> #OK to post
#Julian Herman, 8th November, 2021, Assignment 18

>

> #1)

> $C := \textbf{proc}(a, b, c, d, e)$ **local** *rate* :

$rate := \frac{\frac{b}{a}}{c}$: #rate = number of eggs one chicken lays per day

$print(d \cdot rate \cdot e)$:

end proc:

> $C\left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}, 3, 3\right)$

$$6 \tag{1}$$

>

> #2)

> $W := \textbf{proc}(a, b, k)$**local** C :

$C := \left(\frac{1}{a} - \frac{1}{b}\right) \cdot \left(\frac{1}{k-1}\right)$: #equation simplifies the same every time: $\left\{A + B = \frac{1}{a}, A + C = \frac{1}{b}, B = k \cdot C\right\} \Rightarrow \left\{A + k \cdot C = \frac{1}{a}, A + C = \frac{1}{b}\right\} \Rightarrow$

#⇒ subtracting eq2 from eq1 $\Rightarrow k \cdot C - C = \frac{1}{a} - \frac{1}{b} \Rightarrow C(k-1) = \frac{1}{a} - \frac{1}{b} \Rightarrow C = \left(\frac{1}{a} - \frac{1}{b}\right) \cdot \left(\frac{1}{k-1}\right)$

#C represents the speed of work done by C in the units "cisterns per hour," therefore, the reciprocal of C = number of hours for C alone to fill one cistern

$print\left(\frac{1}{C}\right)$:

end proc:

> $W(4, 5, 2)$

$$20 \tag{2}$$

>

> #3)iii)

> $solve(\{x \cdot (1 - x - y) = 0, x \cdot (3 - 2 \cdot x - y) = 0\}, \{x, y\})$

$$\{x = 0, y = y\}, \{x = 2, y = -1\} \tag{3}$$

> $with(LinearAlgebra)$:

> #For equilibrium point (2,-1):

> $evalf(Eigenvalues([[-2,-2], [-4,-2]]))$

$$\begin{bmatrix} 0.828427124 \\ -4.828427124 \end{bmatrix} \tag{4}$$

> #(2,-1) is UNSTABLE because 0.828427124 is $>$ 0... The real parts of ALL of the eigenvalues must be NEGATIVE for the equilibrium point to be stable.

> *#For equilibrium point (0,y):*

> *Eigenvalues*([[1 − y, 0], [3 − y, 0]])

$$\begin{bmatrix} 0 \\ 1 - y \end{bmatrix} \quad \textbf{(5)}$$

> #(0, y) *is UNSTABLE because the first eigenvalue is always* 0, **and** 0 *is NOT* < 0.(*borderline case when* y = 1 ... *could be called semi − stable because* **then** *the eigenvalues are* [[0], [0]]? *not sure though*)

>

> *#4)*

> **read** *`/Users/julianherman/Documents/Rutgers/Fall 2021/Dynamical Models In Biology/HW/M18.txt`*

> *Help18*( )

Dis2(F,x,y,pt,h,A), SIRS(s,i,beta,gamma,nu,N) **(6)**

> *Dis2*([x·(1 − x − y), x·(3 − 2·x − y)], x, y, [2.01, -1.01], 0.01, 10)[990 ..1000]

[[9.90, [Float(- ∞), Float(- ∞)]], [9.91, [Float(- ∞), Float(- ∞)]], [9.92, [Float(- ∞), Float(- ∞)]], [9.93, [Float(- ∞), Float(- ∞)]], [9.94, [Float(- ∞), Float(- ∞)]], [9.95, [Float(- ∞), Float(- ∞)]], [9.96, [Float(- ∞), Float(- ∞)]], [9.97, [Float(- ∞), Float(- ∞)]], [9.98, [Float(- ∞), Float(- ∞)]], [9.99, [Float(- ∞), Float(- ∞)]], [10.00, [Float(- ∞), Float(- ∞)]]] **(7)**

> *#This confirms that equilibrium point (2,-1) is unstable because a small change (0.01) away from the point, and it tends to (-infinity,-infinity).*

>

> *Dis2*([x·(1 − x − y), x·(3 − 2·x − y)], x, y, [0.01, 10.01], 0.01, 10)[999 ..1000]

[[9.99, [$1.293672666 \times 10^{-43}$, 10.00221360]], [10.00, [$1.177213489 \times 10^{-43}$, 10.00221360]]] **(8)**

> *Dis2*([x·(1 − x − y), x·(3 − 2·x − y)], x, y, [-0.01, -1.01], 0.01, 10)[999 ..1000]

[[9.99, [Float(- ∞), Float(- ∞)]], [10.00, [Float(- ∞), Float(- ∞)]]] **(9)**

> *#For the family of equilibrium points* (0, y) *where y is any real number, it sometimes tends* **to** *the same neighborhood as the point* (0, y) *but* **not** *exactly* = (0, y) *as* **in** *the first case above* **for** (0, 10) ⇒ (0.01, 10.01) *tends* **to** *about the same* ($1.293672666 \times 10^{-43}$, 10.00221360) ... *However*, **in** *the second case* (0, 1) ⇒ (-0.01, -1.01) *tends* **to** (-infinity, -infinity) ... *therefore*, (0, y) *is UNSTABLE*

>

> *#5)*

> *F* ≔ *SIRS*(s, i, 0.01, 1, 1, 50) :

> *L* ≔ *Dis2*(F, s, i, [20, 30], 0.01, 10) :

> *L*[-5 ... -1]

[[9.97, [49.73062516, 0.08865256516]], [9.98, [49.73199151, 0.08820691426]], [9.99, [49.73335085, 0.08776351567]], [10.00, [49.73470322, 0.08732235788]], [10.01, [49.73604867, 0.08688342946]]] **(10)**

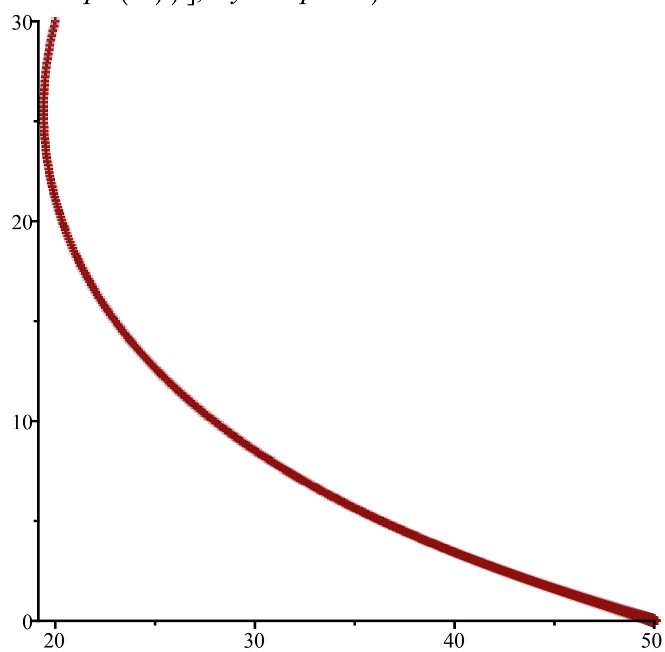
> *#N=50. N-S-I=R... 50-49.73604867-0.08688342946=0.1770679005... not enough iterations yet, but removed and infected go to 0!*

```
> L := Dis2(F, s, i, [20, 30], 0.01, 50) :
> L[-5...-1]
```
[[49.97, [49.99999950, $1.728549414 \times 10^{-10}$]], [49.98, [49.99999950, $1.719906667 \times 10^{-10}$]], [49.99, [49.99999950, $1.711307133 \times 10^{-10}$]], [50.00, [49.99999950, $1.702750598 \times 10^{-10}$]], [50.01, [49.99999950, $1.694236845 \times 10^{-10}$]]] **(11)**

> *#With more iterations: Changed 'A' variable of Dis2() to 50 so there are more iterations... N=50. N-S-I=R... 50-49.99999950-1.694236845* $\times 10^{-10} = 4.998305763 \times 10^{-7}$. *..It is apparent that removed* **and** *infected go* **to** 0!

> *#Plot for this below:*

> *plot*([*seq*(*L*[*i*][2], *i* = 1 ..*nops*(*L*))], *style* = *point*)



> *#With Beta=0.01, gamma=1, nu=1, N=50: infected (vertical axis) goes to 0 and susceptible (horizontal axis) goes to N=50. the epidemic is eradicated! This is because N=50* $< \frac{\nu}{\text{beta}} = 100$

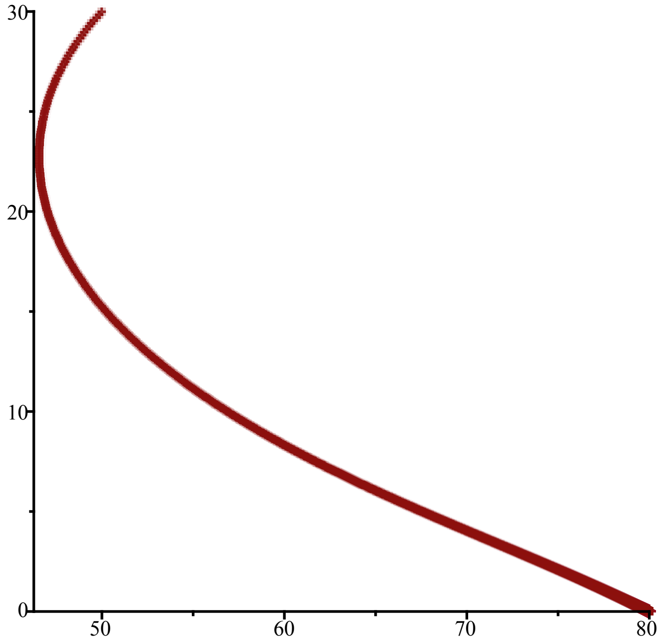
>

```
> F := SIRS(s, i, 0.01, 1, 1, 80) :
> L := Dis2(F, s, i, [50, 30], 0.01, 50) :
> L[-5...-1]
```
[[49.97, [79.99928055, 0.0003197488413]], [49.98, [79.99928198, 0.0003191093206]], [49.99, [79.99928342, 0.0003184710791]], [50.00, [79.99928486, 0.0003178341141]], [50.01, [79.99928629, 0.0003171984232]]] **(12)**

> *#Changed 'A' variable of Dis2() to 50 so there are more iterations... N=80. N-S-I=R... 80 -79.99928629-0.0003171984232=0.0003965115768... It is apparent that the removed and infected go to 0!*

> *#Plot for this below:*

> *plot*([*seq*(*L*[*i*][2], *i* = 1 ..*nops*(*L*))], *style* = *point*)



> *#With Beta=0.01, gamma=1, nu=1, N=80: infected (vertical axis) goes to 0 and susceptible (horizontal axis) goes to N=80. The epidemic is eradicated! This is because N=80* $< \frac{\nu}{\text{beta}} = 100$

>

> $F := SIRS(s, i, 0.01, 1, 1, 120):$

> $L := Dis2(F, s, i, [90, 30], 0.01, 50):$

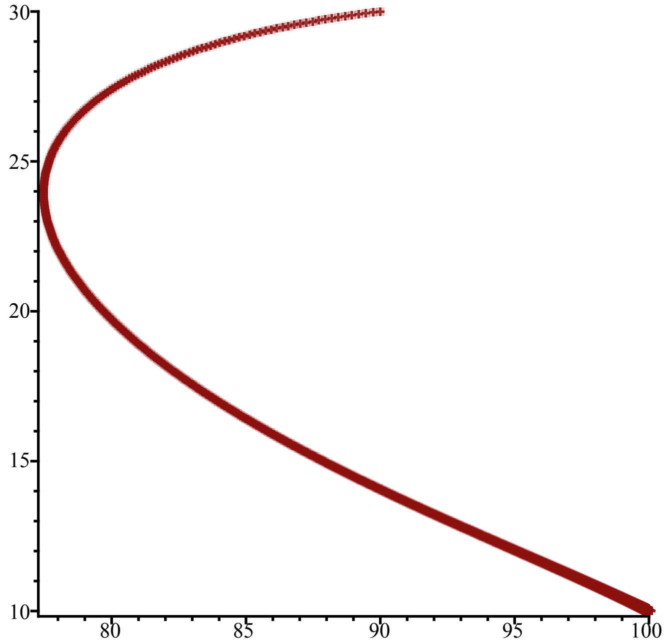
> $L[-5\,...-1]$

[[49.97, [99.99983227, 10.00007241]], [49.98, [99.99983268, 10.00007224]], [49.99, [99.99983308, 10.00007207]], [50.00, [99.99983348, 10.00007190]], [50.01, [99.99983388, 10.00007173]]] **(13)**

> *#Changed 'A' variable of Dis2() to 50 so there are more iterations... N=120. N-S-I=R... 120 -99.99983388-10.00007173=10.00009439. Susceptible=100, Infected=10, Removed=10. The epidemic was not eradicated!*

> *#Plot for this below:*

> $plot([seq(L[i][2], i = 1..nops(L))], style = point)$



> #With Beta=0.01, gamma=1, nu=1, N=120: infected (vertical axis) goes to 10 and susceptible (horizontal axis) goes to N=100. Therefore, removed = N-S-I=120-100-10=10. The epidemic is NOT eradicated! This is because N=120 $> \frac{\nu}{\text{beta}} = 100$.

> #In the long run, with N=120 as in the example above, 10 individuals will be infected!

OK to post

Julian Herman, 11/8/21, Assignment 18

3) i) $x'(t) = x(t)(1 - x(t) - y(t))$

$y'(t) = x(t)(3 - 2 \cdot x(t) - y(t))$

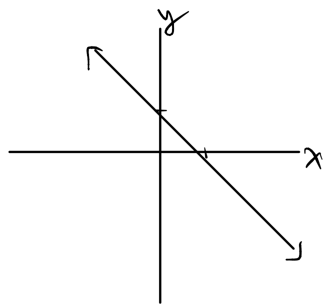
Let $F(x(t), y(t)) = x(t)(1 - x(t) - y(t))$

Then at equilibrium:

$F(x, y) = x(1 - x - y) = 0$

$\boxed{x = 0}$, $1 - x - y = 0$

(eq.1) $\boxed{y = 1 - x}$



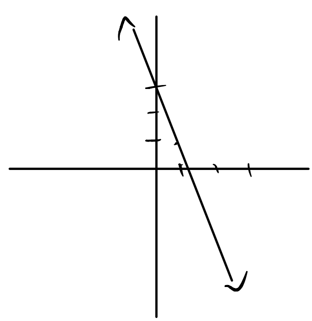
Let $G(x(t), y(t)) = x(t)(3 - 2 \cdot x(t) - y(t))$

Then at equilibrium:

$G(x, y) = x(3 - 2x - y) = 0$

$\boxed{x = 0}$, $3 - 2x - y = 0$

(eq.2) $\boxed{y = 3 - 2x}$



(eq. 1) = (eq. 2)

$1 - x = 3 - 2x$

$x = 2$

$y = 1 - x = 1 - 2 = -1$

E.Q. points:

$(0, y)$ ← when $x = 0$, y can be any real number. $(0, y)$ represents an infinite family of EQ. pt's

$(2, -1)$

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ii) Jacobian $= \begin{bmatrix} F_x , & F_y \\ G_x , & G_y \end{bmatrix} = \begin{bmatrix} 1-2x-y , & -x \\ 3-4x-y , & -x \end{bmatrix}$

more generally: let $x = x_1$, $y = x_2$

$$\begin{bmatrix} 1-2x_1-x_2 , & -x_1 \\ 3-4x_1-x_2 , & -x_1 \end{bmatrix}$$

For EQ. point: $(2, -1)$: $J = \begin{bmatrix} -2 , & -2 \\ -4 , & -2 \end{bmatrix}$

infinite set of equilibrium points $x = 0$, y = anything → $(0, y)$: $J = \begin{bmatrix} 1-y , & 0 \\ 3-y , & 0 \end{bmatrix}$

REFER TO MAPLE SECTION