

HW 17 - Alan Ho

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i)  $x'(t) = 3x(t) - y(t)$ ,  $y'(t) = 2x(t)$ ,  $x(0) = 2$ ,  $y(0) = 3$

ii)  $x''(t) = 3x'(t) - y'(t)$

$$x''(t) = 3x'(t) - 2x(t) \Rightarrow x''(t) - 3x'(t) + 2x(t) = 0$$

$$r^2 - 3r + 2 = 0 \Rightarrow (r-2)(r-1) = 0 \quad r = 2, 1$$

$$x(t) = Ae^{2t} + Be^t \Rightarrow x'(t) = 2Ae^{2t} + Be^t$$

$$x(0) = A + B = 2$$

$$y(t) = 3x(t) - x'(t) = 3[Ae^{2t} + Be^t] - [2Ae^{2t} + Be^t]$$

$$y(0) = 3(A+B) - [2A+B] = A+2B = 3, \quad A+B=2$$

$$2 - B + 2B = 3 \Rightarrow B = 1 \quad A = 1$$

$$\boxed{x(t) = e^{2t} + e^t \quad y(t) = e^{2t} + 2e^t}$$

ii)  $x'(t) = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix} x(t)$   $x(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$$\det \begin{bmatrix} 3-\lambda & -1 \\ 2 & -\lambda \end{bmatrix} = 3\lambda + \lambda^2 + 2 = (\lambda+2)(\lambda+1) = \lambda = -2, -1$$

$$\lambda = -1: \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \quad x = \begin{bmatrix} \frac{1}{2}x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}$$

$$\lambda = -2: \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \quad x = \begin{bmatrix} x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x(t) = A \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} e^{-t} + B \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t}$$

$$x(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix} = A \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} + B \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\frac{1}{2}A + B = 2$$

$$A + B = 3$$

$$\frac{1}{2}A = 1$$

$$A = 2 \quad B = 1$$

$$\boxed{x(t) = e^{-t} + e^{-2t}$$
$$y(t) = 2e^{-t} + e^{-2t}}$$

$$2) \quad x'(t) = x(t) + 9y(t) \quad y'(t) = x(t) \quad x(0) = 6 \quad y(0) = 9$$

$$i) \quad x''(t) = x'(t) + 9y'(t)$$

$$x''(t) = x'(t) + 9x(t) \Rightarrow x''(t) - x'(t) - 9x(t) = 0$$

$$r^2 - r - 9 = 0 \Rightarrow r = \frac{1}{2} + \frac{\sqrt{37}}{2}, \quad \frac{1 - \sqrt{37}}{2}$$

$$x(t) = Ae^{\frac{1+\sqrt{37}}{2}t} + Be^{\frac{1-\sqrt{37}}{2}t}$$

$$x'(t) = \frac{1+\sqrt{37}}{2} Ae^{\frac{1+\sqrt{37}}{2}t} + \frac{1-\sqrt{37}}{2} Be^{\frac{1-\sqrt{37}}{2}t}$$

$$x(0) = 3.54A + (-2.54)B = 6$$

$$y(t) = \frac{x'(t) - x(t)}{9} = \frac{\left(\frac{1+\sqrt{37}}{2} Ae^{\frac{1+\sqrt{37}}{2}t} + \frac{1-\sqrt{37}}{2} Be^{\frac{1-\sqrt{37}}{2}t}\right) - \left(Ae^{\frac{1+\sqrt{37}}{2}t} + Be^{\frac{1-\sqrt{37}}{2}t}\right)}{9}$$

$$y(0) = \frac{3.54A - 2.54B - (3.54A + 2.54A + B)}{9} = \frac{2.54A - 1.54B}{9} = 9$$

$$x(t) = 11.95 e^{\frac{1+\sqrt{37}}{2}t} + 14.29 e^{\frac{1-\sqrt{37}}{2}t}$$

$$y(t) = \frac{\left(\frac{1+\sqrt{37}}{2} (11.95 e^{\frac{1+\sqrt{37}}{2}t}) + \frac{1-\sqrt{37}}{2} (14.29 e^{\frac{1-\sqrt{37}}{2}t})\right) - (11.95 e^{\frac{1+\sqrt{37}}{2}t} + 14.29 e^{\frac{1-\sqrt{37}}{2}t})}{9}$$

$$ii) \quad x'(t) = \begin{bmatrix} 1 & 9 \\ 1 & 0 \end{bmatrix} x(t) \quad x(0) = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$$

$$\lambda = \frac{1+\sqrt{37}}{2}, \quad \frac{1-\sqrt{37}}{2}$$

$$\lambda = \frac{1+\sqrt{37}}{2} : \begin{bmatrix} \frac{9}{\frac{1+\sqrt{37}}{2}} \\ 1 \end{bmatrix} \quad \lambda = \frac{1-\sqrt{37}}{2} : \begin{bmatrix} \frac{9}{\frac{1-\sqrt{37}}{2}} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ 9 \end{bmatrix} = A \begin{bmatrix} \frac{9}{\frac{1+\sqrt{37}}{2}} \\ 1 \end{bmatrix} + B \begin{bmatrix} \frac{9}{\frac{1-\sqrt{37}}{2}} \\ 1 \end{bmatrix}$$

$$3) \quad \begin{aligned} x_1'(t) &= x_1(t) + x_2(t) + x_3(t) & x_2'(t) &= x_1(t) + x_2(t) & x_3'(t) &= x_1(t) \\ x_1(0) &= 1 & x_2(0) &= 2 & x_3(0) &= -1 \end{aligned}$$

$$x'(t) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$\lambda =$

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$$x(t) = -(0.570 + 9.431 \cdot 10^{-10} I) e \dots$$