, 0.01, 10)$

This is a numerical demonstration of the R0 phenomenon in the SIRS model using discretization with mesh size=, 0.01, and letting it run until time t=, 10

with population size, 1000, and fixed parameters nu=, 2, and gamma=, 3

where we change beta from  $0.2 * nu / N$  to  $4 * nu / N$

Recall that the epidemic will persist if beta exceeds  $nu / N$ , that in this case is,  $\frac{1}{500}$

We start with , 200, infected individuals, 0 removed and hence, 800, susceptible

We will show what happens once time is close to, 10

beta is,  $\frac{1}{10}$ , times the threshold value

the long-term behavior is

$[[9.98, [998.9334028, 0.9819978668]], [9.99, [998.9334028, 0.9819978668]], [10.00, [998.9334028, 0.9819978668]], [10.01, [998.9334028, 0.9819978668]]]$

beta is,  $\frac{3}{10}$ , times the threshold value

the long-term behavior is

$[[9.98, [996.4021571, 2.957935239]], [9.99, [996.4021571, 2.957935239]], [10.00, [996.4021571, 2.957935239]], [10.01, [996.4021571, 2.957935239]]]$

beta is,  $\frac{1}{2}$ , times the threshold value

the long-term behavior is

$[[9.98, [993.3444243, 4.949667221]], [9.99, [993.3444243, 4.949667221]], [10.00, [993.3444243, 4.949667221]], [10.01, [993.3444243, 4.949667221]]]$

beta is,  $\frac{7}{10}$ , times the threshold value

the long-term behavior is

$[[9.98, [989.7667603, 6.956997143]], [9.99, [989.7667603, 6.956997143]], [10.00, [989.7667603, 6.956997143]], [10.01, [989.7667603, 6.956997143]]]$

beta is,  $\frac{9}{10}$ , times the threshold value

the long-term behavior is

$[[9.98, [985.6773407, 8.979679729]], [9.99, [985.6773407, 8.979679729]], [10.00, [985.6773407, 8.979679729]], [10.01, [985.6773407, 8.979679729]]]$

beta is,  $\frac{11}{10}$ , times the threshold value

the long-term behavior is

$[[9.98, [981.0859054, 11.01742279]], [9.99, [981.0859054, 11.01742279]], [10.00, [981.0859054, 11.01742279]], [10.01, [981.0859054, 11.01742279]]]$

*beta is,  $\frac{13}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [976.0036901, 13.06988925]], [9.99, [976.0036901, 13.06988925]], [10.00, [976.0036901, 13.06988925]], [10.01, [976.0036901, 13.06988925]]]$

*beta is,  $\frac{3}{2}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [970.4433482, 15.13669951]], [9.99, [970.4433482, 15.13669951]], [10.00, [970.4433482, 15.13669951]], [10.01, [970.4433482, 15.13669951]]]$

*beta is,  $\frac{17}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [964.4188616, 17.21743410]], [9.99, [964.4188616, 17.21743410]], [10.00, [964.4188616, 17.21743410]], [10.01, [964.4188616, 17.21743410]]]$

*beta is,  $\frac{19}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [957.9454447, 19.31163661]], [9.99, [957.9454447, 19.31163661]], [10.00, [957.9454447, 19.31163661]], [10.01, [957.9454447, 19.31163661]]]$

*beta is,  $\frac{21}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [951.0394389, 21.41881679]], [9.99, [951.0394389, 21.41881679]], [10.00, [951.0394389, 21.41881679]], [10.01, [951.0394389, 21.41881679]]]$

*beta is,  $\frac{23}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [943.7182031, 23.53845386]], [9.99, [943.7182031, 23.53845386]], [10.00, [943.7182031, 23.53845386]], [10.01, [943.7182031, 23.53845386]]]$

*beta is,  $\frac{5}{2}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [935.9999984, 25.67000000]], [9.99, [935.9999984, 25.67000000]], [10.00, [935.9999984, 25.67000000]], [10.01, [935.9999984, 25.67000000]]]$

*beta is,  $\frac{27}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [927.9038703, 27.81288384]], [9.99, [927.9038703, 27.81288384]], [10.00, [927.9038703, 27.81288384]], [10.01, [927.9038703, 27.81288384]]]$

*beta is,  $\frac{29}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [919.4495282, 29.96651411]], [9.99, [919.4495282, 29.96651411]], [10.00, [919.4495282, 29.96651411]], [10.01, [919.4495282, 29.96651411]]]$

*beta is,  $\frac{31}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [910.6572255, 32.13028319]], [9.99, [910.6572255, 32.13028319]], [10.00, [910.6572255, 32.13028319]], [10.01, [910.6572255, 32.13028319]]]$

*beta is,  $\frac{33}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [901.5476397, 34.30357076]], [9.99, [901.5476397, 34.30357076]], [10.00, [901.5476397, 34.30357076]], [10.01, [901.5476397, 34.30357076]]]$

*beta is,  $\frac{7}{2}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [892.1417551, 36.48574730]], [9.99, [892.1417551, 36.48574730]], [10.00, [892.1417551, 36.48574730]], [10.01, [892.1417551, 36.48574730]]]$

*beta is,  $\frac{37}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [882.4607475, 38.67617753]], [9.99, [882.4607475, 38.67617753]], [10.00, [882.4607475, 38.67617753]], [10.01, [882.4607475, 38.67617753]]]$

*beta is,  $\frac{39}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [872.5258747, 40.87422371]], [9.99, [872.5258747, 40.87422371]], [10.00, [872.5258747, 40.87422371]], [10.01, [872.5258747, 40.87422371]]]$  (2)

> *SIRSdemo(1000, 200, 7, 3, 0.01, 10)*

*This is a numerical demonstration of the R0 phenomenon in the SIRS model using discretization with mesh size=, 0.01, and letting it run until time t=, 10*

*with population size, 1000, and fixed parameters nu=, 3, and gamma=, 7*

*where we change beta from 0.2\*nu/N to 4\*nu/N*

Recall that the epidemic will persist if beta exceeds  $\nu/N$ , that in this case is,  $\frac{3}{1000}$

We start with 200, infected individuals, 0 removed and hence, 800, susceptible

We will show what happens once time is close to, 10

beta is,  $\frac{1}{10}$ , times the threshold value

the long-term behavior is

[[9.98, [998.9571869, 0.9729968716]], [9.99, [998.9571869, 0.9729968716]], [10.00, [998.9571869, 0.9729968716]], [10.01, [998.9571869, 0.9729968716]]]

beta is,  $\frac{3}{10}$ , times the threshold value

the long-term behavior is

[[9.98, [996.6155905, 2.936908621]], [9.99, [996.6155905, 2.936908621]], [10.00, [996.6155905, 2.936908621]], [10.01, [996.6155905, 2.936908621]]]

beta is,  $\frac{1}{2}$ , times the threshold value

the long-term behavior is

[[9.98, [993.9350689, 4.924545130]], [9.99, [993.9350689, 4.924545130]], [10.00, [993.9350689, 4.924545130]], [10.01, [993.9350689, 4.924545130]]]

beta is,  $\frac{7}{10}$ , times the threshold value

the long-term behavior is

[[9.98, [990.9190693, 6.935665103]], [9.99, [990.9190693, 6.935665103]], [10.00, [990.9190693, 6.935665103]], [10.01, [990.9190693, 6.935665103]]]

beta is,  $\frac{9}{10}$ , times the threshold value

the long-term behavior is

[[9.98, [987.5717147, 8.969979927]], [9.99, [987.5717147, 8.969979927]], [10.00, [987.5717147, 8.969979927]], [10.01, [987.5717147, 8.969979927]]]

beta is,  $\frac{11}{10}$ , times the threshold value

the long-term behavior is

[[9.98, [983.8977865, 11.02715490]], [9.99, [983.8977865, 11.02715490]], [10.00, [983.8977865, 11.02715490]], [10.01, [983.8977865, 11.02715490]]]

beta is,  $\frac{13}{10}$ , times the threshold value

the long-term behavior is

[[9.98, [979.9027040, 13.10681067]], [9.99, [979.9027040, 13.10681067]], [10.00, [979.9027040, 13.10681067]], [10.01, [979.9027040, 13.10681067]]]

*beta is,  $\frac{3}{2}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [975.5925002, 15.20852494]], [9.99, [975.5925002, 15.20852494]], [10.00, [975.5925002, 15.20852494]], [10.01, [975.5925002, 15.20852494]]]$

*beta is,  $\frac{17}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [970.9737953, 17.33183428]], [9.99, [970.9737953, 17.33183428]], [10.00, [970.9737953, 17.33183428]], [10.01, [970.9737953, 17.33183428]]]$

*beta is,  $\frac{19}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [966.0537675, 19.47623623]], [9.99, [966.0537675, 19.47623623]], [10.00, [966.0537675, 19.47623623]], [10.01, [966.0537675, 19.47623623]]]$

*beta is,  $\frac{21}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [960.8401210, 21.64119148]], [9.99, [960.8401210, 21.64119148]], [10.00, [960.8401210, 21.64119148]], [10.01, [960.8401210, 21.64119148]]]$

*beta is,  $\frac{23}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [955.3410529, 23.82612625]], [9.99, [955.3410529, 23.82612625]], [10.00, [955.3410529, 23.82612625]], [10.01, [955.3410529, 23.82612625]]]$

*beta is,  $\frac{5}{2}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [949.5652167, 26.03043478]], [9.99, [949.5652167, 26.03043478]], [10.00, [949.5652167, 26.03043478]], [10.01, [949.5652167, 26.03043478]]]$

*beta is,  $\frac{27}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [943.5216861, 28.25348193]], [9.99, [943.5216861, 28.25348193]], [10.00, [943.5216861, 28.25348193]], [10.01, [943.5216861, 28.25348193]]]$

*beta is,  $\frac{29}{10}$ , times the threshold value*

*the long-term behavior is*

$[[9.98, [937.2199158, 30.49460585]], [9.99, [937.2199158, 30.49460585]], [10.00,$

[937.2199158, 30.49460585]], [10.01, [937.2199158, 30.49460585]]]

*beta is,  $\frac{31}{10}$ , times the threshold value*

*the long-term behavior is*

[[9.98, [930.6697029, 32.75312075]], [9.99, [930.6697029, 32.75312075]], [10.00, [930.6697029, 32.75312075]], [10.01, [930.6697029, 32.75312075]]]

*beta is,  $\frac{33}{10}$ , times the threshold value*

*the long-term behavior is*

[[9.98, [923.8811464, 35.02831970]], [9.99, [923.8811464, 35.02831970]], [10.00, [923.8811464, 35.02831970]], [10.01, [923.8811464, 35.02831970]]]

*beta is,  $\frac{7}{2}$ , times the threshold value*

*the long-term behavior is*

[[9.98, [916.8646074, 37.31947743]], [9.99, [916.8646074, 37.31947743]], [10.00, [916.8646074, 37.31947743]], [10.01, [916.8646074, 37.31947743]]]

*beta is,  $\frac{37}{10}$ , times the threshold value*

*the long-term behavior is*

[[9.98, [909.6306685, 39.62585316]], [9.99, [909.6306685, 39.62585316]], [10.00, [909.6306685, 39.62585316]], [10.01, [909.6306685, 39.62585316]]]

*beta is,  $\frac{39}{10}$ , times the threshold value*

*the long-term behavior is*

[[9.98, [902.1900937, 41.94669340]], [9.99, [902.1900937, 41.94669340]], [10.00, [902.1900937, 41.94669340]], [10.01, [902.1900937, 41.94669340]]] (3)

>

## # Question 2

> **IsStable := proc(M) local EiL, i :**  
EiL := Eigenvalues(evalf(Matrix(M))) :  
evalb(max(seq(coeff(EiL[i], I, 0), i = 1..nops(M))) < 0):  
**end:**

RandNice := **proc(var, K) local ra, i :**  
ra := rand(1..K) :  
[seq((ra() - add(ra() \* var[i], i = 1..nops(var))) \* (ra() - add(ra() \* var[i], i = 1..nops(var))), i = 1..nops(var))]:  
**end:**

```

EquPts :=proc(F, var) local sol, iL :
if nops(F) ≠ nops(var) then
RETURN(FAIL) :
fi:

```

```

sol := {solve({op(F)}, {op(var)})} :
{seq(subs(sol[iL], var), iL = 1 ..nops(sol))} :
end:

```

```

StEquPts :=proc(F, var) local d, pt, E, S, J, i, j, J0, iL, Ei0 :
d := nops(var) :

```

```

if nops(F) ≠ d then
RETURN(FAIL) :
fi:

```

```

E := EquPts(F, var) :
S := {} :

```

```

J := [seq([seq(diff(F[i], var[j]), j = 1 ..d)], i = 1 ..d)] : #J is the general Jacobian

```

```

for pt in E do
J0 := evalf(subs({seq(var[iL] = pt[iL], iL = 1 ..d)}, J)) :
if IsStable(J0) then
S := S union {pt} :
fi:
od:

```

```

S :
end:

```

$$f1 := \text{RandNice}([x, y], 8) \\ f1 := [(1 - 3x - 5y)(5 - 2x - 2y), (2 - 3x - 2y)(8 - 4x - 7y)] \quad (4)$$

>  $f2 := \text{RandNice}([x, y], 8) \\ f2 := [(5 - 3x - 8y)(1 - 8x - 5y), (2 - 3x - 2y)(2 - 4x - 8y)] \quad (5)$

>  $f3 := \text{RandNice}([x, y], 8) \\ f3 := [(3 - 3x - y)(2 - 5x - 4y), (5 - 6x - 2y)(3 - 7x - 8y)] \quad (6)$

>  $\text{EquPts}(f1, [x, y]) \\ \left\{ \left[ 1, \frac{1}{7} \right], \left[ -\frac{19}{4}, \frac{17}{4} \right], \left[ -\frac{1}{26}, \frac{37}{26} \right], \left[ \frac{2}{5}, \frac{2}{5} \right] \right\} \quad (7)$

$$\begin{aligned}
L1 &:= \text{Dis2}\left(fI, x, y, \left[1, \frac{1}{7}\right] + [0.1, 0.1], 0.01, 10\right) \\
L1 &:= [[0.01, [1.1, 0.2428571429]], [0.02, [1.153142857, 0.2180000000]], [0.03, \\
&\quad [1.214189274, 0.1938513690]], [0.04, [1.286739448, 0.1699809021]], [0.05, [1.376313379, \\
&\quad 0.1459352535]], [0.06, [1.491870430, 0.1212989155]], [0.07, [1.649006015, \\
&\quad 0.09589250101]], [0.08, [1.877371535, 0.07042877780]], [0.09, [2.240250377, \\
&\quad 0.04884309203]], [0.10, [2.895910203, 0.04777014036]], [0.11, [4.342690740, \\
&\quad 0.1445253964]], [0.12, [8.830446821, 0.8222787195]], [0.13, [35.29828498, \\
&\quad 7.489411812]], [0.14, [627.2690837, 217.8932591]], [0.15, [235295.6479, 101093.9026]], \\
&\quad [0.16, [3.674730419 \times 10^{10}, 1.712301338 \times 10^{10}]], [0.17, [9.369027533 \times 10^{20}, \\
&\quad 4.506727979 \times 10^{20}]], [0.18, [6.201090375 \times 10^{41}, 3.020930697 \times 10^{41}]], [0.19, \\
&\quad [2.736435845 \times 10^{83}, 1.339882789 \times 10^{83}]], [0.20, [5.344400540 \times 10^{166}, 2.622218548 \\
&\quad \times 10^{166}]], [0.21, [2.040996150 \times 10^{333}, 1.002235312 \times 10^{333}]], [0.22, [2.978080114 \\
&\quad \times 10^{666}, 1.462877371 \times 10^{666}]], [0.23, [6.341740796 \times 10^{1332}, 3.115573839 \times 10^{1332}]], \\
&\quad [0.24, [2.875983828 \times 10^{2665}, 1.412990921 \times 10^{2665}]], [0.25, [5.915016130 \times 10^{5330}, \\
&\quad 2.906151692 \times 10^{5330}]], [0.26, [2.502075409 \times 10^{10661}, 1.229324489 \times 10^{10661}]], [0.27, \\
&\quad [4.477045646 \times 10^{21322}, 2.199678393 \times 10^{21322}]], [0.28, [1.433423982 \times 10^{42645}, \\
&\quad 7.042760961 \times 10^{42644}]], [0.29, [1.469404318 \times 10^{85290}, 7.219545235 \times 10^{85289}]], [0.30, \\
&\quad [1.544097598 \times 10^{170580}, 7.586533453 \times 10^{170579}]], [0.31, [1.705067895 \times 10^{341160}, \\
&\quad 8.377421106 \times 10^{341159}]], [0.32, [2.079100864 \times 10^{682320}, 1.021513817 \times 10^{682320}]], \\
&\quad [0.33, [3.091315508 \times 10^{1364640}, 1.518839997 \times 10^{1364640}]], [0.34, [6.834061515 \\
&\quad \times 10^{2729280}, 3.357743978 \times 10^{2729280}]], [0.35, [3.340027074 \times 10^{5458561}, 1.641038173 \\
&\quad \times 10^{5458561}]], [0.36, [7.977966253 \times 10^{10917122}, 3.919772769 \times 10^{10917122}]], [0.37, \\
&\quad [4.551731235 \times 10^{21834245}, 2.236378494 \times 10^{21834245}]], [0.38, [1.481649373 \times 10^{43668491}, \\
&\quad 7.279711002 \times 10^{43668490}]], [0.39, [1.569940178 \times 10^{87336982}, 7.713505634 \\
&\quad \times 10^{87336981}]], [0.40, [1.762618926 \times 10^{174673964}, 8.660184136 \times 10^{174673963}]], [0.41, \\
&\quad [2.221821059 \times 10^{349347928}, 1.091635816 \times 10^{349347928}]], [0.42, [3.530289970 \\
&\quad \times 10^{698695856}, 1.734519058 \times 10^{698695856}]], [0.43, [8.912775731 \times 10^{1397391712}, \\
&\quad 4.379067866 \times 10^{1397391712}]], [0.44, [5.680913524 \times 10^{2794783425}, 2.791173772 \\
&\quad \times 10^{2794783425}]], [0.45, [2.307961595 \times 10^{5589566851}, 1.133958798 \times 10^{5589566851}]], \\
&\quad [0.46, [3.809336838 \times 10^{11179133702}, 1.871621708 \times 10^{11179133702}]], [0.47, [1.037745776 \\
&\quad \times 10^{22358267405}, 5.098702485 \times 10^{22358267404}]], [0.48, [7.701479602 \times 10^{44716534809}, \\
&\quad 3.783928020 \times 10^{44716534809}]], [0.49, [4.241705964 \times 10^{89433069619}, 2.084055388
\end{aligned}$$

$$\begin{aligned}
& \times 10^{89433069619}]], [0.50, [1.286688267 \times 10^{178866139239}, 6.321818715 \times 10^{178866139238}]], \\
& [0.51, [1.183965105 \times 10^{357732278478}, 5.817114330 \times 10^{357732278477}]], [0.52, [1.002466863 \\
& \times 10^{715464556956}, 4.925368439 \times 10^{715464556955}]], [0.53, [7.186745614 \times 10^{1430929113911}, \\
& 3.531026443 \times 10^{1430929113911}]], [0.54, [3.693658724 \times 10^{2861858227823}, 1.814786181 \\
& \times 10^{2861858227823}]], [0.55, [9.756762953 \times 10^{5723716455646}, 4.793739731 \\
& \times 10^{5723716455646}]], [0.56, [6.807752030 \times 10^{11447432911293}, 3.344817491 \\
& \times 10^{11447432911293}]], [0.57, [3.314359991 \times 10^{22894865822587}, 1.628427302 \\
& \times 10^{22894865822587}]], [0.58, [7.855820956 \times 10^{45789731645174}, 3.859759756 \\
& \times 10^{45789731645174}]], [0.59, [4.413421175 \times 10^{91579463290349}, 2.168423331 \\
& \times 10^{91579463290349}]], [0.60, [1.392973869 \times 10^{183158926580699}, 6.844026252 \\
& \times 10^{183158926580698}]], [0.61, [1.387644313 \times 10^{366317853161398}, 6.817840818 \\
& \times 10^{366317853161397}]], [0.62, [1.377046296 \times 10^{732635706322796}, 6.765770129 \\
& \times 10^{732635706322795}]], [0.63, [1.356092467 \times 10^{1465271412645592}, 6.662818771 \\
& \times 10^{1465271412645591}]], [0.64, [1.315136491 \times 10^{2930542825291184}, 6.461591895 \\
& \times 10^{2930542825291183}]], [0.65, [1.236897971 \times 10^{5861085650582368}, 6.077186639 \\
& \times 10^{5861085650582367}]], [0.66, [1.094107450 \times 10^{11722171301164736}, 5.375621379 \\
& \times 10^{11722171301164735}]], [0.67, [8.560757037 \times 10^{23444342602329471}, 4.206112347 \\
& \times 10^{23444342602329471}]], [0.68, [5.241029015 \times 10^{46888685204658943}, 2.575047599 \\
& \times 10^{46888685204658943}]], [0.69, [1.964379300 \times 10^{93777370409317887}, 9.651482913 \\
& \times 10^{93777370409317886}]], [0.70, [2.759579556 \times 10^{187554740818635774}, 1.355849909 \\
& \times 10^{187554740818635774}]], [0.71, [5.446005290 \times 10^{375109481637271548}, 2.675757529 \\
& \times 10^{375109481637271548}]], [0.72, [2.121037460 \times 10^{750218963274543097}, 1.042118332 \\
& \times 10^{750218963274543097}]], [0.73, [3.217280290 \times 10^{1500437926549086194}, 1.580729635 \\
& \times 10^{1500437926549086194}]], [0.74, [7.402356851 \times 10^{3000875853098172388}, 3.636961590 \\
& \times 10^{3000875853098172388}]], [0.75, [3.918611928 \times 10^{6001751706196344777}, 1.925311270 \\
& \times 10^{6001751706196344777}]], [0.76, [Float( $\infty$ ), Float( $\infty$ )]], [0.77, [Float( $\infty$ ), Float( $\infty$ )]], \\
& [0.78, [Float( $\infty$ ), Float( $\infty$ )]], [0.79, [Float( $\infty$ ), Float( $\infty$ )]], [0.80, [Float( $\infty$ ), \\
& Float( $\infty$ )]], [0.81, [Float( $\infty$ ), Float( $\infty$ )]], [0.82, [Float( $\infty$ ), Float( $\infty$ )]], [0.83, [ \\
& Float( $\infty$ ), Float( $\infty$ )]], [0.84, [Float( $\infty$ ), Float( $\infty$ )]], [0.85, [Float( $\infty$ ), Float( $\infty$ )]], \\
& [0.86, [Float( $\infty$ ), Float( $\infty$ )]], [0.87, [Float( $\infty$ ), Float( $\infty$ )]], [0.88, [Float( $\infty$ ), \\
& Float( $\infty$ )]], [0.89, [Float( $\infty$ ), Float( $\infty$ )]], [0.90, [Float( $\infty$ ), Float( $\infty$ )]], [0.91, [ \\
& Float( $\infty$ ), Float( $\infty$ )]], [0.92, [Float( $\infty$ ), Float( $\infty$ )]], [0.93, [Float( $\infty$ ), Float( $\infty$ )]], \\
& [0.94, [Float( $\infty$ ), Float( $\infty$ )]], [0.95, [Float( $\infty$ ), Float( $\infty$ )]], [0.96, [Float( $\infty$ ),
\end{aligned}$$

















```

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Float(∞), Float(∞)]], [9.96, [Float(∞), Float(∞)]], [9.97, [Float(∞), Float(∞)]],
[9.98, [Float(∞), Float(∞)]], [9.99, [Float(∞), Float(∞)]], [10.00, [Float(∞),
Float(∞)]], [10.01, [Float(∞), Float(∞)]]]

```

$$L2 := Dis2\left(f1, x, y, \left[-\frac{19}{4}, \frac{17}{4}\right] + [0.1, 0.1], 0.01, 10\right)$$

$L2 := [[0.01, [-4.650000000, 4.350000000]], [0.02, [-4.932000000, 4.450000000]], [0.03, -5.055352600, 4.506087160]], [0.04, [-5.124125141, 4.551184594]], [0.05, -5.186430169, 4.599074022]], [0.06, [-5.255712864, 4.654169311]], [0.07, -5.336361871, 4.718704611]], [0.08, [-5.431317787, 4.794809160]], [0.09, -5.543812187, 4.885055460]], [0.10, [-5.677905551, 4.992731634]], [0.11, -5.838884670, 5.122136761]], [0.12, [-6.033780389, 5.279002071]], [0.13, -6.272146346, 5.471133593]], [0.14, [-6.567286920, 5.709429258]], [0.15, -6.938268509, 6.009549155]], [0.16, [-7.413356178, 6.394776868]], [0.17, -8.036184826, 6.901171673]], [0.18, [-8.877518402, 7.587415072]], [0.19, [-10.05933604, 8.555043235]], [0.20, [-11.80879747, 9.993933996]], [0.21, [-14.59378717, 12.29698305]], [0.22, [-19.51853081, 16.39617867]], [0.23, [-29.74220033, 24.97308617]], [0.24, [-57.33553109, 48.33807610]], [0.25, [-176.2794365, 150.1114350]], [0.26, [-1463.118398, 1262.230053]], [0.27, -99255.66453, 86322.31322]], [0.28, [-4.706695062  $\times 10^8$ , 4.102786726  $\times 10^8$ ]], [0.29, [-1.070319476  $\times 10^{16}$ , 9.334649370  $\times 10^{15}$ ]], [0.30, [-5.548840058  $\times 10^{30}$ , 4.839936347  $\times 10^{30}$ ]], [0.31, [-1.492245213  $\times 10^{60}$ , 1.301638921  $\times 10^{60}$ ]], [0.32, [-1.079393108  $\times 10^{119}$ , 9.415277644  $\times 10^{118}$ ]], [0.33, [-5.647732104  $\times 10^{236}$ , 4.926385049  $\times 10^{236}$ ]], [0.34, [-1.546204020  $\times 10^{472}$ , 1.348718311  $\times 10^{472}$ ]], [0.35, [-1.158919192  $\times 10^{943}$ , 1.010898745  $\times 10^{943}$ ]], [0.36, [-6.510676231  $\times 10^{1884}$ , 5.679114342  $\times 10^{1884}$ ]], [0.37, [-2.054811760  $\times 10^{3768}$ , 1.792365434  $\times 10^{3768}$ ]], [0.38, [-2.046745988  $\times 10^{7535}$ , 1.785329845  $\times 10^{7535}$ ]], [0.39, [-2.030709305  $\times 10^{15069}$ , 1.771341410  $\times 10^{15069}$ ]], [0.40, [-1.999011895  $\times 10^{30137}$ , 1.743692480  $\times 10^{30137}$ ]], [0.41, [-1.937093632  $\times 10^{60273}$ , 1.689682596  $\times 10^{60273}$ ]], [0.42, [-1.818951372  $\times 10^{120545}$ , 1.586629799  $\times 10^{120545}$ ]], [0.43, [-1.603843697  $\times 10^{241089}$ , 1.398996281  $\times 10^{241089}$ ]], [0.44, [-1.246935388  $\times 10^{482177}$ , 1.087673301  $\times 10^{482177}$ ]], [0.45, [-7.537160526  $\times 10^{964352}$ , 6.574493231  $\times 10^{964352}$ ]], [0.46, [-2.753819011  $\times 10^{1928704}$ , 2.402093519  $\times 10^{1928704}$ ]], [0.47, [-3.676128219  $\times 10^{3857407}$ , 3.206602817  $\times 10^{3857407}$ ]], [0.48, [-6.550903047  $\times 10^{7714813}$ , 5.714203325  $\times 10^{7714813}$ ]], [0.49, [-2.080281982  $\times 10^{15429626}$ , 1.814582529  $\times 10^{15429626}$ ]], [0.50, [-2.097800951  $\times 10^{30859251}$ , 1.829863929  $\times 10^{30859251}$ ]], [0.51, [-2.133282750  $\times 10^{61718501}$ , 1.860813896  $\times 10^{61718501}$ ]], [0.52, [-2.206056917  $\times 10^{123437001}$ , 1.924293139  $\times 10^{123437001}$ ]], [0.53, [-2.359137714  $\times 10^{246874001}$ , 2.057822022  $\times 10^{246874001}$ ]], [0.54, [-2.697903723  $\times 10^{493748001}$ , 2.353319890]$

$$\begin{aligned}
& \times 10^{493748001}]], [0.55, [-3.528358942 \times 10^{987496001}, 3.077707042 \times 10^{987496001}]], [0.56, \\
& [-6.034834812 \times 10^{1974992001}, 5.264048775 \times 10^{1974992001}]], [0.57, [-1.765430551 \\
& \times 10^{3949984002}, 1.539944807 \times 10^{3949984002}]], [0.58, [-1.510849275 \times 10^{7899968003}, \\
& 1.317879365 \times 10^{7899968003}]], [0.59, [-1.106527327 \times 10^{15799936005}, 9.651985517 \\
& \times 10^{15799936004}]], [0.60, [-5.935320183 \times 10^{31599872008}, 5.177244430 \times 10^{31599872008}]], \\
& [0.61, [-1.707686585 \times 10^{63199744016}, 1.489576062 \times 10^{63199744016}]], [0.62, \\
& [-1.413631444 \times 10^{126399488031}, 1.233078470 \times 10^{126399488031}]], [0.63, [-9.687066021 \\
& \times 10^{252798976060}, 8.449806764 \times 10^{252798976060}]], [0.64, [-4.548878989 \\
& \times 10^{505597952120}, 3.967883397 \times 10^{505597952120}]], [0.65, [-1.003063970 \\
& \times 10^{1011195904240}, 8.749498226 \times 10^{1011195904239}]], [0.66, [-4.877273615 \\
& \times 10^{2022391808478}, 4.254334539 \times 10^{2022391808478}]], [0.67, [-1.153118931 \\
& \times 10^{4044783616956}, 1.005839349 \times 10^{4044783616956}]], [0.68, [-6.445670140 \\
& \times 10^{8089567233910}, 5.622411066 \times 10^{8089567233910}]], [0.69, [-2.013984013 \\
& \times 10^{16179134467820}, 1.756752324 \times 10^{16179134467820}]], [0.70, [-1.966219071 \\
& \times 10^{32358268935639}, 1.715088049 \times 10^{32358268935639}]], [0.71, [-1.874060756 \\
& \times 10^{64716537871277}, 1.634700457 \times 10^{64716537871277}]], [0.72, [-1.702500299 \\
& \times 10^{129433075742553}, 1.485052183 \times 10^{129433075742553}]], [0.73, [-1.405058017 \\
& \times 10^{258866151485105}, 1.225600065 \times 10^{258866151485105}]], [0.74, [-9.569921610 \\
& \times 10^{517732302970208}, 8.347624366 \times 10^{517732302970208}]], [0.75, [-4.439526215 \\
& \times 10^{1035464605940416}, 3.872497468 \times 10^{1035464605940416}]], [0.76, [-9.554173277 \\
& \times 10^{2070929211880831}, 8.333887467 \times 10^{2070929211880831}]], [0.77, [-4.424926840 \\
& \times 10^{4141858423761662}, 3.859762767 \times 10^{4141858423761662}]], [0.78, [-9.491438846 \\
& \times 10^{8283716847523323}, 8.279165645 \times 10^{8283716847523323}]], [0.79, [-4.367007855 \\
& \times 10^{16567433695046646}, 3.809241354 \times 10^{16567433695046646}]], [0.80, [-9.244593271 \\
& \times 10^{33134867390093291}, 8.063847889 \times 10^{33134867390093291}]], [0.81, [-4.142814470 \\
& \times 10^{66269734780186582}, 3.613682583 \times 10^{66269734780186582}]], [0.82, [-8.319760697 \\
& \times 10^{132539469560373163}, 7.257137521 \times 10^{132539469560373163}]], [0.83, [-3.355378647 \\
& \times 10^{265078939120746326}, 2.926820267 \times 10^{265078939120746326}]], [0.84, [-5.457615486 \\
& \times 10^{530157878241492651}, 4.760553530 \times 10^{530157878241492651}]], [0.85, [-1.443862168 \\
& \times 10^{1060315756482985302}, 1.259448040 \times 10^{1060315756482985302}]], [0.86, [-1.010581489 \\
& \times 10^{2120631512965970603}, 8.815071842 \times 10^{2120631512965970602}]], [0.87, [-4.950653597 \\
& \times 10^{4241263025931941204}, 4.318342226 \times 10^{4241263025931941204}]], [0.88, [-1.188077969
\end{aligned}$$

























Float(undefined)]], [10.00, [Float(undefined), Float(undefined)]], [10.01, [Float(undefined), Float(undefined)]]]

➢  $L3 := \text{Dis2}\left(f1, x, y, \left[-\frac{1}{26}, \frac{37}{26}\right] + [0.1, 0.1], 0.01, 10\right)$

$L3 := [[0.01, [0.06153846154, 1.523076923]], [0.02, [0.1112307692, 1.577846154]], [0.03, \quad (10)$

[0.1959276694, 1.668080944]], [0.04, [0.3540615613, 1.827433119]], [0.05,  
 $[0.6931927405, 2.142704022]], [0.06, [1.600694362, 2.900354756]], [0.07, [5.148662806,$   
 $5.497030396]], [0.08, [33.40506280, 23.43572453]], [0.09, [1009.094820, 571.0177239]],$   
 $[0.10, [793764.2705, 413046.2104]], [0.11, [4.661804660 \times 10^{11}, 2.344480074 \times 10^{11}]],$   
 $[0.12, [1.575551638 \times 10^{23}, 7.814346357 \times 10^{22}]], [0.13, [1.785023454 \times 10^{46},$   
 $8.803653211 \times 10^{45}]], [0.14, [2.283709664 \times 10^{92}, 1.123763059 \times 10^{92}]], [0.15,$   
 $[3.733025634 \times 10^{184}, 1.835260360 \times 10^{184}]], [0.16, [9.969423399 \times 10^{368}, 4.899443069$   
 $\times 10^{368}]], [0.17, [7.108780324 \times 10^{737}, 3.493070015 \times 10^{737}]], [0.18, [3.614162733$   
 $\times 10^{1475}, 1.775799480 \times 10^{1475}]], [0.19, [9.341525799 \times 10^{2950}, 4.589798020 \times 10^{2950}]],$   
 $[0.20, [6.240680676 \times 10^{5901}, 3.066221342 \times 10^{5901}]], [0.21, [2.785209045 \times 10^{11803},$   
 $1.368445922 \times 10^{11803}]], [0.22, [5.547642683 \times 10^{23606}, 2.725697397 \times 10^{23606}]], [0.23,$   
 $[2.200946323 \times 10^{47213}, 1.081380002 \times 10^{47213}]], [0.24, [3.464265997 \times 10^{94426},$   
 $1.702080134 \times 10^{94426}]], [0.25, [8.582518070 \times 10^{188852}, 4.216804374 \times 10^{188852}]],$   
 $[0.26, [5.267707980 \times 10^{377705}, 2.588155708 \times 10^{377705}]], [0.27, [1.984429210$   
 $\times 10^{755411}, 9.749993204 \times 10^{755410}]], [0.28, [2.816199686 \times 10^{1510822}, 1.383668789$   
 $\times 10^{1510822}]], [0.29, [5.671776561 \times 10^{3021644}, 2.786684560 \times 10^{3021644}]], [0.30,$   
 $[2.300543494 \times 10^{6043289}, 1.130314101 \times 10^{6043289}]], [0.31, [3.784888743 \times 10^{12086578},$   
 $1.859609753 \times 10^{12086578}]], [0.32, [1.024468140 \times 10^{24173157}, 5.033466172$   
 $\times 10^{24173156}]], [0.33, [7.505664276 \times 10^{48346313}, 3.687719094 \times 10^{48346313}]], [0.34,$   
 $[4.028751596 \times 10^{96692627}, 1.979425623 \times 10^{96692627}]], [0.35, [1.160735326$   
 $\times 10^{193385255}, 5.702980666 \times 10^{193385254}]], [0.36, [9.635153216 \times 10^{386770509},$   
 $4.733989848 \times 10^{386770509}]], [0.37, [6.639103990 \times 10^{773541019}, 3.261956524$   
 $\times 10^{773541019}]], [0.38, [3.152181119 \times 10^{1547082039}, 1.548744798 \times 10^{1547082039}]],$   
 $[0.39, [7.105825658 \times 10^{3094164078}, 3.491268461 \times 10^{3094164078}]], [0.40, [3.610948685$   
 $\times 10^{6188328157}, 1.774148686 \times 10^{6188328157}]], [0.41, [9.324699703 \times 10^{12376656314},$   
 $4.581456336 \times 10^{12376656314}]], [0.42, [6.218160535 \times 10^{24753312629}, 3.055136560$   
 $\times 10^{24753312629}]], [0.43, [2.765133356 \times 10^{49506625259}, 1.358578628 \times 10^{49506625259}]],$   
 $[0.44, [5.467948095 \times 10^{99013250518}, 2.686538573 \times 10^{99013250518}]], [0.45, [2.138163875$

$$\begin{aligned}
& \times 10^{198026501037}, 1.050532964 \times 10^{198026501037}]], [0.46, [3.269446207 \times 10^{396053002074}, \\
& 1.606360043 \times 10^{396053002074}]], [0.47, [7.644350880 \times 10^{792106004148}, 3.755859261 \\
& \times 10^{792106004148}]], [0.48, [4.179010355 \times 10^{1584212008297}, 2.053251479 \\
& \times 10^{1584212008297}]], [0.49, [1.248932927 \times 10^{3168424016595}, 6.136317359 \\
& \times 10^{3168424016594}]], [0.50, [1.115502254 \times 10^{6336848033190}, 5.480739354 \\
& \times 10^{6336848033189}]], [0.51, [8.898834402 \times 10^{12673696066379}, 4.372218143 \\
& \times 10^{12673696066379}]], [0.52, [5.663155295 \times 10^{25347392132759}, 2.782448714 \\
& \times 10^{25347392132759}]], [0.53, [2.293555021 \times 10^{50694784265519}, 1.126880490 \\
& \times 10^{50694784265519}]], [0.54, [3.761928586 \times 10^{101389568531038}, 1.848328856 \\
& \times 10^{101389568531038}]], [0.55, [1.012076439 \times 10^{202779137062077}, 4.972582666 \\
& \times 10^{202779137062076}]], [0.56, [7.325189265 \times 10^{405558274124153}, 3.599047242 \\
& \times 10^{405558274124153}]], [0.57, [3.837336826 \times 10^{811116548248307}, 1.885378797 \\
& \times 10^{811116548248307}]], [0.58, [1.053057448 \times 10^{1622233096496615}, 5.173932531 \\
& \times 10^{1622233096496614}]], [0.59, [7.930422935 \times 10^{3244466192993229}, 3.896413562 \\
& \times 10^{3244466192993229}]], [0.60, [4.497642368 \times 10^{6488932385986459}, 2.209803292 \\
& \times 10^{6488932385986459}]], [0.61, [1.446645297 \times 10^{12977864771972919}, 7.107727289 \\
& \times 10^{12977864771972918}]], [0.62, [1.496636526 \times 10^{25955729543945838}, 7.353346597 \\
& \times 10^{25955729543945837}]], [0.63, [1.601861270 \times 10^{51911459087891676}, 7.870341876 \\
& \times 10^{51911459087891675}]], [0.64, [1.835025158 \times 10^{103822918175783352}, 9.015933916 \\
& \times 10^{103822918175783351}]], [0.65, [2.408109696 \times 10^{207645836351566704}, 1.183164045 \\
& \times 10^{207645836351566704}]], [0.66, [4.147102348 \times 10^{415291672703133408}, 2.037574283 \\
& \times 10^{415291672703133408}]], [0.67, [1.229933776 \times 10^{830583345406266817}, 6.042969818 \\
& \times 10^{830583345406266816}]], [0.68, [1.081821671 \times 10^{1661166690812533634}, 5.315258299 \\
& \times 10^{1661166690812533633}]], [0.69, [8.369578267 \times 10^{3322333381625067267}, 4.112181474 \\
& \times 10^{3322333381625067267}]], [0.70, [5.009557560 \times 10^{6644666763250134535}, 2.461319929 \\
& \times 10^{6644666763250134535}]], [0.71, [Float( $\infty$ ), Float( $\infty$ )]], [0.72, [Float( $\infty$ ), Float( $\infty$ )]], \\
& [0.73, [Float( $\infty$ ), Float( $\infty$ )]], [0.74, [Float( $\infty$ ), Float( $\infty$ )]], [0.75, [Float( $\infty$ ), \\
& Float( $\infty$ )]], [0.76, [Float( $\infty$ ), Float( $\infty$ )]], [0.77, [Float( $\infty$ ), Float( $\infty$ )]], [0.78, [ \\
& Float( $\infty$ ), Float( $\infty$ )]], [0.79, [Float( $\infty$ ), Float( $\infty$ )]], [0.80, [Float( $\infty$ ), Float( $\infty$ )]], \\
& [0.81, [Float( $\infty$ ), Float( $\infty$ )]], [0.82, [Float( $\infty$ ), Float( $\infty$ )]], [0.83, [Float( $\infty$ ), \\
& Float( $\infty$ )]], [0.84, [Float( $\infty$ ), Float( $\infty$ )]], [0.85, [Float( $\infty$ ), Float( $\infty$ )]], [0.86, [ \\
& Float( $\infty$ ), Float( $\infty$ )]], [0.87, [Float( $\infty$ ), Float( $\infty$ )]], [0.88, [Float( $\infty$ ), Float( $\infty$ )]], \\
& [0.89, [Float( $\infty$ ), Float( $\infty$ )]], [0.90, [Float( $\infty$ ), Float( $\infty$ )]], [0.91, [Float( $\infty$ ), \\
& Float( $\infty$ )]]]
\end{aligned}$$



















>  $L4 := Dis2\left(f1, x, y, \left[\frac{2}{5}, \frac{2}{5}\right] + [0.1, 0.1], 0.01, 10\right)$   
 $L4 := [[0.01, [0.5000000000, 0.5000000000]], [0.02, [0.5000000000, 0.4700000000]], [0.03, (11)$   
 $[0.4980890000, 0.4451150000]], [0.04, [0.4948525010, 0.4251036745]], [0.05,$   
 $[0.4907863820, 0.4095004224]], [0.06, [0.4862740291, 0.3977075511]], [0.07,$   
 $[0.4815891868, 0.3890826309]], [0.08, [0.4769135553, 0.3830056454]], [0.09,$   
 $[0.4723592148, 0.3789209391]], [0.10, [0.4679895760, 0.3763566651]], [0.11,$   
 $[0.4638361069, 0.3749279654]], [0.12, [0.4599104209, 0.3743303968]], [0.13,$   
 $[0.4562124256, 0.3743287459]], [0.14, [0.4527355348, 0.3747446273]], [0.15,$   
 $[0.4494698636, 0.3754447726]], [0.16, [0.4464041123, 0.3763308807]], [0.17,$   
 $[0.4435266320, 0.3773312739]], [0.18, [0.4408259970, 0.3783942754]], [0.19,$   
 $[0.4382912874, 0.3794830785]], [0.20, [0.4359122082, 0.3805718403]], [0.21,$   
 $[0.4336791188, 0.3816427446]], [0.22, [0.4315830168, 0.3826838144]], [0.23,$   
 $[0.4296155007, 0.3836872942]], [0.24, [0.4277687255, 0.3846484612]], [0.25,$   
 $[0.4260353570, 0.3855647541]], [0.26, [0.4244085297, 0.3864351385]], [0.27,$   
 $[0.4228818081, 0.3872596456]], [0.28, [0.4214491532, 0.3880390385]], [0.29,$   
 $[0.4201048921, 0.3887745704]], [0.30, [0.4188436921, 0.3894678122]], [0.31,$   
 $[0.4176605372, 0.3901205278]], [0.32, [0.4165507079, 0.3907345864]], [0.33,$   
 $[0.4155097620, 0.3913119001]], [0.34, [0.4145335183, 0.3918543805]], [0.35,$   
 $[0.4136180408, 0.3923639087]], [0.36, [0.4127596249, 0.3928423155]], [0.37,$   
 $[0.4119547839, 0.3932913679]], [0.38, [0.4112002368, 0.3937127611]], [0.39,$   
 $[0.4104928967, 0.3941081143]], [0.40, [0.4098298599, 0.3944789685]], [0.41,$   
 $[0.4092083957, 0.3948267863]], [0.42, [0.4086259366, 0.3951529530]], [0.43,$   
 $[0.4080800691, 0.3954587785]], [0.44, [0.4075685249, 0.3957454997]], [0.45,$   
 $[0.4070891727, 0.3960142835]], [0.46, [0.4066400103, 0.3962662296]], [0.47,$   
 $[0.4062191571, 0.3965023736]], [0.48, [0.4058248473, 0.3967236902]], [0.49,$   
 $[0.4054554228, 0.3969310964]], [0.50, [0.4051093272, 0.3971254543]], [0.51,$   
 $[0.4047850997, 0.3973075744]], [0.52, [0.4044813695, 0.3974782181]], [0.53,$   
 $[0.4041968504, 0.3976381008]], [0.54, [0.4039303356, 0.3977878942]], [0.55,$   
 $[0.4036806931, 0.3979282291]], [0.56, [0.4034468613, 0.3980596975]], [0.57,$   
 $[0.4032278445, 0.3981828551]], [0.58, [0.4030227089, 0.3982982232]], [0.59,$   
 $[0.4028305792, 0.3984062911]], [0.60, [0.4026506346, 0.3985075177]], [0.61,$   
 $[0.4024821056, 0.3986023334]], [0.62, [0.4023242710, 0.3986911420]], [0.63,$   
 $[0.4021764547, 0.3987743220]], [0.64, [0.4020380230, 0.3988522283]], [0.65,$   
 $[0.4019083822, 0.3989251937]], [0.66, [0.4017869757, 0.3989935303]], [0.67,$   
 $[0.4016732820, 0.3990575305]], [0.68, [0.4015668124, 0.3991174684]], [0.69,$   
 $[0.4014671091, 0.3991736009]], [0.70, [0.4013737431, 0.3992261689]], [0.71,$   
 $[0.4012863125, 0.3992753980]], [0.72, [0.4012044408, 0.3993214996]], [0.73,$

[0.4011277752, 0.3993646720]], [0.74, [0.4010559853, 0.3994051007]], [0.75, [0.4009887615, 0.3994429597]], [0.76, [0.4009258138, 0.3994784119]], [0.77, [0.4008668707, 0.3995116101]], [0.78, [0.4008116777, 0.3995426972]], [0.79, [0.4007599966, 0.3995718074]], [0.80, [0.4007116042, 0.3995990660]], [0.81, [0.4006662915, 0.3996245907]], [0.82, [0.4006238626, 0.3996484916]], [0.83, [0.4005841343, 0.3996708718]], [0.84, [0.4005469348, 0.3996918280]], [0.85, [0.4005121032, 0.3997114507]], [0.86, [0.4004794890, 0.3997298246]], [0.87, [0.4004489511, 0.3997470291]], [0.88, [0.4004203575, 0.3997631386]], [0.89, [0.4003935844, 0.3997782227]], [0.90, [0.4003685160, 0.3997923466]], [0.91, [0.4003450438, 0.3998055714]], [0.92, [0.4003230662, 0.3998179542]], [0.93, [0.4003024881, 0.3998295487]], [0.94, [0.4002832205, 0.3998404050]], [0.95, [0.4002651799, 0.3998505700]], [0.96, [0.4002482882, 0.3998600877]], [0.97, [0.4002324723, 0.3998689994]], [0.98, [0.4002176637, 0.3998773436]], [0.99, [0.4002037982, 0.3998851565]], [1.00, [0.4001908158, 0.3998924718]], [1.01, [0.4001786603, 0.3998993212]], [1.02, [0.4001672791, 0.3999057344]], [1.03, [0.4001566228, 0.3999117392]], [1.04, [0.4001466452, 0.3999173616]], [1.05, [0.4001373032, 0.3999226258]], [1.06, [0.4001285563, 0.3999275547]], [1.07, [0.4001203665, 0.3999321697]], [1.08, [0.4001126984, 0.3999364907]], [1.09, [0.4001055188, 0.3999405365]], [1.10, [0.4000987965, 0.3999443246]], [1.11, [0.4000925024, 0.3999478714]], [1.12, [0.4000866093, 0.3999511923]], [1.13, [0.4000810916, 0.3999543017]], [1.14, [0.4000759254, 0.3999572130]], [1.15, [0.4000710883, 0.3999599388]], [1.16, [0.4000665594, 0.3999624910]], [1.17, [0.4000623190, 0.3999648806]], [1.18, [0.4000583487, 0.3999671180]], [1.19, [0.4000546314, 0.3999692129]], [1.20, [0.4000511509, 0.3999711743]], [1.21, [0.4000478921, 0.3999730107]], [1.22, [0.4000448409, 0.3999747302]], [1.23, [0.4000419841, 0.3999763401]], [1.24, [0.4000393093, 0.3999778474]], [1.25, [0.4000368049, 0.3999792587]], [1.26, [0.4000344601, 0.3999805801]], [1.27, [0.4000322646, 0.3999818173]], [1.28, [0.4000302090, 0.3999829757]], [1.29, [0.4000282844, 0.3999840603]], [1.30, [0.4000264824, 0.3999850758]], [1.31, [0.4000247952, 0.3999860266]], [1.32, [0.4000232155, 0.3999869169]], [1.33, [0.4000217364, 0.3999877505]], [1.34, [0.4000203515, 0.3999885309]], [1.35, [0.4000190549, 0.3999892616]], [1.36, [0.4000178409, 0.3999899458]], [1.37, [0.4000167042, 0.3999905863]], [1.38, [0.4000156400, 0.3999911861]], [1.39, [0.4000146436, 0.3999917476]], [1.40, [0.4000137106, 0.3999922734]], [1.41, [0.4000128371, 0.3999927657]], [1.42, [0.4000120192, 0.3999932266]], [1.43, [0.4000112534, 0.3999936581]], [1.44, [0.4000105364, 0.3999940622]], [1.45, [0.4000098651, 0.3999944405]], [1.46, [0.4000092366, 0.3999947947]], [1.47, [0.4000086481, 0.3999951263]], [1.48, [0.4000080971, 0.3999954368]], [1.49,

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[0.4000000509, 0.3999999713]], [2.26, [0.4000000477, 0.3999999731]], [2.27, [0.4000000447, 0.3999999748]], [2.28, [0.4000000419, 0.3999999764]], [2.29, [0.4000000392, 0.3999999779]], [2.30, [0.4000000367, 0.3999999793]], [2.31, [0.4000000344, 0.3999999806]], [2.32, [0.4000000322, 0.3999999819]], [2.33, [0.4000000301, 0.3999999830]], [2.34, [0.4000000282, 0.3999999841]], [2.35, [0.4000000264, 0.3999999851]], [2.36, [0.4000000247, 0.3999999860]], [2.37, [0.4000000231, 0.3999999869]], [2.38, [0.4000000216, 0.3999999878]], [2.39, [0.4000000202, 0.3999999886]], [2.40, [0.4000000189, 0.3999999893]], [2.41, [0.4000000177, 0.3999999900]], [2.42, [0.4000000166, 0.3999999906]], [2.43, [0.4000000155, 0.3999999912]], [2.44, [0.4000000145, 0.3999999918]], [2.45, [0.4000000136, 0.3999999923]], [2.46, [0.4000000127, 0.3999999928]], [2.47, [0.4000000119, 0.3999999933]], [2.48, [0.4000000111, 0.3999999937]], [2.49, [0.4000000104, 0.3999999941]], [2.50, [0.4000000097, 0.3999999945]], [2.51, [0.4000000091, 0.3999999948]], [2.52, [0.4000000085, 0.3999999951]], [2.53, [0.4000000080, 0.3999999954]], [2.54, [0.4000000075, 0.3999999957]], [2.55, [0.4000000070, 0.3999999960]], [2.56, [0.4000000066, 0.3999999963]], [2.57, [0.4000000062, 0.3999999965]], [2.58, [0.4000000058, 0.3999999967]], [2.59, [0.4000000054, 0.3999999969]], [2.60, [0.4000000051, 0.3999999971]], [2.61, [0.4000000048, 0.3999999973]], [2.62, [0.4000000045, 0.3999999975]], [2.63, [0.4000000042, 0.3999999977]], [2.64, [0.4000000039, 0.3999999978]], [2.65, [0.4000000037, 0.3999999979]], [2.66, [0.4000000035, 0.3999999981]], [2.67, [0.4000000033, 0.3999999982]], [2.68, [0.4000000031, 0.3999999983]], [2.69, [0.4000000029, 0.3999999984]], [2.70, [0.4000000027, 0.3999999985]], [2.71, [0.4000000025, 0.3999999986]], [2.72, [0.4000000023, 0.3999999987]], [2.73, [0.4000000022, 0.3999999988]], [2.74, [0.4000000021, 0.3999999988]], [2.75, [0.4000000020, 0.3999999989]], [2.76, [0.4000000019, 0.3999999990]], [2.77, [0.4000000018, 0.3999999990]], [2.78, [0.4000000017, 0.3999999991]], [2.79, [0.4000000016, 0.3999999991]], [2.80, [0.4000000015, 0.3999999991]], [2.81, [0.4000000014, 0.3999999992]], [2.82, [0.4000000013, 0.3999999993]], [2.83, [0.4000000012, 0.3999999993]], [2.84, [0.4000000011, 0.3999999993]], [2.85, [0.4000000010, 0.3999999994]], [2.86, [0.4000000009, 0.3999999994]], [2.87, [0.4000000009, 0.3999999994]], [2.88, [0.4000000009, 0.3999999994]], [2.89, [0.4000000009, 0.3999999994]], [2.90, [0.4000000009, 0.3999999994]], [2.91, [0.4000000009, 0.3999999994]], [2.92, [0.4000000009, 0.3999999994]], [2.93, [0.4000000009, 0.3999999994]], [2.94, [0.4000000009, 0.3999999994]], [2.95, [0.4000000009, 0.3999999994]], [2.96, [0.4000000009, 0.3999999994]], [2.97, [0.4000000009, 0.3999999994]], [2.98, [0.4000000009, 0.3999999994]], [2.99, [0.4000000009, 0.3999999994]], [3.00, [0.4000000009, 0.3999999994]], [3.01,



















```
[0.4000000009, 0.3999999994]], [9.86, [0.4000000009, 0.3999999994]], [9.87,
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[0.4000000009, 0.3999999994]]]
```

> # Stable

>  $EquPts(f2, [x, y])$

$$\left\{ [-8, 13], \left[ -3, \frac{7}{4} \right], \left[ -\frac{1}{22}, \frac{3}{11} \right], \left[ \frac{1}{3}, \frac{1}{2} \right] \right\} \quad (12)$$

>

>  $L5 := Dis2(f2, x, y, [-8, 13] + [0.1, 0.1], 0.01, 10) : print([op(nops(L5)) - 5 .. nops(L5), L5]) :$

```
[[9.96, [Float(∞), Float(∞)]], [9.97, [Float(∞), Float(∞)]], [9.98, [Float(∞),
Float(∞)]], [9.99, [Float(∞), Float(∞)]], [10.00, [Float(∞), Float(∞)]], [10.01, [
Float(∞), Float(∞)]]] \quad (13)
```

>  $L6 := Dis2\left(f2, x, y, \left[-3, \frac{7}{4}\right] + [0.1, 0.1], 0.01, 10\right) : print([op(nops(L6)) - 5 .. nops(L6), L6]) :$

```
[[9.96, [Float(∞), Float(∞)]], [9.97, [Float(∞), Float(∞)]], [9.98, [Float(∞),
Float(∞)]], [9.99, [Float(∞), Float(∞)]], [10.00, [Float(∞), Float(∞)]], [10.01, [
Float(∞), Float(∞)]]] \quad (14)
```

>  $L7 := Dis2\left(f2, x, y, \left[-\frac{1}{22}, \frac{3}{11}\right] + [0.1, 0.1], 0.01, 10\right) : print([op(nops(L7)) - 5 .. nops(L7), L7]) :$

```
[[9.96, [-0.04545454593, 0.2727272734]], [9.97, [-0.04545454593, 0.2727272734]], [9.98, (15)
[-0.04545454593, 0.2727272734]], [9.99, [-0.04545454593, 0.2727272734]], [10.00,
[-0.04545454593, 0.2727272734]], [10.01, [-0.04545454593, 0.2727272734]]]
```

> # Stable

>  $L8 := Dis2\left(f2, x, y, \left[\frac{1}{3}, \frac{1}{2}\right] + [0.1, 0.1], 0.01, 10\right) : print([op(nops(L8)) - 5 .. nops(L8), L8])$

```
[[9.96, [Float(∞), Float(∞)]], [9.97, [Float(∞), Float(∞)]], [9.98, [Float(∞),
Float(∞)]], [9.99, [Float(∞), Float(∞)]], [10.00, [Float(∞), Float(∞)]], [10.01, [
Float(∞), Float(∞)]]] \quad (16)
```

>  $EquPts(f3, [x, y])$

$$\left\{ \left[ \frac{1}{3}, \frac{1}{12} \right], \left[ \frac{8}{7}, -\frac{13}{14} \right], \left[ \frac{21}{17}, -\frac{12}{17} \right] \right\} \quad (17)$$

>

$$\begin{aligned} > L9 := & \text{Dis2}\left(f3, x, y, \left[\frac{1}{3}, \frac{1}{12}\right] + [0.1, 0.1], 0.01, 10\right) : \text{print}([op(nops(L9)) - 5 .. nops(L9)], \\ & [[9.96, [0.3333333353, 0.08333333161]], [9.97, [0.3333333353, 0.08333333161]], [9.98, \\ & [0.3333333353, 0.08333333161]], [9.99, [0.3333333353, 0.08333333161]], [10.00, \\ & [0.3333333353, 0.08333333161]], [10.01, [0.3333333353, 0.08333333161]]]) \end{aligned} \quad (18)$$

> # Stable

$$\begin{aligned} > L10 := & \text{Dis2}\left(f3, x, y, \left[\frac{8}{7}, -\frac{13}{14}\right] + [0.1, 0.1], 0.01, 10\right) : \text{print}([op(nops(L10)) - 5 \\ & .. nops(L10)], L10)] \\ & [[9.96, [\text{Float}(\infty), \text{Float}(\infty)]], [9.97, [\text{Float}(\infty), \text{Float}(\infty)]], [9.98, [\text{Float}(\infty), \\ & \text{Float}(\infty)]], [9.99, [\text{Float}(\infty), \text{Float}(\infty)]], [10.00, [\text{Float}(\infty), \text{Float}(\infty)]], [10.01, [ \\ & \text{Float}(\infty), \text{Float}(\infty)]]] \end{aligned} \quad (19)$$

$$\begin{aligned} > L11 := & \text{Dis2}\left(f3, x, y, \left[\frac{21}{17}, -\frac{12}{17}\right] + [0.1, 0.1], 0.01, 10\right) : \text{print}([op(nops(L11)) - 5 \\ & .. nops(L11)], L11)] \\ & [[9.96, [\text{Float}(\infty), \text{Float}(\infty)]], [9.97, [\text{Float}(\infty), \text{Float}(\infty)]], [9.98, [\text{Float}(\infty), \\ & \text{Float}(\infty)]], [9.99, [\text{Float}(\infty), \text{Float}(\infty)]], [10.00, [\text{Float}(\infty), \text{Float}(\infty)]], [10.01, [ \\ & \text{Float}(\infty), \text{Float}(\infty)]]] \end{aligned} \quad (20)$$

>

# Question 3

$$\begin{aligned} > \text{EquPts}(SIRS(s, i, \beta, \gamma, \nu, N), [s, i]) \\ & \left\{ [N, 0], \left[ \frac{\nu}{\beta}, \frac{\gamma(N\beta - \nu)}{\beta(\gamma + \nu)} \right] \right\} \end{aligned} \quad (21)$$

> # Seady state about of suseptible and infected are the same

# Question 4

$$\begin{aligned} > \text{Chemostat} := & \text{proc}(N, C, a1, a2) : \\ & \left[ a1 \cdot \left( \frac{C}{(1+C)} \cdot N \right) - N, -\frac{C}{(1+C)} \cdot N - C + a2 \right] : \\ & \text{end}: \\ > \text{EquPts}(\text{Chemostat}(N, C, a1, a2), [N, C]) \\ & \left\{ [0, a2], \left[ \frac{a1(a2a1 - a2 - 1)}{a1 - 1}, \frac{1}{a1 - 1} \right] \right\} \end{aligned} \quad (22)$$

>

Max Mekhanikov - HW 19 - Okay to post

i)  $\beta = 0.3 \cdot \frac{\nu}{N} \rightarrow R = N - IN - S$

$$R = 1000 - 996.7 - 2.979$$

$$R = 6.279$$

$$\beta = 0.9 \cdot \frac{\nu}{N} \rightarrow R = 1000 - 988.3315 - 8.99$$
$$R = 2.678$$

$$\beta = 3.9 \cdot \frac{\nu}{N} \rightarrow R = 1000 - 914.6283 - 40$$
$$R = 45.372$$

ii)  $\beta = 0.3 \frac{\nu}{N} \rightarrow R = 1000 - 996.402 - 2.958$   
$$R = 0.64$$

$$\beta = 0.9 \frac{\nu}{N} \rightarrow R = 1000 - 985.6773 - 8.9797$$
$$R = 5.343$$

$$\beta = 3.9 \frac{\nu}{N} \rightarrow R = 1000 - 872.5259 - 40.8742$$
$$R = 86.60$$

iii)  $\beta = 0.3 \frac{\nu}{N} \rightarrow 1000 - 946.62 - 2.94$   
$$R = 0.44$$

$$\beta = 0.9 \frac{\nu}{N} \rightarrow R = 3.46 \quad \beta = 3.9 \frac{\nu}{N} \rightarrow R = 55.86$$