```
> #OK to post
  #Julian Herman, 8th November, 2021, Assignment 19
> read `/Users/julianherman/Documents/Rutgers/Fall 2021/Dynamical Models In Biology/HW/M19.
      txt`
> Help19( )
```

SIRSdemo(N,IN,gamma,nu,h,A),e.g. SIRSdemo(100,20,1, 1,0.01, 10); EquPts(F,var), StEquPts(F, var) , IsStable(M), RandNice(var,K) **(1)**

```
> #1)
> #i)
> SIRSdemo(1000, 200, 3, 1, 0.01, 10)
```

This is a numerical demonstration of the R0 phenomenon in the SIRS model using discretization with mesh size=, 0.01, *and letting it run until time t=*, 10

with population size, 1000, *and fixed parameters nu=*, 1, *and gamma=*, 3

*where we change beta from 0.2*nu/N to 4*nu/N*

Recall that the epidemic will persist if beta exceeds nu/N, that in this case is, $\frac{1}{1000}$

We start with , 200, *infected individuals, 0 removed and hence*, 800, *susceptible*

We will show what happens once time is close to, 10

beta is, $\frac{1}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [998.9666995, 0.9909989667]], [9.99, [998.9666995, 0.9909989667]], [10.00, [998.9666995, 0.9909989667]], [10.01, [998.9666995, 0.9909989667]]]

beta is, $\frac{3}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [996.7009881, 2.978970309]], [9.99, [996.7009881, 2.978970309]], [10.00, [996.7009881, 2.978970309]], [10.01, [996.7009881, 2.978970309]]]

beta is, $\frac{1}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [994.1715221, 4.974854288]], [9.99, [994.1715221, 4.974854288]], [10.00, [994.1715221, 4.974854288]], [10.01, [994.1715221, 4.974854288]]]

beta is, $\frac{7}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [991.3807432, 6.978577656]], [9.99, [991.3807432, 6.978577656]], [10.00, [991.3807432, 6.978577656]], [10.01, [991.3807432, 6.978577656]]]

beta is, $\frac{9}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [988.3315033, 8.990054852]], [9.99, [988.3315033, 8.990054852]], [10.00, [988.3315033, 8.990054852]], [10.01, [988.3315033, 8.990054852]]]

beta is, $\frac{11}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [985.0270559, 11.00918827]], [9.99, [985.0270559, 11.00918827]], [10.00, [985.0270559, 11.00918827]], [10.01, [985.0270559, 11.00918827]]]

beta is, $\frac{13}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [981.4710448, 13.03586861]], [9.99, [981.4710448, 13.03586861]], [10.00, [981.4710448, 13.03586861]], [10.01, [981.4710448, 13.03586861]]]

beta is, $\frac{3}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [977.6674922, 15.06997519]], [9.99, [977.6674922, 15.06997519]], [10.00, [977.6674922, 15.06997519]], [10.01, [977.6674922, 15.06997519]]]

beta is, $\frac{17}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [973.6207848, 17.11137641]], [9.99, [973.6207848, 17.11137641]], [10.00, [973.6207848, 17.11137641]], [10.01, [973.6207848, 17.11137641]]]

beta is, $\frac{19}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [969.3356593, 19.15993017]], [9.99, [969.3356593, 19.15993017]], [10.00, [969.3356593, 19.15993017]], [10.01, [969.3356593, 19.15993017]]]

beta is, $\frac{21}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [964.8171858, 21.21548438]], [9.99, [964.8171858, 21.21548438]], [10.00, [964.8171858, 21.21548438]], [10.01, [964.8171858, 21.21548438]]]

beta is, $\frac{23}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [960.0707508, 23.27787743]], [9.99, [960.0707508, 23.27787743]], [10.00, [960.0707508, 23.27787743]], [10.01, [960.0707508, 23.27787743]]]

beta is, $\frac{5}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [955.1020392, 25.34693877]], [9.99, [955.1020392, 25.34693877]], [10.00, [955.1020392, 25.34693877]], [10.01, [955.1020392, 25.34693877]]]

beta is, $\frac{27}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [949.9170149, 27.42248950]], [9.99, [949.9170149, 27.42248950]], [10.00, [949.9170149, 27.42248950]], [10.01, [949.9170149, 27.42248950]]]

beta is, $\frac{29}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [944.5219011, 29.50434292]], [9.99, [944.5219011, 29.50434292]], [10.00, [944.5219011, 29.50434292]], [10.01, [944.5219011, 29.50434292]]]

beta is, $\frac{31}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [938.9231598, 31.59230516]], [9.99, [938.9231598, 31.59230516]], [10.00, [938.9231598, 31.59230516]], [10.01, [938.9231598, 31.59230516]]]

beta is, $\frac{33}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [933.1274712, 33.68617582]], [9.99, [933.1274712, 33.68617582]], [10.00, [933.1274712, 33.68617582]], [10.01, [933.1274712, 33.68617582]]]

beta is, $\frac{7}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [927.1417118, 35.78574860]], [9.99, [927.1417118, 35.78574860]], [10.00, [927.1417118, 35.78574860]], [10.01, [927.1417118, 35.78574860]]]

beta is, $\frac{37}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [920.9729335, 37.89081195]], [9.99, [920.9729335, 37.89081195]], [10.00, [920.9729335, 37.89081195]], [10.01, [920.9729335, 37.89081195]]]

beta is, $\frac{39}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [914.6283415, 40.00114971]], [9.99, [914.6283415, 40.00114971]], [10.00, **(2)** [914.6283415, 40.00114971]], [10.01, [914.6283415, 40.00114971]]]

> *#With nu=1 and gamma=3*

> *#For beta = 0.3* $\cdot \frac{N}{nu}$ *: At time* $t = 10$, *there are* 998.9666995 *susceptible*, 0.9909989667 *infected*, **and** 0.0423015333 *removed* (*removed* $= N - s - i = 1000 - 998.9666995 - 0.9909989667 = 0.0423015333$).

> *#For beta = 0.9* $\frac{N}{nu}$ *: At time* $t = 10$, *there are* 988.3315033 *susceptible*, 8.990054852 *infected*, **and** $1000 - 988.3315033 - 8.990054852 = 2.678441848$ *removed*.

> *#For beta = 3.9* $\frac{N}{nu}$ *: At time* $t = 10$, *there are* 914.6283415 *susceptible*, 40.00114971 *infected*, **and** $1000 - 914.6283415 - 40.00114971 = 45.37050879$ *removed*.

>

> *#ii)*

> *SIRSdemo*(1000, 200, 3, 2, 0.01, 10)

This is a numerical demonstration of the R0 phenomenon in the SIRS model using discretization with mesh size=, 0.01, *and letting it run until time t=*, 10

with population size, 1000, *and fixed parameters nu=*, 2, *and gamma=*, 3

*where we change beta from 0.2*nu/N to 4*nu/N*

Recall that the epidemic will persist if beta exceeds nu/N, that in this case is, $\frac{1}{500}$

We start with , 200, *infected individuals, 0 removed and hence*, 800, *susceptible*

We will show what happens once time is close to, 10

beta is, $\frac{1}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [998.9334028, 0.9819978668]], [9.99, [998.9334028, 0.9819978668]], [10.00, [998.9334028, 0.9819978668]], [10.01, [998.9334028, 0.9819978668]]]

beta is, $\frac{3}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [996.4021571, 2.957935239]], [9.99, [996.4021571, 2.957935239]], [10.00, [996.4021571, 2.957935239]], [10.01, [996.4021571, 2.957935239]]]

beta is, $\frac{1}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [993.3444243, 4.949667221]], [9.99, [993.3444243, 4.949667221]], [10.00, [993.3444243, 4.949667221]], [10.01, [993.3444243, 4.949667221]]]

beta is, $\frac{7}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [989.7667603, 6.956997143]], [9.99, [989.7667603, 6.956997143]], [10.00, [989.7667603, 6.956997143]], [10.01, [989.7667603, 6.956997143]]]

beta is, $\frac{9}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [985.6773407, 8.979679729]], [9.99, [985.6773407, 8.979679729]], [10.00, [985.6773407, 8.979679729]], [10.01, [985.6773407, 8.979679729]]]

beta is, $\frac{11}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [981.0859054, 11.01742279]], [9.99, [981.0859054, 11.01742279]], [10.00, [981.0859054, 11.01742279]], [10.01, [981.0859054, 11.01742279]]]

beta is, $\frac{13}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [976.0036901, 13.06988925]], [9.99, [976.0036901, 13.06988925]], [10.00, [976.0036901, 13.06988925]], [10.01, [976.0036901, 13.06988925]]]

beta is, $\frac{3}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [970.4433482, 15.13669951]], [9.99, [970.4433482, 15.13669951]], [10.00, [970.4433482, 15.13669951]], [10.01, [970.4433482, 15.13669951]]]

beta is, $\frac{17}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [964.4188616, 17.21743410]], [9.99, [964.4188616, 17.21743410]], [10.00, [964.4188616, 17.21743410]], [10.01, [964.4188616, 17.21743410]]]

beta is, $\frac{19}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [957.9454447, 19.31163661]], [9.99, [957.9454447, 19.31163661]], [10.00, [957.9454447, 19.31163661]], [10.01, [957.9454447, 19.31163661]]]

beta is, $\frac{21}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [951.0394389, 21.41881679]], [9.99, [951.0394389, 21.41881679]], [10.00, [951.0394389, 21.41881679]], [10.01, [951.0394389, 21.41881679]]]

$$\text{beta is, } \frac{23}{10}\text{, times the threshold value}$$

the long-term behavior is

[[9.98, [943.7182031, 23.53845386]], [9.99, [943.7182031, 23.53845386]], [10.00, [943.7182031, 23.53845386]], [10.01, [943.7182031, 23.53845386]]]

$$\text{beta is, } \frac{5}{2}\text{, times the threshold value}$$

the long-term behavior is

[[9.98, [935.9999984, 25.67000000]], [9.99, [935.9999984, 25.67000000]], [10.00, [935.9999984, 25.67000000]], [10.01, [935.9999984, 25.67000000]]]

$$\text{beta is, } \frac{27}{10}\text{, times the threshold value}$$

the long-term behavior is

[[9.98, [927.9038703, 27.81288384]], [9.99, [927.9038703, 27.81288384]], [10.00, [927.9038703, 27.81288384]], [10.01, [927.9038703, 27.81288384]]]

$$\text{beta is, } \frac{29}{10}\text{, times the threshold value}$$

the long-term behavior is

[[9.98, [919.4495282, 29.96651411]], [9.99, [919.4495282, 29.96651411]], [10.00, [919.4495282, 29.96651411]], [10.01, [919.4495282, 29.96651411]]]

$$\text{beta is, } \frac{31}{10}\text{, times the threshold value}$$

the long-term behavior is

[[9.98, [910.6572255, 32.13028319]], [9.99, [910.6572255, 32.13028319]], [10.00, [910.6572255, 32.13028319]], [10.01, [910.6572255, 32.13028319]]]

$$\text{beta is, } \frac{33}{10}\text{, times the threshold value}$$

the long-term behavior is

[[9.98, [901.5476397, 34.30357076]], [9.99, [901.5476397, 34.30357076]], [10.00, [901.5476397, 34.30357076]], [10.01, [901.5476397, 34.30357076]]]

$$\text{beta is, } \frac{7}{2}\text{, times the threshold value}$$

the long-term behavior is

[[9.98, [892.1417551, 36.48574730]], [9.99, [892.1417551, 36.48574730]], [10.00, [892.1417551, 36.48574730]], [10.01, [892.1417551, 36.48574730]]]

$$\text{beta is, } \frac{37}{10}\text{, times the threshold value}$$

the long-term behavior is

[[9.98, [882.4607475, 38.67617753]], [9.99, [882.4607475, 38.67617753]], [10.00, [882.4607475, 38.67617753]], [10.01, [882.4607475, 38.67617753]]]

beta is, $\frac{39}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [872.5258747, 40.87422371]], [9.99, [872.5258747, 40.87422371]], [10.00, [872.5258747, 40.87422371]], [10.01, [872.5258747, 40.87422371]]] **(3)**

> *#With nu=2 and gamma=3*

> *#For beta =* $0.3 \cdot \frac{N}{\text{nu}}$ *: At time* $t = 10$, *there are* 996.4021571 *susceptible*, 2.957935239 *infected*, **and** 0.639907661 *removed*.

> *#For beta =* $0.9 \frac{N}{\text{nu}}$ *: At time* $t = 10$, *there are* 985.6773407 *susceptible*, 8.979679729 *infected*, **and** 5.342979571 *removed*.

> *#For beta =* $3.9 \frac{N}{\text{nu}}$ *: At time* $t = 10$, *there are* 872.5258747 *susceptible*, 40.87422371 *infected*, **and** 86.59990159 *removed*.

>

> *#iii)*

> *SIRSdemo*(1000, 200, 7, 3, 0.01, 10)

This is a numerical demonstration of the R0 phenomenon in the SIRS model using discretization with mesh size=, 0.01, *and letting it run until time t=*, 10

with population size, 1000, *and fixed parameters nu=*, 3, *and gamma=*, 7

*where we change beta from 0.2*nu/N to 4*nu/N*

Recall that the epidemic will persist if beta exceeds nu/N, that in this case is, $\frac{3}{1000}$

We start with , 200, *infected individuals, 0 removed and hence*, 800, *susceptible*

We will show what happens once time is close to, 10

beta is, $\frac{1}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [998.9571869, 0.9729968716]], [9.99, [998.9571869, 0.9729968716]], [10.00, [998.9571869, 0.9729968716]], [10.01, [998.9571869, 0.9729968716]]]

beta is, $\frac{3}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [996.6155905, 2.936908621]], [9.99, [996.6155905, 2.936908621]], [10.00, [996.6155905, 2.936908621]], [10.01, [996.6155905, 2.936908621]]]

beta is, $\frac{1}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [993.9350689, 4.924545130]], [9.99, [993.9350689, 4.924545130]], [10.00, [993.9350689, 4.924545130]], [10.01, [993.9350689, 4.924545130]]]

beta is, $\frac{7}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [990.9190693, 6.935665103]], [9.99, [990.9190693, 6.935665103]], [10.00, [990.9190693, 6.935665103]], [10.01, [990.9190693, 6.935665103]]]

beta is, $\frac{9}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [987.5717147, 8.969979927]], [9.99, [987.5717147, 8.969979927]], [10.00, [987.5717147, 8.969979927]], [10.01, [987.5717147, 8.969979927]]]

beta is, $\frac{11}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [983.8977865, 11.02715490]], [9.99, [983.8977865, 11.02715490]], [10.00, [983.8977865, 11.02715490]], [10.01, [983.8977865, 11.02715490]]]

beta is, $\frac{13}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [979.9027040, 13.10681067]], [9.99, [979.9027040, 13.10681067]], [10.00, [979.9027040, 13.10681067]], [10.01, [979.9027040, 13.10681067]]]

beta is, $\frac{3}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [975.5925002, 15.20852494]], [9.99, [975.5925002, 15.20852494]], [10.00, [975.5925002, 15.20852494]], [10.01, [975.5925002, 15.20852494]]]

beta is, $\frac{17}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [970.9737953, 17.33183428]], [9.99, [970.9737953, 17.33183428]], [10.00, [970.9737953, 17.33183428]], [10.01, [970.9737953, 17.33183428]]]

beta is, $\frac{19}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [966.0537675, 19.47623623]], [9.99, [966.0537675, 19.47623623]], [10.00, [966.0537675, 19.47623623]], [10.01, [966.0537675, 19.47623623]]]

beta is, $\frac{21}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [960.8401210, 21.64119148]], [9.99, [960.8401210, 21.64119148]], [10.00, [960.8401210, 21.64119148]], [10.01, [960.8401210, 21.64119148]]]

beta is, $\frac{23}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [955.3410529, 23.82612625]], [9.99, [955.3410529, 23.82612625]], [10.00, [955.3410529, 23.82612625]], [10.01, [955.3410529, 23.82612625]]]

beta is, $\frac{5}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [949.5652167, 26.03043478]], [9.99, [949.5652167, 26.03043478]], [10.00, [949.5652167, 26.03043478]], [10.01, [949.5652167, 26.03043478]]]

beta is, $\frac{27}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [943.5216861, 28.25348193]], [9.99, [943.5216861, 28.25348193]], [10.00, [943.5216861, 28.25348193]], [10.01, [943.5216861, 28.25348193]]]

beta is, $\frac{29}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [937.2199158, 30.49460585]], [9.99, [937.2199158, 30.49460585]], [10.00, [937.2199158, 30.49460585]], [10.01, [937.2199158, 30.49460585]]]

beta is, $\frac{31}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [930.6697029, 32.75312075]], [9.99, [930.6697029, 32.75312075]], [10.00, [930.6697029, 32.75312075]], [10.01, [930.6697029, 32.75312075]]]

beta is, $\frac{33}{10}$, *times the threshold value*

the long-term behavior is

[[9.98, [923.8811464, 35.02831970]], [9.99, [923.8811464, 35.02831970]], [10.00, [923.8811464, 35.02831970]], [10.01, [923.8811464, 35.02831970]]]

beta is, $\frac{7}{2}$, *times the threshold value*

the long-term behavior is

[[9.98, [916.8646074, 37.31947743]], [9.99, [916.8646074, 37.31947743]], [10.00, [916.8646074, 37.31947743]], [10.01, [916.8646074, 37.31947743]]]

$$\textit{beta is}, \frac{37}{10}, \textit{times the threshold value}$$

the long-term behavior is

[[9.98, [909.6306685, 39.62585316]], [9.99, [909.6306685, 39.62585316]], [10.00, [909.6306685, 39.62585316]], [10.01, [909.6306685, 39.62585316]]]

$$\textit{beta is}, \frac{39}{10}, \textit{times the threshold value}$$

the long-term behavior is

[[9.98, [902.1900937, 41.94669340]], [9.99, [902.1900937, 41.94669340]], [10.00, [902.1900937, 41.94669340]], [10.01, [902.1900937, 41.94669340]]] **(4)**

\>

\> *#With nu=3 and gamma=7*

\> *#For beta* $= 0.3 \cdot \frac{N}{\mathrm{nu}}$: *At time* $t = 10$, *there are* 996.6155905 *susceptible*, 2.936908621 *infected*, **and** 0.447500879 *removed*.

\> *#For beta* $= 0.9 \frac{N}{\mathrm{nu}}$: *At time* $t = 10$, *there are* 987.5717147 *susceptible*, 8.969979927 *infected*, **and** 3.458305373 *removed*.

\> *#For beta* $= 3.9 \frac{N}{\mathrm{nu}}$: *At time* $t = 10$, *there are* 902.1900937 *susceptible*, 41.94669340 *infected*, **and** 55.8632129 *removed*.

\>

\> *Help19*( )

SIRSdemo(N,IN,gamma,nu,h,A),e.g. SIRSdemo(100,20,1, 1,0.01, 10); EquPts(F,var), StEquPts(F, var) , IsStable(M), RandNice(var,K) **(5)**

\> *#2)*

\> *#For the first F:*

\> $F := \mathit{RandNice}([x, y], 8)$

$$F := [(3 - 3x - y)(2 - 5x - 4y), (5 - 6x - 2y)(3 - 7x - 8y)] \quad \textbf{(6)}$$

\> *#i)*

\> $\mathit{EquPts}(F, [x, y])$

$$\left\{\left[\frac{1}{3}, \frac{1}{12}\right], \left[\frac{8}{7}, -\frac{13}{14}\right], \left[\frac{21}{17}, -\frac{12}{17}\right]\right\} \quad \textbf{(7)}$$

\> *#ii)*

$\mathit{StEquPts}(F, [x, y])$

$$\left\{\left[\frac{1}{3}, \frac{1}{12}\right]\right\} \quad \textbf{(8)}$$

\> *#iii)*

$\mathit{Dis2}\left(F, x, y, \left[\frac{1}{3} + 0.1, \frac{1}{12} + 0.1\right], 0.01, 10\right)[998..1000]$

[[9.98, [0.3333333353, 0.08333333161]], [9.99, [0.3333333353, 0.08333333161]], [10.00, **(9)**

$[0.3333333353, 0.08333333161]]]$

> #EQ point $\left[\frac{1}{3}, \frac{1}{12}\right]$ *is stable; a small deviation* $[x\_eq + 0.1, y\_eq + 0.1]$ **and** *it still returns* **to** $\left[\frac{1}{3}, \frac{1}{12}\right]$

> $Dis2\left(F, x, y, \left[\frac{8}{7} + 0.1, \frac{-13}{14} + 0.1\right], 0.01, 10\right)[998..1000]$

$[[9.98, [\mathrm{Float}(\infty), \mathrm{Float}(\infty)]], [9.99, [\mathrm{Float}(\infty), \mathrm{Float}(\infty)]], [10.00, [\mathrm{Float}(\infty), \mathrm{Float}(\infty)]]]$ **(10)**

> #EQ point $\left[\frac{8}{7}, -\frac{13}{14}\right]$ *is unstable; a small deviation* $[x\_eq + 0.1, y\_eq + 0.1]$ **and** *it goes* **to** [Float(∞), Float(∞)]

> $Dis2\left(F, x, y, \left[\frac{21}{17} + 0.1, \frac{-12}{17} + 0.1\right], 0.01, 10\right)[998..1000]$

$[[9.98, [\mathrm{Float}(\infty), \mathrm{Float}(\infty)]], [9.99, [\mathrm{Float}(\infty), \mathrm{Float}(\infty)]], [10.00, [\mathrm{Float}(\infty), \mathrm{Float}(\infty)]]]$ **(11)**

> #EQ point $\left[\frac{21}{17}, -\frac{12}{17}\right]$ *is unstable; a small deviation* $[x\_eq + 0.1, y\_eq + 0.1]$ **and** *it goes* **to** [Float(∞), Float(∞)]

>

> *#For the second F:*

> $F := RandNice([x, y], 8)$

$$F := [(2 - 2x - 3y)(4 - 7x - 5y), (8 - x - 6y)(4 - x - 7y)] \quad \textbf{(12)}$$

> *#i)*

> $EquPts(F, [x, y])$

$$\left\{\left[-\frac{16}{37}, \frac{52}{37}\right], \left[-\frac{4}{3}, \frac{14}{9}\right], \left[\frac{2}{11}, \frac{6}{11}\right]\right\} \quad \textbf{(13)}$$

> *#ii)*

> $StEquPts(F, [x, y])$

$$\emptyset \quad \textbf{(14)}$$

> *#iii)*

> $Dis2\left(F, x, y, \left[-\frac{16}{37} + 0.1, \frac{52}{37} + 0.1\right], 0.01, 10\right)[998..1000]$

$[[9.98, [\mathrm{Float}(\infty), \mathrm{Float}(\infty)]], [9.99, [\mathrm{Float}(\infty), \mathrm{Float}(\infty)]], [10.00, [\mathrm{Float}(\infty), \mathrm{Float}(\infty)]]]$ **(15)**

> #EQ point $\left[-\frac{16}{37}, \frac{52}{37}\right]$ *is unstable; a small deviation* $[x\_eq + 0.1, y\_eq + 0.1]$ **and** *it goes* **to** [Float(∞), Float(∞)]

> $Dis2\left(F, x, y, \left[-\frac{4}{3} + 0.1, \frac{14}{9} + 0.1\right], 0.01, 10\right)[998..1000]$

$[[9.98, [\mathrm{Float}(\infty), \mathrm{Float}(\infty)]], [9.99, [\mathrm{Float}(\infty), \mathrm{Float}(\infty)]], [10.00, [\mathrm{Float}(\infty), \mathrm{Float}(\infty)]]]$ **(16)**

> #EQ point $\left[-\frac{4}{3}, \frac{14}{9}\right]$ *is unstable; a small deviation* [*x_eq* + 0.1, *y_eq* + 0.1] **and** *it goes* **to** [Float(∞), Float(∞)]

> $Dis2\left(F, x, y, \left[\frac{2}{11} + 0.1, \frac{6}{11} + 0.1\right], 0.01, 10\right)[998..1000]$

[[9.98, [Float(∞), Float(∞)]], [9.99, [Float(∞), Float(∞)]], [10.00, [Float(∞), Float(∞)]]] **(17)**

> #EQ point $\left[\frac{2}{11}, \frac{6}{11}\right]$ *is unstable; a small deviation* [*x_eq* + 0.1, *y_eq* + 0.1] **and** *it goes* **to** [Float(∞), Float(∞)]

>

> *#For the third F:*

> $F := RandNice([x, y], 8)$

$$F := [(6 - 3x - y)(3 - 7x - 2y), (4 - x - 5y)(2 - 3x - 7y)] \quad \textbf{(18)}$$

> *#i)*

> $EquPts(F, [x, y])$

$$\left\{\left[\frac{7}{33}, \frac{25}{33}\right], \left[\frac{13}{7}, \frac{3}{7}\right], \left[\frac{17}{43}, \frac{5}{43}\right], \left[\frac{20}{9}, -\frac{2}{3}\right]\right\} \quad \textbf{(19)}$$

> *#ii)*

> $StEquPts(F, [x, y])$

$$\left\{\left[\frac{17}{43}, \frac{5}{43}\right]\right\} \quad \textbf{(20)}$$

> *#iii)*

> $Dis2\left(F, x, y, \left[\frac{7}{33} + 0.1, \frac{25}{33} + 0.1\right], 0.01, 10\right)[998..1000]$

[[9.98, [Float(∞), Float(∞)]], [9.99, [Float(∞), Float(∞)]], [10.00, [Float(∞), Float(∞)]]] **(21)**

> #EQ point $\left[\frac{7}{33}, \frac{25}{33}\right]$ *is unstable; a small deviation* [*x_eq* + 0.1, *y_eq* + 0.1] **and** *it goes* **to** [Float(∞), Float(∞)]

> $Dis2\left(F, x, y, \left[\frac{13}{7} + 0.1, \frac{3}{7} + 0.1\right], 0.01, 10\right)[998..1000]$

[[9.98, [Float(∞), Float(∞)]], [9.99, [Float(∞), Float(∞)]], [10.00, [Float(∞), Float(∞)]]] **(22)**

> #EQ point $\left[\frac{13}{7}, \frac{3}{7}\right]$ *is unstable; a small deviation* [*x_eq* + 0.1, *y_eq* + 0.1] **and** *it goes* **to** [Float(∞), Float(∞)]

> $Dis2\left(F, x, y, \left[\frac{17}{43} + 0.1, \frac{5}{43} + 0.1\right], 0.01, 10\right)[998..1000]$

[[9.98, [0.3953488370, 0.1162790700]], [9.99, [0.3953488370, 0.1162790700]], [10.00, [0.3953488370, 0.1162790700]]] **(23)**

> #EQ point $\left[\frac{17}{43}, \frac{5}{43}\right]$ *is stable; a small deviation* $[x\_eq + 0.1, y\_eq + 0.1]$ **and** *it returns back* **to** $\left[\frac{17}{43}, \frac{5}{43}\right]$

> $evalf\left(\left[\frac{17}{43}, \frac{5}{43}\right]\right)$

$$[0.3953488372, 0.1162790698] \quad (24)$$

> $Dis2\left(F, x, y, \left[\frac{20}{9} + 0.1, \frac{-2}{3} + 0.1\right], 0.01, 10\right)[998..1000]$

[[9.98, [Float(∞), Float(∞)]], [9.99, [Float(∞), Float(∞)]], [10.00, [Float(∞), Float(∞)]]] **(25)**

> #EQ point $\left[\frac{20}{9}, -\frac{2}{3}\right]$ *is unstable; a small deviation* $[x\_eq + 0.1, y\_eq + 0.1]$ **and** *it goes* **to** [Float(∞), Float(∞)]

>

>

>

> *#3)*

> $EquPts(SIRS(s, i, \text{beta}, \text{gamma}, \text{nu}, N), [s, i])$

$$\left\{[N, 0], \left[\frac{\nu}{\beta}, \frac{\gamma(N\beta - \nu)}{\beta(\gamma + \nu)}\right]\right\} \quad (26)$$

> *#eq (29a): The first eq point from the above where: Susceptible = N, Infected = 0, Removed = N-S-I=N-N-0=0.*

> *#eq (29b): The second eq point from the above where:*

> #Susceptible $= \frac{\nu}{\beta}$

> #Infected $= \frac{\gamma(N\beta - \nu)}{\beta(\gamma + \nu)}$

> *#For the form in equation (29b):*

> $simplify\left(subs\left(\text{beta} = \frac{\text{nu}}{S}, \frac{\gamma(N\beta - \nu)}{\beta(\gamma + \nu)}\right)\right)$

$$\frac{\gamma(N - S)}{\gamma + \nu} \quad (27)$$

> #Infected $= \frac{\gamma(N - S)}{\gamma + \nu}$, *where* S = *Susceptible equation above* $= \frac{\nu}{\beta}$

> *#Removed = N-S-I...*

> $simplify\left(N - S - \frac{\gamma(N - S)}{\gamma + \nu}\right)$

$$\frac{\nu(N - S)}{\gamma + \nu} \quad (28)$$

> #Removed $= \frac{\nu\,(N-S)}{\gamma+\nu} = \nu\cdot\frac{I}{\gamma}$ *where I represents the equation* **for** *Infected* : $\frac{\gamma\,(N-S)}{\gamma+\nu}$, $S = \frac{\nu}{\beta}$.

> #These equations, derived from the output of EquPts() are now in the form of eq (29b).

> #Both equilibrium points confirm equations (29a) and (29b).

>

>

>

> #4)

> $Chemostat := \mathbf{proc}(N, C, a1, a2) : \left[a1\cdot\left(\frac{C}{1+C}\right)\cdot N - N, -\left(\frac{C}{1+C}\right)\cdot N - C + a2\right] :\mathbf{end}:$

> $simplify(EquPts(Chemostat(N, C, a1, a2), [N, C]))$

$$\left\{[0, a2], \left[\frac{a1\,(a2\,a1 - a2 - 1)}{a1-1}, \frac{1}{a1-1}\right]\right\} \qquad \textbf{(29)}$$

> #For N= $\frac{a1\,(a2\,a1 - a2 - 1)}{a1-1}$ **in** *the second eq point, it can be expressed equivalently* **to** *eq* $\cdot(25\,a)$ ...

> # $\frac{a1\,(a2\,a1 - a2 - 1)}{a1-1} = a1\cdot\left(\frac{a2(a1-1)-1}{(a1-1)}\right) = a1\cdot\left(a2 - \frac{1}{(a1-1)}\right)$

> #this makes the second eq point [N,C]= $\left[a1\cdot\left(a2 - \frac{1}{(a1-1)}\right), \frac{1}{a1-1}\right]$ *just as eq* $\cdot(25\,a)$ **in** *the book.*

> #the first eq point [N,C]= [0,a2] matches eq ·(25b) from the book.

>

>

>

> #5) EXTRA CREDIT:

> #i)

```
> Orb3 := proc(F, x, y, z, pt0, K1, K2) local pt, L, i :
  pt := pt0 :
  for i to K1 − 1 do: pt := subs({x = pt[1], y = pt[2], z = pt[3]}, F) od:
  L := [ ]:
  for i from K1 to K2 do: L := [op(L), pt] : pt := normal(subs({x = pt[1], y = pt[2], z
      = pt[3]}, F))od:
  L
  end proc :
```

```
> Dis3 := proc(F, x, y, z, pt, h, A) local L, i :
  L := Orb3([x + h·F[1], y + h·F[2], z + h·F[3]], x, y, z, pt, 0, trunc(A/h)) :
  L := [seq([i*h, [L[i][1], L[i][2], L[i][3]]], i = 1..nops(L))] :
  end:
```

>

> $F := RandNice([x, y, z], 10)$

$F := [(6 - 8\,x - 8\,y - 7\,z)\,(9 - 6\,x - y - 2\,z), (9 - 5\,x - 3\,y - 2\,z)\,(5 - 3\,x - 9\,y - 8\,z),$ **(30)**

$(5-4x-3y-3z)(9-3x-8y-7z)]$

> $E := EquPts(F, [x, y, z])$

$$E := \left\{[2, 1, -2], [6, 19, -23], \left[-\frac{3}{5}, \frac{194}{5}, -\frac{214}{5}\right], \left[-\frac{3}{5}, \frac{312}{25}, -\frac{318}{25}\right], \left[\frac{8}{13}, -\frac{47}{13}, \frac{58}{13}\right], \left[\frac{8}{19}, -\frac{97}{19}, \frac{118}{19}\right], \left[\frac{10}{7}, \frac{5}{7}, -\frac{1}{7}\right], \left[\frac{34}{11}, -\frac{17}{11}, -\frac{10}{11}\right]\right\} \quad (31)$$

> $StEquPts(F, [x, y, z])$

$$\left\{\left[\frac{34}{11}, -\frac{17}{11}, -\frac{10}{11}\right]\right\} \quad (32)$$

> $Dis3\left(F, x, y, z, \left[\frac{34}{11} + 0.1, -\frac{17}{11} + 0.1, -\frac{10}{11} + 0.1\right], 0.01, 10\right)[-3..-1]$

$[[9.99, [3.090909093, -1.545454549, -0.9090909076]], [10.00, [3.090909093, -1.545454549, -0.9090909076]], [10.01, [3.090909093, -1.545454549, -0.9090909076]]]$ **(33)**

> $evalf\left(\left[\frac{34}{11}, -\frac{17}{11}, -\frac{10}{11}\right]\right)$

$$[3.090909091, -1.545454545, -0.9090909091] \quad (34)$$

> # $\left[\frac{34}{11}, -\frac{17}{11}, -\frac{10}{11}\right]$ *is the ONLY STABLE equilibrium point!*

> *#Below is the result of running Dis3() on all equilibrium points* $[x\_eq + 0.1, y\_eq + 0.1, z\_eq + 0.1]$:

> **for** i **in** E **do**: $print('EQ\ Point'\ i, 'Last\ value\ of\ Dis3'\ Dis3(F, x, y, z, [i[1] + 0.1, i[2] + 0.1, i[3] + 0.1], 0.01, 10)[-1][2])$ **od**:

EQ Point $[2, 1, -2]$, *Last value of Dis3* $[3.090909093, -1.545454544, -0.9090909128]$

EQ Point $[6, 19, -23]$, *Last value of Dis3* $[\mathrm{Float}(\infty), \mathrm{Float}(\infty), \mathrm{Float}(\infty)]$

EQ Point $\left[-\frac{3}{5}, \frac{194}{5}, -\frac{214}{5}\right]$, *Last value of Dis3* $[\mathrm{Float}(\infty), \mathrm{Float}(\infty), \mathrm{Float}(\infty)]$

EQ Point $\left[-\frac{3}{5}, \frac{312}{25}, -\frac{318}{25}\right]$, *Last value of Dis3* $[\mathrm{Float}(\infty), \mathrm{Float}(\infty), \mathrm{Float}(\infty)]$

EQ Point $\left[\frac{8}{13}, -\frac{47}{13}, \frac{58}{13}\right]$, *Last value of Dis3* $[\mathrm{Float}(\infty), \mathrm{Float}(\infty), \mathrm{Float}(\infty)]$

EQ Point $\left[\frac{8}{19}, -\frac{97}{19}, \frac{118}{19}\right]$, *Last value of Dis3* $[\mathrm{Float}(\infty), \mathrm{Float}(\infty), \mathrm{Float}(\infty)]$

EQ Point $\left[\frac{10}{7}, \frac{5}{7}, -\frac{1}{7}\right]$, *Last value of Dis3* $[\mathrm{Float}(\infty), \mathrm{Float}(\infty), \mathrm{Float}(\infty)]$

EQ Point $\left[\frac{34}{11}, -\frac{17}{11}, -\frac{10}{11}\right]$, *Last value of Dis3* $[3.090909093, -1.545454549, -0.9090909076]$ **(35)**

> *#Clearly, for all points excluding the only stable equilibrium point (and except for [2,1,-2]), a small change of* $[x\_eq + 0.1, y\_eq + 0.1, z\_eq + 0.1]$ **and** *the system eventually goes* **to** $[\mathrm{Float}(\infty), \mathrm{Float}(\infty), \mathrm{Float}(\infty)]$ *therefore they are UNSTABLE!*

In the case of the equilibrium point $[2, 1, -2]$,
it is still UNSTABLE because the small change caused the system **to** *eventually converge* **to** *the stable equilibrium point* $\left[\frac{34}{11}, -\frac{17}{11}, -\frac{10}{11}\right] = [3.090909091, -1.545454545, -0.9090909091]$ **and** *NOT itself.*

>

> #Let's try for another F:

> $F := RandNice([x, y, z], 10)$

$$F := [(2 - 2x - 4y - z)(2 - 5x - 6y - 10z), (5 - 5x - y - 2z)(8 - 10x - 2y - 5z), (1 - 3x - 9y - 2z)(2 - 3x - 5y - 4z)] \quad \textbf{(36)}$$

> $E := EquPts(F, [x, y, z])$

$$E := \left\{\left[\frac{10}{3}, \frac{1}{3}, -6\right], \left[\frac{28}{9}, \frac{1}{9}, -\frac{14}{3}\right], \left[\frac{43}{45}, \frac{1}{9}, -\frac{16}{45}\right], \left[\frac{53}{46}, \frac{1}{46}, -\frac{9}{23}\right], \left[\frac{61}{53}, \frac{1}{53}, -\frac{41}{106}\right], \left[\frac{90}{79}, -\frac{17}{79}, -\frac{19}{79}\right], \left[\frac{194}{205}, \frac{4}{41}, -\frac{68}{205}\right], \left[\frac{552}{605}, -\frac{19}{121}, -\frac{98}{605}\right]\right\} \quad \textbf{(37)}$$

> $StEquPts(F, [x, y, z])$

$$\left\{\left[\frac{28}{9}, \frac{1}{9}, -\frac{14}{3}\right]\right\} \quad \textbf{(38)}$$

> **for** i **in** E **do**: $print('EQ\ Point'\ i, 'Last\ value\ of\ Dis3'\ Dis3(F, x, y, z, [i[1] + 0.1, i[2] + 0.1, i[3] + 0.1], 0.01, 10)[-1][2])$ **od**:

$$EQ\ Point \left[\frac{10}{3}, \frac{1}{3}, -6\right], Last\ value\ of\ Dis3\ [\mathrm{Float}(\infty), \mathrm{Float}(\infty), \mathrm{Float}(\infty)]$$

$$EQ\ Point \left[\frac{28}{9}, \frac{1}{9}, -\frac{14}{3}\right], Last\ value\ of\ Dis3\ [3.111111117, 0.1111111112, -4.666666678]$$

$$EQ\ Point \left[\frac{43}{45}, \frac{1}{9}, -\frac{16}{45}\right], Last\ value\ of\ Dis3\ [\mathrm{Float}(\infty), \mathrm{Float}(\infty), \mathrm{Float}(\infty)]$$

$$EQ\ Point \left[\frac{53}{46}, \frac{1}{46}, -\frac{9}{23}\right], Last\ value\ of\ Dis3\ [\mathrm{Float}(\infty), \mathrm{Float}(\infty), \mathrm{Float}(\infty)]$$

$$EQ\ Point \left[\frac{61}{53}, \frac{1}{53}, -\frac{41}{106}\right], Last\ value\ of\ Dis3\ [\mathrm{Float}(\infty), \mathrm{Float}(\infty), \mathrm{Float}(\infty)]$$

$$EQ\ Point \left[\frac{90}{79}, -\frac{17}{79}, -\frac{19}{79}\right], Last\ value\ of\ Dis3\ [\mathrm{Float}(\infty), \mathrm{Float}(\infty), \mathrm{Float}(\infty)]$$

$$EQ\ Point \left[\frac{194}{205}, \frac{4}{41}, -\frac{68}{205}\right], Last\ value\ of\ Dis3\ [\mathrm{Float}(\infty), \mathrm{Float}(\infty), \mathrm{Float}(\infty)]$$

$$EQ\ Point \left[\frac{552}{605}, -\frac{19}{121}, -\frac{98}{605}\right], Last\ value\ of\ Dis3\ [\mathrm{Float}(\infty), \mathrm{Float}(\infty), \mathrm{Float}(\infty)] \quad \textbf{(39)}$$

> $evalf\left(\left[\frac{28}{9}, \frac{1}{9}, -\frac{14}{3}\right]\right)$

$$[3.111111111, 0.1111111111, -4.666666667] \quad \textbf{(40)}$$

> #Again, only the stable equilibrium point $\left[\frac{28}{9}, \frac{1}{9}, -\frac{14}{3}\right] = [3.111111111, 0.1111111111,$

-4.666666667] *converged* **to** *itself ....with all other equilibrium points* : *a small change* **and** *the system went off* **to** [Float(∞), Float(∞), Float(∞)], *therefore they are UNSTABLE*!

>