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Julian Herman, 11/1/21, Assignment 17

1) $x'(t) = 3x(t) - y(t)$ $\quad x(0) = 2$

$y'(t) = 2x(t)$ $\quad y(0) = 3$

i) $x''(t) = 3x'(t) - \underbrace{y'(t)}_{=2x(t)}$

$\Rightarrow x''(t) - 3x'(t) + 2x(t) = 0$

$$r^2 - 3r + 2 = 0$$

$$(r-2)(r-1) = 0$$

$$r_1 = 2, \; r_2 = 1$$

$$x(t) = Ae^{2t} + Be^{t}$$

(eq. 1) $x(0) = A + B = 2$

$$x'(t) = 2Ae^{2t} + Be^{t}$$

$$x'(t) = 3x(t) - y(t)$$

$$2Ae^{2t} + Be^{t} = 3Ae^{2t} + 3Be^{t} - y(t)$$

$$\Rightarrow y(t) = Ae^{2t} + 2Be^{t}$$

$y(0) = 3$

(eq. 2) $y(0) = A + 2B = 3$

(eq. 1) $A + B = 2 \rightarrow A = 2 - B$

(eq. 2) $A + 2B = 3 \rightarrow 2 - B + 2B = 3$

$\boxed{B = 1}$ $\boxed{A = 2 - B = 2 - 1 = 1}$

$$\Rightarrow \boxed{\begin{aligned} x(t) &= e^{2t} + e^{t} \\ y(t) &= e^{2t} + 2e^{t} \end{aligned}}$$

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ii) $x'(t) = 3 \cdot x(t) - y(t)$

$y'(t) = 2 \cdot x(t) - 0 \cdot y(t)$

Let $X(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ be a vector-function

initial conditions:

$$X'(t) = \underbrace{\begin{bmatrix} 3, & -1 \\ 2, & 0 \end{bmatrix}}_{=A} X(t) \qquad X(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} 3-\lambda, & -1 \\ 2, & -\lambda \end{bmatrix}\right) = 0 \qquad \lambda^2 - 3\lambda + 2 = 0$$

$(\lambda - 1)(\lambda - 2) = 0$

$\lambda_1 = 1, \quad \lambda_2 = 2$

$\lambda_1 = 1$: find corresponding eigenvector

$$(\vec{A} - \lambda_1 \vec{I})\vec{v_1} = \vec{0}$$

$$\begin{bmatrix} 2, & -1 \\ 2, & -1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2a - b = 0 \Rightarrow b = 2a \Rightarrow \vec{v_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\lambda_2 = 2: \begin{bmatrix} 1, & -1 \\ 2 & -2 \end{bmatrix} \overbrace{\begin{bmatrix} a \\ b \end{bmatrix}}^{\vec{v_2}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$a - b = 0 \Rightarrow a = b \Rightarrow \vec{v_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$X(t) = A \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{t} + B \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t}$$

Initial condition

$$X(0) = A \begin{bmatrix} 1 \\ 2 \end{bmatrix} + B \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} A + B \\ 2A + B \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$A + B = 2 \rightarrow B = 2 - A$$

$$2A + B = 3 \rightarrow 2A + \overbrace{2 - A}^{} = 3$$

$$\boxed{A = 1 \rightarrow B = 2 - 1 = 1}$$

$$\Rightarrow X(t) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{t} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t}$$

SAME!

$$x(t) = e^{t} + e^{2t}$$

$$y(t) = 2e^{t} + e^{2t}$$

2.) $x'(t) = 2 \cdot x(t)$ $\quad x(0) = 7$

$y'(t) = 4 \cdot x(t)$ $\quad y(0) = 6$

i) $x'(t) - 2x(t) = 0$

$r - 2 = 0$

$r = 2$

$x(t) = Ae^{2t}$

$x(0) = A = 7 \rightarrow \boxed{x(t) = 7e^{2t}}$

$y'(t) = 4 \cdot 7e^{2t} = 28e^{2t}$

$\int y'(t)\,dt = 28 \int e^{2t}\,dt$

$y(t) = 28 \cdot \frac{e^{2t}}{2} + c$

$y(t) = 14e^{2t} + c$

$y(0) = 14 + c = 6 \Rightarrow c = -8$

$\Rightarrow \boxed{y(t) = 14e^{2t} - 8}$

ii) $x'(t) = 2 \cdot x(t) + 0 \cdot y(t)$

$y'(t) = 4 \cdot x(t) + 0 \cdot y(t)$

Let $X(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$

$$X'(t) = \begin{bmatrix} 2, & 0 \\ 4, & 0 \end{bmatrix} X(t)$$

$$\det\left(\begin{bmatrix} 2-\lambda, & 0 \\ 4, & -\lambda \end{bmatrix}\right) = 0 \qquad \lambda^2 - 2\lambda = 0$$

$$\lambda(\lambda - 2) = 0$$

don't want 0 as a solution $\longrightarrow$ $\lambda_1 = 0, \quad \lambda_2 = 2$

$\lambda_1 = 0$: find eigenvector

$$\begin{bmatrix} 2, & 0 \\ 4, & 0 \end{bmatrix} \overset{\vec{v}_1}{\begin{bmatrix} a \\ b \end{bmatrix}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \left.\begin{aligned} 2a + 0 \cdot b = 0 \\ 4a + 0b = 0 \end{aligned}\right\} \begin{aligned} a = 0 \\ \text{let } b = 1 \end{aligned}$$

$\lambda_2 = 2$: find corresponding eigenvector

$$\begin{bmatrix} 0, & 0 \\ 4, & -2 \end{bmatrix} \overset{\vec{v}_2}{\begin{bmatrix} a \\ b \end{bmatrix}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow 4a - 2b = 0$$

$$4a = 2b$$

$$2a = b$$

$$\vec{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$X(t) = A \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} + B \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cancel{e^{0t}}$$

initial condition $X(0) = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

$$X(0) = A \begin{bmatrix} 1 \\ 2 \end{bmatrix} + B \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

$$A = 7$$

$$2A + B = 6 \rightarrow 14 + B = 6 \rightarrow B = -8$$

$$X(t) = 7\begin{bmatrix}1\\2\end{bmatrix}e^{2t} - 8\begin{bmatrix}0\\1\end{bmatrix}$$

SAME AS: $x(t) = 7e^{2t}$

$y(t) = 14e^{2t} - 8$

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3) Let $X(t) = \begin{bmatrix}x_1(t)\\x_2(t)\\x_3(t)\end{bmatrix}$

$$X'(t) = \begin{bmatrix}1, & 1, & 1\\1, & 1, & 0\\1, & 0, & 0\end{bmatrix} X(t)$$

$$\det\left(\begin{bmatrix}1-\lambda, & 1, & 1\\1, & 1-\lambda, & 0\\1, & 0, & -\lambda\end{bmatrix}\right) = 0$$

$$(1-\lambda)(\lambda^2-\lambda) - (-\lambda) + (\lambda-1) = 0$$

$$\lambda^2 \cancel{-\lambda} - \lambda^3 + \lambda^2 \cancel{+\lambda} + \lambda - 1 = 0$$

$$-\lambda^3 + 2\lambda^2 + \lambda - 1 = 0$$

refer to maple

> *#OK to post*
#Julian Herman, November 1st, 2021, Assignment 17

>

> *#1)*

> $dsolve(\{diff(x(t), t) = 3\cdot x(t) - y(t), diff(y(t), t) = 2\cdot x(t), x(0) = 2, y(0) = 3\}, \{x(t), y(t)\})$

$$\left\{x(t) = e^{t} + e^{2t}, y(t) = 2\,e^{t} + e^{2t}\right\} \tag{1}$$

>

> *#2)*

> $dsolve(\{diff(x(t), t) = 2\cdot x(t), diff(y(t), t) = 4\cdot x(t), x(0) = 7, y(0) = 6\}, \{x(t), y(t)\})$

$$\left\{x(t) = 7\,e^{2t}, y(t) = 14\,e^{2t} - 8\right\} \tag{2}$$

>

> *#3)*

> $evalf(dsolve(\{diff(x(t), t) = x(t) + y(t) + z(t), diff(y(t), t) = x(t) + y(t), diff(z(t), t) = x(t), x(0) = 1, y(0) = 2, z(0) = -1\}, \{x(t), y(t), z(t)\}))$

$$\begin{aligned}\{x(t) &= -(0.5697026303 + 9.164188886\times 10^{-10}\,I)\,e^{(0.5549581324 - 4.760383402\times 10^{-10}\,I)\,t} \\ &+ (0.3971667823 - 1.584703633\times 10^{-10}\,I)\,e^{(-0.8019377366 + 1.336718457\times 10^{-10}\,I)\,t} \\ &+ (1.172535850 - 1.800998826\times 10^{-10}\,I)\,e^{(2.246979605 + 7.972616167\times 10^{-10}\,I)\,t}, y(t) \\ &= (1.280110189 + 2.343690660\times 10^{-9}\,I)\,e^{(0.5549581324 - 4.760383402\times 10^{-10}\,I)\,t} \\ &+ (-0.2204109358 + 2.240066821\times 10^{-10}\,I)\,e^{(-0.8019377366 + 1.336718457\times 10^{-10}\,I)\,t} \\ &+ (0.9403007426 - 1.140497572\times 10^{-9}\,I)\,e^{(2.246979605 + 7.972616167\times 10^{-10}\,I)\,t}, z(t) = \\ &-(1.026568667 + 2.153914464\times 10^{-9}\,I)\,e^{(0.5549581324 - 4.760383402\times 10^{-10}\,I)\,t} \\ &+ (-0.4952588764 + 1.651271691\times 10^{-12}\,I)\,e^{(-0.8019377366 + 1.336718457\times 10^{-10}\,I)\,t} \\ &+ (0.5218275450 + 1.069774777\times 10^{-9}\,I)\,e^{(2.246979605 + 7.972616167\times 10^{-10}\,I)\,t}\}\end{aligned} \tag{3}$$

> $with(LinearAlgebra):$

> $evalf(Eigenvectors(Matrix([[1, 1, 1], [1, 1, 0], [1, 0, 0]])))$

$$\left[\begin{array}{c} 2.246979605 + 1.\times 10^{-10}\,I \\ -0.8019377358 - 1.866025404\times 10^{-10}\,I \\ 0.5549581322 - 1.339745960\times 10^{-11}\,I \end{array}\right], [[2.246979634 + 1.514675242\times 10^{-9}\,I, -0.8019377350 + 3.686305552\times 10^{-10}\,I, 0.5549581323 - 2.254559307\times 10^{-11}\,I], [1.801937769 + 1.888769131\times 10^{-9}\,I, 0.4450418682 - 5.947994638\times 10^{-10}\,I, -1.246979604 + 5.809451696\times 10^{-11}\,I], [1., 1., 1.]] \tag{4}$$

> $evalf(solve(-x^3 + 2\cdot x^2 + x - 1 = 0, x))$

$$2.246979605 + 1.\times 10^{-10}\,I, -0.8019377358 - 1.866025404\times 10^{-10}\,I, 0.5549581322 - 1.339745960\times 10^{-11}\,I \tag{5}$$

> *#the above are the eigenvalues*

> #these numbers are not easy to work with, the result is shown above by dsolve()

>

> #4)

> #i)

>

read `/Users/julianherman/Documents/Rutgers/Fall 2021/Dynamical Models In Biology/HW/M17.txt`

> $Help17(\ )$

HW3g(u,v,w,M), HW2g(u,v,M) **(6)**

> $F := HW2g(u, v, [[1, 1, 1], [1, 1, 1], [1, 1, 1]])$

$$F := \left[u^2 + u\,v + \frac{1}{4}\,v^2, -2\,u\,v - 2\,u^2 + 2\,u - \frac{1}{2}\,v^2 + v\right] \tag{7}$$

> $Help13(\ )$

RT2(x,y,d,K), Orb2(F,x,y,pt0,K1,K2), FP2(F,x,y), SFP2(F,x,y), PlotOrb2(L), FP2drz(F,x,y), SFP2drz(F,x,y) **(8)**

> $H := Orb2(F, u, v, [a, b], 1, 5)$

$$H := \left[[a, b], \left[a^2 + a\,b + \frac{1}{4}\,b^2, -2\,a\,b - 2\,a^2 + 2\,a - \frac{1}{2}\,b^2 + b\right], \left[a^2 + a\,b + \frac{1}{4}\,b^2, -2\,a\,b - 2\,a^2 + 2\,a - \frac{1}{2}\,b^2 + b\right], \left[a^2 + a\,b + \frac{1}{4}\,b^2, -2\,a\,b - 2\,a^2 + 2\,a - \frac{1}{2}\,b^2 + b\right], \left[a^2 + a\,b + \frac{1}{4}\,b^2, -2\,a\,b - 2\,a^2 + 2\,a - \frac{1}{2}\,b^2 + b\right]\right] \tag{9}$$

> $evalb(H[2][1] = H[3][1]$ **and** $H[2][2] = H[3][2])$

true **(10)**

> #The values are the same after one generation!

> #ii)

> $r := rand(0.0..1.0):$

> $M := [[r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )]]:$
$Orb2(HW2g(u, v, M), u, v, [.33, .33], 1000, 1005)$

[[0.8618836443, 0.1361473036], [0.8618836443, 0.1361473036], [0.8618836443, 0.1361473036], [0.8618836443, 0.1361473036], [0.8618836443, 0.1361473036], [0.8618836443, 0.1361473036]] **(11)**

> #stabilized, but allele aa has basically died out (super low frequency)

> $M := [[r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )]]:$
$Orb2(HW2g(u, v, M), u, v, [.33, .33], 1000, 1005)$

[[0.04946057537, 0.3953578214], [0.04946057537, 0.3953578214], [0.04946057537, 0.3953578214], [0.04946057537, 0.3953578214], [0.04946057537, 0.3953578214], [0.04946057537, 0.3953578214]] **(12)**

> #stabilized, but allele AA has basically died out (super low frequency)

> $M := [[r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )]]:$
$Orb2(HW2g(u, v, M), u, v, [.33, .33], 1000, 1005)$

(13)

$$[[1.952367577 \times 10^{-1067}, 4.374163758 \times 10^{-534}], [1.640251814 \times 10^{-1068}, 1.267854766 \times 10^{-534}], [1.378032518 \times 10^{-1069}, 3.674886899 \times 10^{-535}], [1.157732982 \times 10^{-1070}, 1.065168825 \times 10^{-535}], [9.726516890 \times 10^{-1072}, 3.087400122 \times 10^{-536}], [8.171584666 \times 10^{-1073}, 8.948853263 \times 10^{-537}]] \quad (13)$$

> *#stabilized, AA,Aa have died out*

> $M := [[r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )]]:$
$Orb2(HW2g(u, v, M), u, v, [.33, .33], 1000, 1005)$

$$[[1.000000004, -3.799163265 \times 10^{-9}], [1.000000004, -3.799163265 \times 10^{-9}], [1.000000004, -3.799163265 \times 10^{-9}], [1.000000004, -3.799163265 \times 10^{-9}], [1.000000004, -3.799163265 \times 10^{-9}], [1.000000004, -3.799163265 \times 10^{-9}]] \quad (14)$$

> *#stabilized, but allele Aa has basically died out*

> $M := [[r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )]]:$
$Orb2(HW2g(u, v, M), u, v, [.33, .33], 1000, 1005)$

$$[[0.8001831078, 0.1885889137], [0.8001831080, 0.1885889137], [0.8001831078, 0.1885889137], [0.8001831080, 0.1885889137], [0.8001831078, 0.1885889137], [0.8001831080, 0.1885889137]] \quad (15)$$

> *#stabilized*

> $M := [[r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )]]:$
$Orb2(HW2g(u, v, M), u, v, [.33, .33], 1000, 1005)$

$$[[1.000000000, -1.863390510 \times 10^{-210}], [1.000000000, -1.154296673 \times 10^{-210}], [1.000000000, -7.150411055 \times 10^{-211}], [1.000000000, -4.429396658 \times 10^{-211}], [1.000000000, -2.743835928 \times 10^{-211}], [1.000000000, -1.699697765 \times 10^{-211}]] \quad (16)$$

> *#stabilized, but Aa has basically died out*

> $M := [[r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )]]:$
$Orb2(HW2g(u, v, M), u, v, [.33, .33], 1000, 1005)$

$$[[0.09334806033, 0.4582823533], [0.09334806033, 0.4582823533], [0.09334806033, 0.4582823533], [0.09334806033, 0.4582823533], [0.09334806033, 0.4582823533], [0.09334806033, 0.4582823533]] \quad (17)$$

> *#stabilized*

> $M := [[r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )]]:$
$Orb2(HW2g(u, v, M), u, v, [.33, .33], 1000, 1005)$

$$[[0.9999999998, -6.964095425 \times 10^{-10}], [0.9999999992, 7.588333396 \times 10^{-10}], [0.9999999993, 4.051359363 \times 10^{-10}], [0.9999999993, 1.346940947 \times 10^{-9}], [0.9999999999, 7.191225699 \times 10^{-10}], [1.000000000, -7.467070095 \times 10^{-10}]] \quad (18)$$

> *#Aa and aa have died out*

> $M := [[r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )], [r(\ ), r(\ ), r(\ )]]:$
$Orb2(HW2g(u, v, M), u, v, [.33, .33], 1000, 1005)$

$$[[0.1961221903, 0.5484550781], [0.1961221905, 0.5484550778], [0.1961221903, 0.5484550781], [0.1961221905, 0.5484550778], [0.1961221903, 0.5484550781], [0.1961221905, 0.5484550778]] \quad (19)$$

> *#stabilized and all alleles present in reasonable frequency*

> $M := [[r(\,), r(\,), r(\,)], [r(\,), r(\,), r(\,)], [r(\,), r(\,), r(\,)]]:$
> $Orb2(HW2g(u, v, M), u, v, [.33, .33], 1000, 1005)$

$$[[4.391550777\times10^{-280}, 3.452544434\times10^{-140}], [2.293353359\times10^{-280}, 2.494972770\times10^{-140}], [1.197633796\times10^{-280}, 1.802985955\times10^{-140}], [6.254276967\times10^{-281}, 1.302923380\times10^{-140}], [3.266105260\times10^{-281}, 9.415543861\times10^{-141}], [1.705623787\times10^{-281}, 6.804119687\times10^{-141}]] \quad \textbf{(20)}$$

> *#stabilized but alleles AA, Aa have died out*