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Jeton Hida, Assignment 17, November 1, 2021

Question 1

$x'(t) = 3x(t) - y(t)$

$y'(t) = 2x(t)$ $\quad x(0) = 2,\ y(0) = 3$

$x''(t) = 3x'(t) - y'(t)$

$x''(t) = 3x'(t) - 2x(t)$

$x''(t) - 3x'(t) + 2x(t) = 0$

$\lambda^2 - 3\lambda + 2 = 0$

$(\lambda - 2)(\lambda - 1) = 0$

$\lambda = 2,\ \lambda = 1$

$x_1(t) = e^{2t} \quad x_2(t) = e^t$

$x(t) = C_1 e^{2t} + C_2 e^t \rightarrow x(0) = C_1 + C_2 = 2$

$x'(t) = C_1 2e^{2t} + C_2 e^t \quad x'(0) = 3x(0) - y(0) = 2C_1 + C_2 = 3$

$C_1 = 1,\ C_2 = 1$

$x(t) = e^{2t} + e^t$

$y(t) = 2e^{2t} + e^t - (3e^{2t} + 3e^t) = -e^{2t} - 2e^t = 2e^t + e^{2t}$

ii. $x'(t) = 3x(t) + -1y(t)$

$y'(t) = 2x(t) + 0y(t)$

$$X(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

$$X'(t) = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix} X(t) \qquad X(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\det(A-\lambda I) = \begin{bmatrix} 3-\lambda & -1 \\ 2 & -\lambda \end{bmatrix} \qquad \det(A-\lambda I) = (3-\lambda)(-\lambda) + 2 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\lambda_1 = 2, \ \lambda_2 = 1$$

$\lambda = 2$

$$\begin{bmatrix} 3-2 & -1 \\ 2 & -2 \end{bmatrix} \vec{V_1} = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \vec{V_1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\vec{V_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\lambda_2 = 1$

$$\begin{bmatrix} 3-1 & -1 \\ 2 & -1 \end{bmatrix} \vec{V_2} = \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \vec{V_2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\vec{V_2} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$X(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{t}$$

$$X(0) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2(0)} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{0} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} c_1 + c_2 \\ c_1 + 2c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad \begin{matrix} c_2 = 1 \\ c_1 = 1 \end{matrix}$$

$$X(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{t}$$

$x'(t) = x(t) + 7y(t)$

$y'(t) = 0$ $\qquad x(0) = 6 \qquad y(0) = 8$

```
> #Question 1 iii.
> dsolve({diff(x(t),t)=3*x(t)-y(t),diff(y(t),t)=2*x(t),x(0)=2,y(0)=3}
  ,{x(t),y(t)})
```

$$\{x(t) = e^{t} + e^{2\,t}, y(t) = 2\,e^{t} + e^{2\,t}\} \quad (1)$$

```
> #Question 2 x'(t) = x(t) + 7*y(t) y'(t) = 0

> #Method 1
  #x''(t) = x'(t) + 7*y'(t), -> x''(t) - x'(t) = 0
  solve(x^2-x=0,x)
```

$$0, 1 \quad (2)$$

```
> #x1(t) = 1, x2(t) = exp(t)
  #x(t) = c1 + c2*exp(t), x'(t) = c2*exp(t)
  #x(0)=c1 + c2 = 6
  #x'(0) = c2 = 6 + 7*8 = 62, c1 = 6 - 62 = -56
  #x(t) = -56 + 62*exp(t)
  #y(t) = 8

> #Method 2
  #x'(t) = 1*x(t)+7*y(t)
  #y'(t) = 0*x(t)+0*y(t)
  with (LinearAlgebra)
```

[*&x*, *Add*, *Adjoint*, *BackwardSubstitute*, *BandMatrix*, *Basis*, *BezoutMatrix*, *BidiagonalForm*, (3)
BilinearForm, *CARE*, *CharacteristicMatrix*, *CharacteristicPolynomial*, *Column*,
ColumnDimension, *ColumnOperation*, *ColumnSpace*, *CompanionMatrix*,
CompressedSparseForm, *ConditionNumber*, *ConstantMatrix*, *ConstantVector*, *Copy*,
CreatePermutation, *CrossProduct*, *DARE*, *DeleteColumn*, *DeleteRow*, *Determinant*,
Diagonal, *DiagonalMatrix*, *Dimension*, *Dimensions*, *DotProduct*, *EigenConditionNumbers*,
Eigenvalues, *Eigenvectors*, *Equal*, *ForwardSubstitute*, *FrobeniusForm*,
FromCompressedSparseForm, *FromSplitForm*, *GaussianElimination*, *GenerateEquations*,
GenerateMatrix, *Generic*, *GetResultDataType*, *GetResultShape*, *GivensRotationMatrix*,
GramSchmidt, *HankelMatrix*, *HermiteForm*, *HermitianTranspose*, *HessenbergForm*,
HilbertMatrix, *HouseholderMatrix*, *IdentityMatrix*, *IntersectionBasis*, *IsDefinite*,
IsOrthogonal, *IsSimilar*, *IsUnitary*, *JordanBlockMatrix*, *JordanForm*, *KroneckerProduct*,
LA_Main, *LUDecomposition*, *LeastSquares*, *LinearSolve*, *LyapunovSolve*, *Map*, *Map2*,
MatrixAdd, *MatrixExponential*, *MatrixFunction*, *MatrixInverse*, *MatrixMatrixMultiply*,
MatrixNorm, *MatrixPower*, *MatrixScalarMultiply*, *MatrixVectorMultiply*,
MinimalPolynomial, *Minor*, *Modular*, *Multiply*, *NoUserValue*, *Norm*, *Normalize*, *NullSpace*,
OuterProductMatrix, *Permanent*, *Pivot*, *PopovForm*, *ProjectionMatrix*, *QRDecomposition*,
RandomMatrix, *RandomVector*, *Rank*, *RationalCanonicalForm*, *ReducedRowEchelonForm*,
Row, *RowDimension*, *RowOperation*, *RowSpace*, *ScalarMatrix*, *ScalarMultiply*,
ScalarVector, *SchurForm*, *SingularValues*, *SmithForm*, *SplitForm*,
StronglyConnectedBlocks, *SubMatrix*, *SubVector*, *SumBasis*, *SylvesterMatrix*, *SylvesterSolve*,
ToeplitzMatrix, *Trace*, *Transpose*, *TridiagonalForm*, *UnitVector*, *VandermondeMatrix*,
VectorAdd, *VectorAngle*, *VectorMatrixMultiply*, *VectorNorm*, *VectorScalarMultiply*,

ZeroMatrix, ZeroVector, Zip]

```
> A:=Matrix([[1,7],[0,0]])
```

$$A := \begin{bmatrix} 1 & 7 \\ 0 & 0 \end{bmatrix} \tag{4}$$

```
> Eigenvalues(A)
```

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{5}$$

```
> Eigenvectors(A)
```

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -7 & 1 \\ 1 & 0 \end{bmatrix} \tag{6}$$

```
> #X(t) = c1[-7,1] + c2[1,0]exp(t) [-7,1] & [1,0] are column vectors
  corresponding to eigenvalues 0 and 1 respectively.
  #X(0) = -7c1 + c2 =6
  #X(0) = c1 = 8 -> c1 = 8 , c2 = 62
  #X(t) = 8[-7,1] + 62[1,0]exp(t)

> #Method 3

> dsolve({diff(x(t),t)=x(t)+7*y(t), diff(y(t),t)=0,x(0)=6,y(0)=8},{x
  (t),y(t)})
```

$$\{x(t) = -56 + 62\,e^{t}, y(t) = 8\} \tag{7}$$

```
> #Question 3

> A:=Matrix([[1,1,1],[1,1,0],[1,0,0]])
```

$$A := \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{8}$$

```
> evalf(Eigenvectors(A))
```

$$\begin{bmatrix} 2.246979605 + 1.\,10^{-10}\,I \\ -0.8019377358 - 1.866025404\,10^{-10}\,I \\ 0.5549581322 - 1.339745960\,10^{-11}\,I \end{bmatrix}, [[2.246979634 + 1.514675242\,10^{-9}\,I, \tag{9}$$

$$-0.8019377350 + 3.686305552\,10^{-10}\,I, 0.5549581323 - 2.254559307\,10^{-11}\,I],$$

$$[1.801937769 + 1.888769131\,10^{-9}\,I, 0.4450418682 - 5.947994638\,10^{-10}\,I,$$

$$-1.246979604 + 5.809451696\,10^{-11}\,I],$$

$$[1., 1., 1.]]$$

```
> evalf(dsolve({diff(x(t),t)=x(t)+y(t)+z(t),diff(y(t),t)=x(t)+y(t),
  diff(z(t),t)=x(t),x(0)=1,y(0)=2,z(0)=-1},{x(t),y(t),z(t)}))
```

$$\{x(t) = -(0.5697026293 + 5.309060149\,10^{-10}\,I)\,e^{(0.5549581324 - 4.760383402\,10^{-10}\,I)\,t} \tag{10}$$

$$+ (0.3971667826 - 2.996607211\,10^{-11}\,I)\,e^{(-0.8019377366 + 1.336718457\,10^{-10}\,I)\,t}$$

$$+ (1.172535850 - 1.800998826\,10^{-10}\,I)\,e^{(2.246979605 + 7.972616167\,10^{-10}\,I)\,t}, y(t)$$

$$= (1.280110188 + 1.629777932\,10^{-9}\,I)\,e^{(0.5549581324 - 4.760383402\,10^{-10}\,I)\,t}$$
$$+ (-0.2204109361 + 1.437113238\,10^{-10}\,I)\,e^{(-0.8019377366 + 1.336718457\,10^{-10}\,I)\,t}$$
$$+ (0.9403007426 - 1.140497572\,10^{-9}\,I)\,e^{(2.246979605 + 7.972616167\,10^{-10}\,I)\,t},\ z(t) =$$
$$-(1.026568665 + 1.475697371\,10^{-9}\,I)\,e^{(0.5549581324 - 4.760383402\,10^{-10}\,I)\,t}$$
$$- (0.4952588769 + 8.752174075\,10^{-11}\,I)\,e^{(-0.8019377366 + 1.336718457\,10^{-10}\,I)\,t}$$
$$+ (0.5218275450 + 1.069774777\,10^{-9}\,I)\,e^{(2.246979605 + 7.972616167\,10^{-10}\,I)\,t}\Big\}$$

```
> #Tried solving with method (ii.), numbers were not easy to work
  with
```

```
> #Question 4
  read "/Users/jeton/Desktop/Math 336/M17.txt"
```

```
> M:=Matrix([[1,1,1],[1,1,1],[1,1,1]])
```

$$M := \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{11}$$

```
> F:=HW2g(u,v,M)
```

$$F := \left[u^2 + u\,v + \frac{1}{4}\,v^2,\ -2\,u\,v - 2\,u^2 + 2\,u - \frac{1}{2}\,v^2 + v\right] \tag{12}$$

```
> print(Orb2)
```

```
proc(F, x, y, pt0, K1, K2)                                        (13)
    local pt, L, i;
    pt := pt0;
    for i to K1 - 1 do pt := subs({x = pt[1], y = pt[2]}, F) end do;
    L := [];
    for i from K1 to K2 do
        L := [op(L), pt]; pt := normal(subs({x = pt[1], y = pt[2]}, F))
    end do;
    L
end proc
```

```
> Orb2(F,u,v,[a,b],1,10)
```

$$\left[[a, b], \left[a^2 + a\,b + \frac{1}{4}\,b^2,\ -2\,a\,b - 2\,a^2 + 2\,a - \frac{1}{2}\,b^2 + b\right], \left[a^2 + a\,b + \frac{1}{4}\,b^2,\ -2\,a\,b\right. \right. \tag{14}$$
$$\left. - 2\,a^2 + 2\,a - \frac{1}{2}\,b^2 + b\right], \left[a^2 + a\,b + \frac{1}{4}\,b^2,\ -2\,a\,b - 2\,a^2 + 2\,a - \frac{1}{2}\,b^2 + b\right], \left[a^2$$
$$+ a\,b + \frac{1}{4}\,b^2,\ -2\,a\,b - 2\,a^2 + 2\,a - \frac{1}{2}\,b^2 + b\right], \left[a^2 + a\,b + \frac{1}{4}\,b^2,\ -2\,a\,b - 2\,a^2$$
$$+ 2\,a - \frac{1}{2}\,b^2 + b\right], \left[a^2 + a\,b + \frac{1}{4}\,b^2,\ -2\,a\,b - 2\,a^2 + 2\,a - \frac{1}{2}\,b^2 + b\right], \left[a^2 + a\,b$$

$$+\frac{1}{4}b^2,\ -2ab-2a^2+2a-\frac{1}{2}b^2+b\Big],\Big[a^2+ab+\frac{1}{4}b^2,\ -2ab-2a^2+2a-\frac{1}{2}b^2+b\Big],\Big[a^2+ab+\frac{1}{4}b^2,\ -2ab-2a^2+2a-\frac{1}{2}b^2+b\Big]\Big]$$

```
> M:=RandomMatrix(3,3)
```

$$M := \begin{bmatrix} 86 & -15 & 77 \\ -19 & -89 & -80 \\ -53 & 66 & -19 \end{bmatrix} \tag{15}$$

```
> F:=HW2g(u,v,M)
```

$$F := \left[\frac{344u^2-68uv-89v^2}{4\left(43u^2-82uv-94v^2+62u+24v-19\right)},\ -\frac{48u^2+68uv+75v^2-48u+14v}{2\left(43u^2-82uv-94v^2+62u+24v-19\right)}\right] \tag{16}$$

```
> Orb2(F,u,v,[2,2],1,5)
```

$$\left[[2,2],\left[-\frac{187}{379},\frac{348}{379}\right],\left[-\frac{709531}{4250887},\frac{2896134}{4250887}\right],\left[\frac{108395334107257}{833148330438746},\frac{207657934658556}{416574165219373}\right],\left[\frac{3592691760386692126520380139218}{18685004750316528873820856821109},\frac{8535596781599272575331318125768}{18685004750316528873820856821109}\right]\right] \tag{17}$$

```
> M:=RandomMatrix(3,3)
```

$$M := \begin{bmatrix} 63 & 75 & 42 \\ -90 & 33 & -86 \\ -11 & 27 & -77 \end{bmatrix} \tag{18}$$

```
> F:=HW2g(u,v,M)
```

$$F := \left[-\frac{3\left(84u^2-10uv+11v^2\right)}{4\left(45u^2+141uv-15v^2-185u-95v+77\right)},\ \frac{62u^2+18uv-92v^2-62u+59v}{2\left(45u^2+141uv-15v^2-185u-95v+77\right)}\right] \tag{19}$$

```
> Orb2(F,u,v,[3,1],1,5)
```

$$\left[[3,1],\left[-\frac{737}{320},\frac{131}{160}\right],\left[-\frac{12078055}{13245697},\frac{7242363}{13245697}\right],\left[-\frac{41116731850425627}{109747590760334992},\frac{18223208649146851}{54873795380167496}\right],\left[-\frac{1838641327921555557843431481614 4345}{175299730222934097309096936631650217},\frac{336715724085282561354213062998 40923}{175299730222934097309096936631650217}\right]\right] \tag{20}$$

```
> M:=RandomMatrix(3,3)
```

(21)

$$M := \begin{bmatrix} 11 & -89 & 50 \\ 39 & 24 & 30 \\ 80 & 67 & -28 \end{bmatrix} \tag{21}$$

```
> F:=HW2g(u,v,M)
```

$$F := \left[-\frac{11\,u^2 - 25\,u\,v + 6\,v^2}{147\,u^2 + 333\,u\,v + 101\,v^2 - 186\,u - 153\,v + 28}, \frac{260\,u^2 + 407\,u\,v + 73\,v^2 - 260\,u - 97\,v}{2\left(147\,u^2 + 333\,u\,v + 101\,v^2 - 186\,u - 153\,v + 28\right)} \right] \tag{22}$$

```
> Orb2(F,u,v,[10,10],1,5)
```

$$\left[[10, 10], \left[\frac{400}{27369}, \frac{35215}{54738} \right], \left[\frac{6743317350}{84447574153}, \frac{96203146565}{168895148306} \right], \left[\frac{8364879185689014215950}{23913030229761361583077 9}, \frac{30578012950861475904413 5}{47826060459522723166155 8} \right], \left[\frac{14539898884814925865473355815732300491734447895 0}{20901698004568930984031482856620073182604068240 91}, \frac{24284666703314841055901825073259340268955948606 95}{41803396009137861968062965713240146365208136481 82} \right] \right] \tag{23}$$

```
> M:=RandomMatrix(3,3)
```

$$M := \begin{bmatrix} 71 & 23 & -54 \\ -20 & -53 & 40 \\ 34 & 65 & -81 \end{bmatrix} \tag{24}$$

```
> F:=HW2g(u,v,M)
```

$$F := \left[\frac{284\,u^2 + 6\,u\,v - 53\,v^2}{4\left(10\,u^2 - 244\,u\,v - 239\,v^2 + 142\,u + 267\,v - 81\right)}, \frac{40\,u^2 - 62\,u\,v - 158\,v^2 - 40\,u + 105\,v}{2\left(10\,u^2 - 244\,u\,v - 239\,v^2 + 142\,u + 267\,v - 81\right)} \right] \tag{25}$$

```
> Orb2(F,u,v,[0,1],1,5)
```

$$\left[[0, 1], \left[\frac{1}{4}, \frac{1}{2} \right], \left[-\frac{21}{26}, \frac{9}{13} \right], \left[\frac{8817}{3958}, \frac{5072}{1979} \right], \left[-\frac{612889777}{4461796202}, \frac{3968539346}{15616286707} \right] \right] \tag{26}$$

```
> M:=RandomMatrix(3,3)
```

$$M := \begin{bmatrix} -49 & -11 & 97 \\ -33 & -85 & 10 \\ -61 & -81 & -72 \end{bmatrix} \tag{27}$$

```
> F:=HW2g(u,v,M)
```

$$F := \left[\frac{196\,u^2 + 88\,u\,v + 85\,v^2}{4\left(157\,u^2 + 153\,u\,v + 86\,v^2 - 180\,u - 73\,v + 72\right)}, \right. \tag{28}$$

$$\frac{72\,u^2+45\,u\,v+14\,v^2-72\,u+71\,v}{2\left(157\,u^2+153\,u\,v+86\,v^2-180\,u-73\,v+72\right)}\Bigg]$$

```
> Orb2(F,u,v,[2,1],1,5)
```

$$\left[[2,1],\left[\frac{1045}{2636},\frac{319}{1318}\right],\left[\frac{76826530}{190151387},\frac{17699627}{190151387}\right],\left[\frac{260628977709924329}{711798607020155212},-\frac{64464493771497457}{355899303510077606}\right],\left[\frac{2942419903596064006010228398402614442}{1673883852974195769321421075421612026 7},-\frac{8130751364237042542800609833650537129}{167388385297419576932142107542161202 67}\right]\right] \tag{29}$$

```
> M:=RandomMatrix(3,3)
```

$$M:=\begin{bmatrix}-60 & 63 & 48\\ 66 & -48 & -13\\ 29 & -79 & -96\end{bmatrix} \tag{30}$$

```
> F:=HW2g(u,v,M)
```

$$F:=\left[\frac{3\left(40\,u^2-43\,u\,v+8\,v^2\right)}{2\left(233\,u^2+48\,u\,v+52\,v^2-269\,u-100\,v+96\right)},\frac{154\,u^2-67\,u\,v-44\,v^2-154\,u+92\,v}{2\left(233\,u^2+48\,u\,v+52\,v^2-269\,u-100\,v+96\right)}\right] \tag{31}$$

```
> Orb2(F,u,v,[1,1],1,5)
```

$$\left[[1,1],\left[\frac{1}{8},-\frac{19}{120}\right],\left[\frac{72429}{2367434},-\frac{449099}{2367434}\right],\left[\frac{9666158187903}{1216799615608678},-\frac{130109157591889}{1216799615608678}\right],\left[\frac{579731168204293825627293329727}{311294327173987263023179812278182},-\frac{17022654281048787373555960764049}{311294327173987263023179812278182}\right]\right] \tag{32}$$

```
> M:=RandomMatrix(3,3)
```

$$M:=\begin{bmatrix}-98 & 27 & -69\\ 80 & 23 & 20\\ 85 & 70 & -7\end{bmatrix} \tag{33}$$

```
> F:=HW2g(u,v,M)
```

$$F:=\left[\frac{392\,u^2-214\,u\,v-23\,v^2}{4\left(121\,u^2+13\,u\,v+74\,v^2-30\,u-104\,v+7\right)},\frac{32\,u^2+15\,u\,v+67\,v^2-32\,u-90\,v}{2\left(121\,u^2+13\,u\,v+74\,v^2-30\,u-104\,v+7\right)}\right] \tag{34}$$

```
> Orb2(F,u,v,[2,9],1,5)
```

$$\left[[2,9],\left[-\frac{4147}{22892},\frac{4951}{11446}\right],\left[-\frac{3318482738}{8255310705},\frac{5429669773}{8255310705}\right],\left[\right. \tag{35}$$

$$-\frac{749467103184881979 2717}{331519418225930166308}, \frac{1100113063713336799629}{165759709112965083154}\Bigg],$$

$$\Bigg[\frac{635904692810388013138162447249556217460507223 0}{693989413252097316688996032451812067047565955 9}, \frac{944150320076770216972659398846892665582272919}{693989413252097316688996032451812067047565955 9}\Bigg]\Bigg]$$

```
> M:=RandomMatrix(3,3)
```

$$M := \begin{bmatrix} 37 & -46 & 73 \\ 31 & 97 & 90 \\ -15 & -22 & 33 \end{bmatrix} \tag{36}$$

```
> F:=HW2g(u,v,M)
```

$$F := \left[\frac{148\,u^2 - 30\,u\,v + 97\,v^2}{4\left(12\,u^2 - 75\,u\,v + 62\,v^2 - 8\,u + 2\,v + 33\right)}, -\frac{116\,u^2 + 199\,u\,v - 29\,v^2 - 116\,u - 68\,v}{2\left(12\,u^2 - 75\,u\,v + 62\,v^2 - 8\,u + 2\,v + 33\right)}\right] \tag{37}$$

```
> Orb2(F,u,v,[10,8],1,5)
```

$$\Bigg[[10, 8], \Bigg[-\frac{4652}{863}, \frac{11980}{863}\Bigg], \Bigg[\frac{4699072748}{13415152313}, \frac{6489758240}{13415152313}\Bigg], \tag{38}$$

$$\Bigg[\frac{1609625016747630236048}{6197738864182830783473}, \frac{2911971035843877183000}{6197738864182830783473}\Bigg],$$

$$\Bigg[\frac{2663388703475781466856727212685957542627272 48}{1429167674606735954106258390827257633326417073}, \frac{6985372001285626274158914403373170106189292 00}{1429167674606735954106258390827257633326417073}\Bigg]\Bigg]$$

```
> M:=RandomMatrix(3,3)
```

$$M := \begin{bmatrix} -27 & 10 & -20 \\ -48 & 88 & 62 \\ 28 & -53 & -70 \end{bmatrix} \tag{39}$$

```
> F:=HW2g(u,v,M)
```

$$F := \left[\frac{27\,u^2 + 19\,u\,v - 22\,v^2}{105\,u^2 + 195\,u\,v - 9\,v^2 - 148\,u - 149\,v + 70}, \frac{16\,u^2 + 63\,u\,v - 79\,v^2 - 16\,u - 9\,v}{2\left(105\,u^2 + 195\,u\,v - 9\,v^2 - 148\,u - 149\,v + 70\right)}\right] \tag{40}$$

```
> Orb2(F,u,v,[20,10],1,5)
```

$$\Bigg[[20, 10], \Bigg[\frac{310}{1893}, \frac{1069}{15144}\Bigg], \Bigg[\frac{191291338}{9235721847}, -\frac{190488501}{6157147898}\Bigg], \Bigg[\tag{41}$$

$$-\frac{351745175404494018}{1160967199405221956065}, -\frac{1579964036134103755}{1393160639286266347278}\Bigg], \Bigg[$$

$$-\frac{935922623404887231195274279084121224522}{3406947176076012974235873334181769243634540605},$$

$$\frac{726666807430287423098004800906616825608519}{681389435215202594847174666836353848726908121 0}\Bigg]\Bigg]$$

```
> M:=RandomMatrix(3,3)
```

$$M := \begin{bmatrix} -97 & -65 & 25 \\ -13 & -80 & 33 \\ 10 & -39 & -65 \end{bmatrix} \tag{42}$$

```
> F:=HW2g(u,v,M)
```

$$F := \left[\frac{97\,u^2+39\,u\,v+20\,v^2}{197\,u^2+237\,u\,v+139\,v^2-165\,u-124\,v+65},\right.$$

$$\left.\frac{35\,u^2+71\,u\,v+37\,v^2-35\,u+3\,v}{197\,u^2+237\,u\,v+139\,v^2-165\,u-124\,v+65}\right] \tag{43}$$

```
> Orb2(F,u,v,[300,8],1,5)
```

$$\left[[300,\,8],\left[\frac{8824880}{18257269},\frac{3312292}{18257269}\right],\left[\frac{8913633343835520}{11377825846342729},-\frac{250660882166628}{11377825846342729}\right],\right. \tag{44}$$

$$\left[\frac{76210463095318496314084527278006 40}{71657010588968828965121513730266 09},\right.$$

$$\left.-\frac{93363822407674584910350219391 3788}{71657010588968828965121513730266 09}\right],$$

$$\left[\frac{53737305457764787983661300072801833170042811678520318127569817801616 00}{50331467640750775701214882122609762125263011444626162160658680650140 49},\right.$$

$$\left.\left.-\frac{37154740151245271700548392041187070780123075861143562565671399275606 8}{50331467640750775701214882122609762125263011444626162160658680650140 49}\right]\right]$$