HW 17

OK to POST

$x'(t) = 3x(t) - y(t)$, $y'(t) = 2x(t)$; $x(0)=2$, $y(0)=3$

i) $x''(t) = 3 - y'(t)$

$x''(t) = 3 - 2x(t)$

$x''(t) + 2x(t) - 3 = 0$

$r^2 + 2 - 3 = 0$

$r^2 - 1 = 0$

$x(t) = e^t + e^{2t}$ $\quad$ $y(t) = 2e^t + e^{2t}$

ii) $x'(t) = 3 \cdot x(t) - (1)\, y(t)$

$y'(t) = 2 \cdot x(t) - 0\, y(t)$

$$z(t) := \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

$$z'(t) = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix} z(t) \quad , \; z(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Eigen values $2, 1$

Eigen vectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, Respectively

So, $x(t) = A\begin{bmatrix}1\\1\end{bmatrix}e^{2t} + B\begin{bmatrix}1\\2\end{bmatrix}e^{t}$

$$x(0) = \begin{bmatrix}2\\3\end{bmatrix}$$

So, $\begin{bmatrix}A+B\\A+2B\end{bmatrix}\begin{bmatrix}Q\\3\end{bmatrix}$

$$A + B = 2$$
$$A + 2B = 3$$

$$A = 2 - B$$
$$2 - B + 2B = 3$$
$$2 - B = 3$$
$$-B = 1$$

$$\boxed{\begin{aligned} B &= -1 \\ A &= 3 \end{aligned}}$$

$$x(t) = 3\begin{bmatrix}1\\1\end{bmatrix}e^{2t} + (-1)\begin{bmatrix}1\\2\end{bmatrix}e^{t}$$

iii) * Maple code *

2) $x'(t) = 1(x(t)) + 8(y(t))$

$y'(t) = 0 - 1(y(t))$

$x(0) = 1$

$y(0) = 9$

$x''(t) = x'(t) + 8y'(t)$

$x''(t) = x'(t) + 8y(t)$

$r^2 = r + 8$

$r^2 - r - 8$ $\quad r = \frac{1+\sqrt{33}}{2}, \; r = \frac{1-\sqrt{33}}{2}$

$A e^{1+\sqrt{33}/2t} + B e^{\frac{1-\sqrt{33}}{2}t}$

$1 = A + B$

$9 = A - B$

ii) $z'(t) = 1z(t) + 8y(t)$
$y'(t) = 0 - y(t)$

$z(0) = \begin{bmatrix} 1 \\ 9 \end{bmatrix}$

$z'(t) = \begin{bmatrix} 1 & 8 \\ 0 & -1 \end{bmatrix}$ $\quad z = 1, -1$

Eigenvectors : $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

So, $A\begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{t} + B\begin{bmatrix} -4 \\ 1 \end{bmatrix} e^{-t}$

$A - 4B = 1$
$B = 9$
$A - 36 = 1$
$A = 37$

so $37\begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{t} + 9\begin{bmatrix} -4 \\ 1 \end{bmatrix} e^{t}$

iii) Maple code

3) $x'(t) = x_1(t) + x_2(t) + x_3(t)$

$x_2'(t) = x_1(t) + x_2(t)$

$x_3'(t) = x_1(t)$

$$x'(t) = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$$

$$x(0) = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Eigen vectors and Eigen values in Maple

$$x'(t) = A v_1 e^{r_1 t} + B v_2 e^{r_2 t} + C v_3 e^{r_3 t}$$

These values are large so they have been computed with maple