

> #Please do not post homework  
#Shreya Ghosh, 11-01-2021, Assignment 17

> read "/Users/shreyaghosh/Documents/M17.txt"

> Help17( )

$$HW3g(u,v,w,M), HW2g(u,v,M) \quad (1)$$

> #1iii.

$$dsolve(\{diff(x(t), t) = 3 \cdot x(t) - y(t), diff(y(t), t) = 2 \cdot x(t), x(0) = 2, y(0) = 3\}, \{x(t), y(t)\});$$
$$\{x(t) = e^t + e^{2t}, y(t) = 2e^t + e^{2t}\} \quad (2)$$

> #2iii.

$$evalf(dsolve(\{diff(x(t), t) = x(t) + 8 \cdot y(t), diff(y(t), t) = 5 \cdot x(t), x(0) = 5, y(0) = 2\}, \{x(t), y(t)\}));$$

$$\{x(t) = 3.958004251 e^{6.844288770 t} + 1.041995748 e^{-5.844288770 t}, y(t) = 2.891464975 e^{6.844288770 t} - 0.891464975 e^{-5.844288770 t}\} \quad (3)$$

> #3.

$$evalf(dsolve(\{diff(x(t), t) = x(t) + y(t) + z(t), diff(y(t), t) = x(t) + y(t), diff(z(t), t) = x(t), x(0) = 1, y(0) = 2, z(0) = -1\}, \{x(t), y(t), z(t)\}));$$

$$\{x(t) = -(0.5697026303 + 8.907180302 \times 10^{-10} I) e^{(0.5549581324 - 4.760383402 \times 10^{-10} I) t} + (0.3971667823 - 1.841712215 \times 10^{-10} I) e^{(-0.8019377366 + 1.336718457 \times 10^{-10} I) t} + (1.172535850 - 1.800998826 \times 10^{-10} I) e^{(2.246979605 + 7.972616167 \times 10^{-10} I) t}, y(t) = (1.280110189 + 2.254385490 \times 10^{-9} I) e^{(0.5549581324 - 4.760383402 \times 10^{-10} I) t} + (-0.2204109358 + 2.329371991 \times 10^{-10} I) e^{(-0.8019377366 + 1.336718457 \times 10^{-10} I) t} + (0.9403007426 - 1.140497572 \times 10^{-9} I) e^{(2.246979605 + 7.972616167 \times 10^{-10} I) t}, z(t) = -(1.026568667 + 2.028913658 \times 10^{-9} I) e^{(0.5549581324 - 4.760383402 \times 10^{-10} I) t} + (-0.4952588764 + 3.734690814 \times 10^{-11} I) e^{(-0.8019377366 + 1.336718457 \times 10^{-10} I) t} + (0.5218275450 + 1.069774777 \times 10^{-9} I) e^{(2.246979605 + 7.972616167 \times 10^{-10} I) t}\} \quad (4)$$

> #4i.

$$HW2g(u, v, [[1, 1, 1], [1, 1, 1], [1, 1, 1]])$$

$$\left[ u^2 + u v + \frac{1}{4} v^2, -2 u v - 2 u^2 + 2 u - \frac{1}{2} v^2 + v \right] \quad (5)$$

$$> Orb2\left(\left[ u^2 + u v + \frac{1}{4} v^2, -2 u v - 2 u^2 + 2 u - \frac{1}{2} v^2 + v \right], u, v, [a, b], 1, 3\right)$$

$$\left[ [a, b], \left[ a^2 + a b + \frac{1}{4} b^2, -2 a b - 2 a^2 + 2 a - \frac{1}{2} b^2 + b \right], \left[ a^2 + a b + \frac{1}{4} b^2, -2 a b - 2 a^2 + 2 a - \frac{1}{2} b^2 + b \right] \right] \quad (6)$$

> #2ii.

$$\text{with}(LinearAlgebra)$$

$$[\&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, \quad (7)$$

*BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA\_Main, LUdecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRdecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]*

> *evalf(Eigenvalues([[1, 8], [5, 0]]))*

$$\begin{bmatrix} 6.844288770 \\ -5.844288770 \end{bmatrix} \quad (8)$$

> *evalf(Eigenvectors([[1, 8], [5, 0]]))*

$$\begin{bmatrix} 6.844288770 \\ -5.844288770 \end{bmatrix}, \begin{bmatrix} 1.368857754 & -1.168857754 \\ 1. & 1. \end{bmatrix} \quad (9)$$

> #3.  
*evalf(Eigenvectors([[1, 1, 1], [1, 1, 0], [1, 0, 0]]))*

$$\begin{bmatrix} 2.246979605 + 1. \times 10^{-10} I \\ -0.8019377358 - 1.866025404 \times 10^{-10} I \\ 0.5549581322 - 1.339745960 \times 10^{-11} I \end{bmatrix}, \begin{bmatrix} [2.246979634 + 1.514675242 \times 10^{-9} I, \\ -0.8019377350 + 3.686305552 \times 10^{-10} I, 0.5549581323 - 2.254559307 \times 10^{-11} I], \\ [1.801937769 + 1.888769131 \times 10^{-9} I, 0.4450418682 - 5.947994638 \times 10^{-10} I, \\ -1.246979604 + 5.809451696 \times 10^{-11} I], \\ [1., 1., 1.] \end{bmatrix} \quad (10)$$

> #4ii.

HW2g(u, v, [[1, 2, .3], [2, 3, 1], [4, 3, 2]])

$$\left[ -\frac{4. u^2 + 8. u v + 3. v^2}{4 (1.300000000 u^2 + 0.3000000000 u v - 1. v^2 - 0.3000000000 u - 2.)}, \right. \\ \left. \frac{4.300000000 u v + 4.300000000 u^2 - 4.300000000 u + 0.5000000000 v^2 - 2. v}{1.300000000 u^2 + 0.3000000000 u v - 1. v^2 - 0.3000000000 u - 2.} \right] \quad (11)$$

> evalf(Orb2( [ [ -\frac{4. u^2 + 8. u v + 3. v^2}{4 (1.300000000 u^2 + 0.3000000000 u v - 1. v^2 - 0.3000000000 u - 2.)}, \frac{4.300000000 u v + 4.300000000 u^2 - 4.300000000 u + 0.5000000000 v^2 - 2. v}{1.300000000 u^2 + 0.3000000000 u v - 1. v^2 - 0.3000000000 u - 2.} ], u, v, [a, b], 1, 5 ) )

>

## HW 17

1.  $x'(t) = 3x(t) - y(t)$ ,  $y'(t) = 2x(t)$ ;  $x(0) = 2$ ,  $y(0) = 3$

i)  $x'(t) = 3x(t) - y(t)$

$$x''(t) = 3x'(t) - y'(t)$$

$$x''(t) = 3x'(t) - 2x(t)$$

$$x''(t) - 3x'(t) + 2x(t) = 0$$

$$r^2 - 3r + 2 = 0$$

$$(r-2)(r-1) = 0 \Rightarrow r = 2, 1$$

$$x(t) = Ae^{2t} + Be^t \quad y'(t) = 2Ae^{2t} + 2Be^t$$

$$2 = A + B \quad y(t) = Ae^{2t} + 2Be^t$$

$$2 = A + 1 \quad 3 = A + 2B$$

$$A = 1 \quad \underline{-(2 = A + B)}$$

$$1 = B$$

$$x(t) = e^{2t} + e^t, \quad y(t) = e^{2t} + 2e^t$$

ii)  $x'(t) = 3x(t) - y(t)$ ,  $y'(t) = 2x(t) + 0y(t)$

$$X(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

$$X'(t) = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix} X(t) \quad X(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$(3-\lambda)(-\lambda) + 2 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$(\lambda-2)(\lambda-1) = 0 \quad \lambda = 1, 2$$

$$v_1 = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} 3-1 & -1 \\ 2 & 0-1 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2a_1 - b_1 = 0$$

$$2a_1 = b_1 \Rightarrow v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3-2 & -1 \\ 2 & 0-2 \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$a_2 - b_2 = 0 \Rightarrow a_2 = b_2 \Rightarrow v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$X(t) = A \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + B \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^t$$

$$\begin{bmatrix} A + 2B \\ A + B \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$A + 2B = 2$$

$$\underline{-(A + B = 3)}$$

$$B = -1$$

$$2 \quad x'(t) = x(t) + 8y(t), \quad y'(t) = 5x(t); \quad x(0) = 5, \quad y(0) = 2$$

$$i) \quad x''(t) = x'(t) + 8y'(t)$$

$$x''(t) = x'(t) + 40x(t)$$

$$r^2 - r - 40 = 0$$

$$r = \frac{1 \pm \sqrt{1+160}}{2} = \frac{1 \pm \sqrt{161}}{2} = \frac{1}{2} \pm \frac{\sqrt{161}}{2} \Rightarrow r = 6.844, -5.844$$

$$x(t) = A e^{6.844t} + B e^{-5.844t} \quad y(t) = \frac{5}{6.844} A e^{6.844t} - \frac{5}{5.844} B e^{-5.844t}$$

$$5 = A + B$$

$$2 = 0.73056A - 0.8555B$$

$$B = 5 - A$$

$$2 = 0.73056A - 0.8555(5 - A)$$

$$B = 5 - 3.9579$$

$$6.27789 = 1.586145A$$

$$B = 1.04204$$

$$A = 3.9579$$

$$x(t) = 3.9579 e^{6.844t} + 1.04204 e^{-5.844t}, \quad y(t) = 2.8915 e^{6.844t} - 0.8915 e^{-5.844t}$$

$$ii) \quad X'(t) = \begin{bmatrix} 1 & 8 \\ 5 & 0 \end{bmatrix} X(t), \quad X(0) = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\lambda = 6.844, -5.844$$

$$\text{Eigenvectors: } \begin{bmatrix} 1.3688 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1.1688 \\ 1 \\ 1 \end{bmatrix}$$

$$X(t) = A \begin{bmatrix} 1.3688 \\ 1 \\ 1 \end{bmatrix} e^{6.844t} + B \begin{bmatrix} -1.1688 \\ 1 \\ 1 \end{bmatrix} e^{-5.844t}$$

$$5 = 1.3688A - 1.1688B \quad 2 = A + B$$

$$5 = 1.3688A - (1.1688)(2 - A)$$

$$A = 2.89155 \quad B = -0.89155$$

$$X(t) = 2.89155 \begin{bmatrix} 1.3688 \\ 1 \\ 1 \end{bmatrix} e^{6.844t} - 0.89155 \begin{bmatrix} -1.1688 \\ 1 \\ 1 \end{bmatrix} e^{-5.844t}$$

$$x(t) = 3.9579 e^{6.844t} + 1.042 e^{-5.844t}$$

$$y(t) = 2.89155 e^{6.844t} - 0.89155 e^{-5.844t}$$

$$3. \quad x_1'(t) = x_1(t) + x_2(t) + x_3(t) \quad x_1(0) = 1$$

$$x_2'(t) = x_1(t) + x_2(t) \quad x_2(0) = 2$$

$$x_3'(t) = x_1(t) \quad x_3(0) = -1$$

$$X'(t) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\text{Eigenvectors: } \begin{bmatrix} 2.24697 + 1.5146 \times 10^{-9}i \\ 1.80193 + 1.88876 \times 10^{-9}i \\ 1 \end{bmatrix}, \begin{bmatrix} -0.8019377 + 3.68630 \times 10^{-10}i \\ 0.445041 - 5.94799 \times 10^{-10}i \\ 1 \end{bmatrix}, \begin{bmatrix} 0.55495 - 2.25455 \times 10^{-11}i \\ -1.246979 + 5.80945 \times 10^{-11}i \\ 1 \end{bmatrix}$$

$$1 = (2.24697 + 1.5146 \times 10^{-9}i)A + (-0.8019377 + 3.68630 \times 10^{-10}i)B + (0.55495 - 2.25455 \times 10^{-11}i)C$$