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Homework 17

1 $x'(t) = 3x(t) - y(t)$ $\quad x(0)=2 \quad y(0)=3$

$y'(t) = 2x(t)$

i $\left(y'(t) = 2x(t)\right)\frac{d}{dt} \rightarrow y''(t) - 2x'(t) = 0$ $\quad x''(t) = 3x'(t) - y'(t)$

$x'(t) = \frac{3}{2}y''(t) - y(t)$ $\quad 2x(t) + 3x'(t) + x''(t) = 0$

Char equation $r^2 + 3r + 2 = 0 \quad r = 2, 1$

Gen solution $= \underline{Ae^{2t} + Be^{t}} = x(t)$

$x'(t) = 2Ae^{2t} + Be^{t}$ $\quad y'(t) = 2Ae^{2t} + 2Be^{t}$

$2 = A + B \quad A = 1$ $\quad \underline{y(t) = Ae^{2t} + 2Be^{t}}$

$3 = A + 2B \quad B = 1$

$$\boxed{\begin{aligned} x(t) &= e^{2t} + e^{t} \\ y(t) &= e^{2t} + 2e^{t} \end{aligned}}$$

ii $x'(t) = 3 \cdot x(t) - 1 \cdot (y(t))$

$y'(t) = 2 \cdot x(t) + 0 \cdot y(t)$ $\rightarrow \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$

$A \cdot \vec{v}$

$\det[A - I\lambda] = (3-\lambda)(-\lambda) + 2$

$\lambda^2 - 3\lambda + 2 \rightarrow \lambda = 2, 1$

for $\lambda = 2$ $\quad$ for $\lambda = 1$

$\begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \quad v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\vec{x(t)} = A\begin{bmatrix} 1 \\ 2 \end{bmatrix}e^{t} + B\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{2t}$ $\quad x(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \rightarrow$

$$\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}A+B\\2A+B\end{bmatrix} \quad A=1 \quad B=1$$

$$\vec{x}(t) = \begin{bmatrix}1\\2\end{bmatrix}e^{t} + \begin{bmatrix}1\\1\end{bmatrix}e^{2t}$$

$$x(t) = e^{t} + e^{2t} \qquad y(t) = 2e^{t} + e^{2t}$$

2. $x'(t) = x(t) + 3y(t)$ $\qquad x(0)=9$

$y'(t) = 2x(t)$ $\qquad y(0)=5$

$$\frac{d}{dt}\left(x'(t) = x(t) + 3y(t)\right) = x''(t) = x'(t) + 3y'(t)$$

$$x''(t) - x'(t) - 6x(t) = 0$$

Char equation $= r^2 - r - 6 = 0 \qquad r = -2, 3$

Gen sol $\quad x(t) = Ae^{-2t} + Be^{3t}$

$$\frac{x'(t) - x(t)}{3} = y(t) = \frac{-2Ae^{-2t} + 3Be^{3t} - Ae^{-2t} - Be^{3t}}{3} = -Ae^{-2t} + \frac{2Be^{3t}}{3}$$

$y(0)=5 \quad x(0)=9$

$9 = A + B \qquad 14 = 5B/3$

$5 = -A + \frac{2B}{3} \qquad B = 42/5$

i $\quad x(t) = \frac{3e^{-2t}}{5} + \frac{42}{5}e^{3t} \qquad y(t) = \frac{-3e^{-2t}}{5} + \frac{28e^{3t}}{5}$

i) $\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$
A

$\det(A - I\lambda) = \lambda^2 - \lambda - 6 = 0$

$\lambda_1 = -2, \quad \lambda_2 = 3$

λ_1: $\begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}$ $\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

λ_2: $\begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$\vec{x}(t) = A\begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-2t} + B\begin{bmatrix} 3 \\ 2 \end{bmatrix} e^{3t}$

$x(0) = \begin{bmatrix} 9 \\ 5 \end{bmatrix} = \begin{bmatrix} A + 3B \\ -A + 2B \end{bmatrix}$ $B = \frac{14}{5}$ $A = 3/5$

$\vec{x}(t) = \frac{3}{5}\begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-2t} + \frac{14}{5} e^{3t} \cdot \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

ii) $x(t) = 3e^{-2t} + \frac{42}{5} e^{3t}$ $y(t) = -5e^{-2t} + \frac{28}{5} e^{3t}$

3 $x_1'(t) = x_1(t) + x_2(t) + x_3(t)$

$x_2'(t) = x_1(t) + x_2(t)$

$x_3'(t) = x_1(t)$

$\vec{x}(0) = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$x_1(t) = 1.174\, e^{2.24t} + 0.349\, e^{-0.802t} - 0.569\, e^{0.555t}$

$x_2(t) = 0.94\, e^{2.24t} - 0.222\, e^{-.802t} + 1.278\, e^{0.555t}$

$x_3(t) = 0.523\, e^{2.24t} - 0.498\, e^{-.802t} - 1.0248\, e$