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> # Max Mekhanikov - HW 17 - [redacted]
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> # Question 1
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> dsolve({diff(x(t),t) = 3·x(t) - y(t), diff(y(t),t) = 2·x(t), x(0) = 2, y(0) = 3}, {x(t), y(t)})
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$$\{x(t) = e^{t} + e^{2t},\ y(t) = 2e^{t} + e^{2t}\} \tag{1}$$

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> # Question 2
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> dsolve({diff(x(t),t) = x(t) + 8·y(t), diff(y(t),t) = 4·x(t), x(0) = 3, y(0) = 9}, {x(t), y(t)})
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$$\left\{x(t) = \left(\frac{\sqrt{129}}{8} + \frac{1}{8}\right)\left(\frac{9}{2} + \frac{5\sqrt{129}}{86}\right)e^{\frac{(1+\sqrt{129})t}{2}} + \left(-\frac{\sqrt{129}}{8} + \frac{1}{8}\right)\left(\frac{9}{2} - \frac{5\sqrt{129}}{86}\right)e^{-\frac{(-1+\sqrt{129})t}{2}},\ y(t) = \left(\frac{9}{2} + \frac{5\sqrt{129}}{86}\right)e^{\frac{(1+\sqrt{129})t}{2}} + \left(\frac{9}{2} - \frac{5\sqrt{129}}{86}\right)e^{-\frac{(-1+\sqrt{129})t}{2}}\right\} \tag{2}$$

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> # Question 3
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> A := <<1, 1, 1>|<1, 1, 0>|<1, 0, 0>>
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$$A := \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{3}$$

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> with(LinearAlgebra):
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> Eigenvalues(A)
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$$\left[\left[\frac{(28 + 84\,I\sqrt{3})^{1/3}}{6} + \frac{14}{3(28 + 84\,I\sqrt{3})^{1/3}} + \frac{2}{3}\right],\ \left[-\frac{(28 + 84\,I\sqrt{3})^{1/3}}{12} - \frac{7}{3(28 + 84\,I\sqrt{3})^{1/3}} + \frac{2}{3} + \frac{I\sqrt{3}\left(\frac{(28 + 84\,I\sqrt{3})^{1/3}}{6} - \frac{14}{3(28 + 84\,I\sqrt{3})^{1/3}}\right)}{2}\right],\ \left[-\frac{(28 + 84\,I\sqrt{3})^{1/3}}{12} - \frac{7}{3(28 + 84\,I\sqrt{3})^{1/3}} + \frac{2}{3}\right] \tag{4}$$

$$-\frac{I\sqrt{3}\left(\frac{(28+84I\sqrt{3})^{1/3}}{6}-\frac{14}{3(28+84I\sqrt{3})^{1/3}}\right)}{2}\Bigg]\Bigg]$$

> *Eigenvectors*(A)

$$\left[\left[\frac{(28+84I\sqrt{3})^{1/3}}{6}+\frac{14}{3(28+84I\sqrt{3})^{1/3}}+\frac{2}{3}\right],\right.$$

$$\left[-\frac{(28+84I\sqrt{3})^{1/3}}{12}-\frac{7}{3(28+84I\sqrt{3})^{1/3}}+\frac{2}{3}\right.$$

$$\left.+\frac{I\sqrt{3}\left(\frac{(28+84I\sqrt{3})^{1/3}}{6}-\frac{14}{3(28+84I\sqrt{3})^{1/3}}\right)}{2}\right],$$

$$\left[-\frac{(28+84I\sqrt{3})^{1/3}}{12}-\frac{7}{3(28+84I\sqrt{3})^{1/3}}+\frac{2}{3}\right.$$

$$\left.\left.-\frac{I\sqrt{3}\left(\frac{(28+84I\sqrt{3})^{1/3}}{6}-\frac{14}{3(28+84I\sqrt{3})^{1/3}}\right)}{2}\right]\right],\left[\left[-1\Big/\right.\right.$$

$$\left.\left.\left(\left(\frac{(28+84I\sqrt{3})^{1/3}}{6}+\frac{14}{3(28+84I\sqrt{3})^{1/3}}+\frac{2}{3}\right)^2-\frac{(28+84I\sqrt{3})^{1/3}}{3}\right.\right.$$

(5)

$$-\frac{28}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}-\frac{7}{3}\Bigg),-1\Bigg/\Bigg(\Bigg(\Bigg(-\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{12}$$

$$-\frac{7}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$

$$+\frac{\mathrm{I}\sqrt{3}\left(\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}$$

$$+\frac{14}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}-\frac{7}{3}-\mathrm{I}\sqrt{3}\left(\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}\right.$$

$$\left.-\frac{14}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}\right)\Bigg),-1\Bigg/\Bigg(\Bigg(\Bigg(-\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{12}$$

$$-\frac{7}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$

$$-\frac{\mathrm{I}\sqrt{3}\left(\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}$$

$$+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{7}{3}+I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}\right.$$

$$\left.\left.-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)\right)\Bigg],$$

$$\left[-\left(\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}\right)^2\right.\right.$$

$$\left.\left.-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{8}{3}\right)\Bigg/\right.$$

$$\left(\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}\right)^2-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{3}\right.$$

$$\left.-\frac{28}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{7}{3}\right),-\left(\left(\left(-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}\right.\right.\right.$$

$$-\frac{7}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$

$$+\frac{\mathrm{I}\sqrt{3}\left(\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^{2}+\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{12}$$

$$+\frac{7}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}-\frac{8}{3}$$

$$-\frac{\mathrm{I}\sqrt{3}\left(\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)\Big/\Big(\Big(\Big($$

$$-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$

$$+\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}$$

$$+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{7}{3}-I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}\right.$$

$$\left.-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)\Bigg),-\Bigg(\Bigg(-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}$$

$$-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$
$$-\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}$$

$$+\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{8}{3}$$
$$+\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)\Bigg/\Bigg(\Bigg($$

$$-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$
$$-\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}$$
$$+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{7}{3}+I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}\right.$$
$$\left.\left.\left.-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)\right)\right],$$
$$\left[\begin{matrix}1,\,1,\,1\end{matrix}\right]\Bigg]$$

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> dsolve({diff(x(t),t) = x(t)+y(t)+z(t), diff(y(t),t) = x(t)+y(t), diff(z(t),t) = x(t),
    x(0) = 1, y(0) = 2, z(0) = -1}, {x(t), y(t), z(t)})
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$$\left\{x(t)=\left(\left(-105670656+76608\left(28+84\,I\sqrt{3}\right)^{4/3}-2585856\left(28+84\,I\sqrt{3}\right)^{2/3}\right.\right.$$

$$-24192\,I\left(28+84\,I\sqrt{3}\right)^{4/3}\sqrt{3}+338688\,I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{1/3}$$

$$+18\,I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{8/3}+1008\,I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{5/3}+24385536\,I\sqrt{3}$$

$$-193536\,I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{2/3}+6\left(28+84\,I\sqrt{3}\right)^{8/3}-4368\left(28+84\,I\sqrt{3}\right)^{5/3}$$

$$+8015616\left(28+84\,I\sqrt{3}\right)^{1/3}\Bigg)$$

$$e^{-\frac{\left(I\left(28+84\,I\sqrt{3}\right)^{2/3}\sqrt{3}+\left(28+84\,I\sqrt{3}\right)^{2/3}-28\,I\sqrt{3}-8\left(28+84\,I\sqrt{3}\right)^{1/3}+28\right)t}{12\left(28+84\,I\sqrt{3}\right)^{1/3}}}\Bigg)\Bigg/$$

$$\left(3\left(I\left(28+84\,I\sqrt{3}\right)^{4/3}\sqrt{3}+56\,I\left(28+84\,I\sqrt{3}\right)^{1/3}\sqrt{3}-840\,I\sqrt{3}\right.\right.$$

$$\left.+504\right)\left(18\left(28+84\,I\sqrt{3}\right)^{4/3}-2\,I\left(28+84\,I\sqrt{3}\right)^{4/3}\sqrt{3}+1568\,I\left(28\right.\right.$$

$$\left.\left.+84\,I\sqrt{3}\right)^{1/3}\sqrt{3}\right)\right)+\Bigg(\Bigg(24192\,I\left(28+84\,I\sqrt{3}\right)^{4/3}\sqrt{3}+6\left(28+84\,I\sqrt{3}\right)^{8/3}$$

$$+3048192\,I\left(28+84\,I\sqrt{3}\right)^{1/3}\sqrt{3}+18\,I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{8/3}-20160\left(28\right.$$

$$+ 84\,I\sqrt{3}\,\Big)^{4/3} + 1008\,I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{5/3} - 4368\,\left(28 + 84\,I\sqrt{3}\right)^{5/3}$$

$$- 24385536\,I\sqrt{3} + 1257984\,I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{2/3} + 2596608\,\left(28 + 84\,I\sqrt{3}\right)^{1/3}$$

$$+ 105670656 + 607488\,\left(28 + 84\,I\sqrt{3}\right)^{2/3}\Big)$$

$$e^{\frac{\left(I\left(28 + 84\,I\sqrt{3}\right)^{2/3}\sqrt{3} - \left(28 + 84\,I\sqrt{3}\right)^{2/3} - 28\,I\sqrt{3} + 8\left(28 + 84\,I\sqrt{3}\right)^{1/3} - 28\right)t}{12\left(28 + 84\,I\sqrt{3}\right)^{1/3}}}\Bigg)\Bigg/$$

$$\Big(3\,\Big(I\left(28 + 84\,I\sqrt{3}\right)^{4/3}\sqrt{3} + 56\,I\left(28 + 84\,I\sqrt{3}\right)^{1/3}\sqrt{3} - 840\,I\sqrt{3}$$

$$+ 504\Big)\Big(18\left(28 + 84\,I\sqrt{3}\right)^{4/3} - 2\,I\left(28 + 84\,I\sqrt{3}\right)^{4/3}\sqrt{3} + 1568\,I\,(28$$

$$+ 84\,I\sqrt{3}\,\Big)^{1/3}\sqrt{3}\Big)\Big) + \Bigg(\Big(18\left(28 + 84\,I\sqrt{3}\right)^{4/3} - 2\,I\left(28 + 84\,I\sqrt{3}\right)^{4/3}\sqrt{3}$$

$$+ 16128\,I\sqrt{3} + 288\,I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{2/3} + 1568\,I\left(28 + 84\,I\sqrt{3}\right)^{1/3}\sqrt{3} - 32256$$

$$+ 1440\left(28 + 84\,I\sqrt{3}\right)^{2/3}\Big)\,e^{\frac{\left(\left(28 + 84\,I\sqrt{3}\right)^{2/3} + 4\left(28 + 84\,I\sqrt{3}\right)^{1/3} + 28\right)t}{6\left(28 + 84\,I\sqrt{3}\right)^{1/3}}}\Bigg)\Bigg/$$

$$\left(3\left(18\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-2\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}+1568\,\mathrm{I}\left(28\right.\right.\right.$$

$$\left.\left.\left.+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}\right)\right),\ y(t)=\left(\left(\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}-\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\right.\right.$$

$$+56\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}-48\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}-1008\,\mathrm{I}\sqrt{3}-56\left(28\right.$$

$$\left.+84\,\mathrm{I}\sqrt{3}\right)^{1/3}-336\Big)\left(-105670656+76608\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-2585856\left(28\right.$$

$$+84\,\mathrm{I}\sqrt{3}\Big)^{2/3}-24192\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}+338688\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}$$

$$+18\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{8/3}+1008\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{5/3}+24385536\,\mathrm{I}\sqrt{3}$$

$$-193536\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}+6\left(28+84\,\mathrm{I}\sqrt{3}\right)^{8/3}-4368\left(28+84\,\mathrm{I}\sqrt{3}\right)^{5/3}$$

$$\left.+8015616\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\right)$$

$$\left.\mathrm{e}^{-\frac{\left(\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\sqrt{3}+\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}-28\,\mathrm{I}\sqrt{3}-8\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}+28\right)t}{12\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}}\right)\Big/$$

$$\left(216\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\left(\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}+56\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}\right.\right.$$

$$\left.-840\,\mathrm{I}\sqrt{3}+504\right)\left(18\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-2\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}\right.$$

$$\left.\left.+1568\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}\right)\right)-\left(\left(\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}+\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+56\,\mathrm{I}\left(28\right.\right.\right.$$

$$\left.\left.+1344\right)\left(24192\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}+6\left(28+84\,\mathrm{I}\sqrt{3}\right)^{8/3}+3048192\,\mathrm{I}\left(28\right.\right.\right.$$

$$\left.\left.+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}+18\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{8/3}-20160\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\right.$$

$$\left.+1008\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{5/3}-4368\left(28+84\,\mathrm{I}\sqrt{3}\right)^{5/3}-24385536\,\mathrm{I}\sqrt{3}\right.$$

$$\left.+1257984\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}+2596608\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}+105670656\right.$$

$$\left.+607488\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\right)$$

$$\mathrm{e}^{\frac{\left(\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\sqrt{3}-\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}-28\,\mathrm{I}\sqrt{3}+8\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}-28\right)t}{12\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}}\Bigg)\Bigg/$$

$$\left(216\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\left(\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}+56\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}\right.\right.$$

$$\left.-840\,\mathrm{I}\sqrt{3}+504\right)\left(18\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-2\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}\right.$$

$$\left.+1568\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}\right)\Bigg)+\Bigg(\Bigg(\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-24\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}+168\,\mathrm{I}\sqrt{3}+$$

$$+16128\,\mathrm{I}\sqrt{3}+288\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}+1568\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}-32256$$

$$\left.+1440\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\right)\mathrm{e}^{\frac{\left(\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}+4\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}+28\right)t}{6\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}}\Bigg)\Bigg/$$

$$\left(108\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\left(18\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-2\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}\right.\right.$$

$$\left.\left.+1568\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}\right)\right),\ z(t)=-\Bigg(\Bigg(\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}-\left(28\right.$$

$$\left.+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-112\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}-24\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}-336\,\mathrm{I}\sqrt{3}$$

$$+112\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}-1680\Big)\left(-105670656+76608\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\right.$$

$$- 2585856 \left(28 + 84\,I\sqrt{3}\right)^{2/3} - 24192\,I \left(28 + 84\,I\sqrt{3}\right)^{4/3}\sqrt{3}$$

$$+ 338688\,I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{1/3} + 18\,I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{8/3} + 1008\,I\sqrt{3}\,\left(28$$

$$+ 84\,I\sqrt{3}\right)^{5/3} + 24385536\,I\sqrt{3} - 193536\,I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{2/3} + 6\,\left(28$$

$$+ 84\,I\sqrt{3}\right)^{8/3} - 4368\,\left(28 + 84\,I\sqrt{3}\right)^{5/3} + 8015616\,\left(28 + 84\,I\sqrt{3}\right)^{1/3}\Bigg)$$

$$e^{-\frac{\left(I\left(28 + 84\,I\sqrt{3}\right)^{2/3}\sqrt{3} + \left(28 + 84\,I\sqrt{3}\right)^{2/3} - 28\,I\sqrt{3} - 8\left(28 + 84\,I\sqrt{3}\right)^{1/3} + 28\right)t}{12\left(28 + 84\,I\sqrt{3}\right)^{1/3}}}\Bigg) \Bigg/$$

$$\left(216\left(28 + 84\,I\sqrt{3}\right)^{2/3}\left(I\left(28 + 84\,I\sqrt{3}\right)^{4/3}\sqrt{3} + 56\,I\left(28 + 84\,I\sqrt{3}\right)^{1/3}\sqrt{3}\right.\right.$$

$$- 840\,I\sqrt{3} + 504\Big)\left(18\left(28 + 84\,I\sqrt{3}\right)^{4/3} - 2\,I\left(28 + 84\,I\sqrt{3}\right)^{4/3}\sqrt{3}\right.$$

$$\left.\left. + 1568\,I\left(28 + 84\,I\sqrt{3}\right)^{1/3}\sqrt{3}\right)\right) + \Bigg(\Big(I\left(28 + 84\,I\sqrt{3}\right)^{4/3}\sqrt{3} + \left(28 + 84\,I\sqrt{3}\right)^{4/3} - 112\,I\,(28$$

$$- 336\Big)\left(24192\,I\left(28 + 84\,I\sqrt{3}\right)^{4/3}\sqrt{3} + 6\left(28 + 84\,I\sqrt{3}\right)^{8/3} + 3048192\,I\,(28\right.$$

$$+ 84\,I\sqrt{3}\,\Big)^{1/3}\sqrt{3} + 18\,I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{8/3} - 20160\left(28 + 84\,I\sqrt{3}\right)^{4/3}$$

$$+ 1008\,I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{5/3} - 4368\left(28 + 84\,I\sqrt{3}\right)^{5/3} - 24385536\,I\sqrt{3}$$

$$+ 1257984\,I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{2/3} + 2596608\left(28 + 84\,I\sqrt{3}\right)^{1/3} + 105670656$$

$$+ 607488\left(28 + 84\,I\sqrt{3}\right)^{2/3}\Big)$$

$$e^{\frac{\left(I\left(28 + 84\,I\sqrt{3}\right)^{2/3}\sqrt{3} - \left(28 + 84\,I\sqrt{3}\right)^{2/3} - 28\,I\sqrt{3} + 8\left(28 + 84\,I\sqrt{3}\right)^{1/3} - 28\right)t}{12\left(28 + 84\,I\sqrt{3}\right)^{1/3}}}\Bigg) \Big/$$

$$\Big(216\left(28 + 84\,I\sqrt{3}\right)^{2/3}\Big(I\left(28 + 84\,I\sqrt{3}\right)^{4/3}\sqrt{3} + 56\,I\left(28 + 84\,I\sqrt{3}\right)^{1/3}\sqrt{3}$$

$$- 840\,I\sqrt{3} + 504\Big)\Big(18\left(28 + 84\,I\sqrt{3}\right)^{4/3} - 2\,I\left(28 + 84\,I\sqrt{3}\right)^{4/3}\sqrt{3}$$

$$+ 1568\,I\left(28 + 84\,I\sqrt{3}\right)^{1/3}\sqrt{3}\Big)\Big) - \Bigg(\Big(\left(28 + 84\,I\sqrt{3}\right)^{4/3} - 12\left(28 + 84\,I\sqrt{3}\right)^{2/3} - 336\,I\sqrt{3} -$$

$$+ 16128\,I\sqrt{3} + 288\,I\sqrt{3}\left(28 + 84\,I\sqrt{3}\right)^{2/3} + 1568\,I\left(28 + 84\,I\sqrt{3}\right)^{1/3}\sqrt{3} - 32256$$

$$+ 1440\left(28 + 84\,I\sqrt{3}\right)^{2/3}\Big)\,e^{\frac{\left(\left(28 + 84\,I\sqrt{3}\right)^{2/3} + 4\left(28 + 84\,I\sqrt{3}\right)^{1/3} + 28\right)t}{6\left(28 + 84\,I\sqrt{3}\right)^{1/3}}}\Bigg) \Big/$$

$$\Big(108\left(28 + 84\,I\sqrt{3}\right)^{2/3}\Big(18\left(28 + 84\,I\sqrt{3}\right)^{4/3} - 2\,I\left(28 + 84\,I\sqrt{3}\right)^{4/3}\sqrt{3}$$

$$+ 1568\,I\left(28 + 84\,I\sqrt{3}\right)^{1/3}\sqrt{3}\Big)\Big)\Big\}$$

```
> # Question 4
> M := Matrix([[1, 1, 1], [1, 1, 1], [1, 1, 1]])
```

$$M := \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{7}$$

```
> RT2 := proc(x, y, d, K) local ra, i, j, f, g :
  ra := rand(1 ..K) : #random integer from -K to K
  f := add(add(ra( ) *x^i*y^j, j = 0 ..d-i), i = 0 ..d) / add(add(ra( ) *x^i*y^j, j = 0 ..d-i), i
       = 0 ..d) :
  g := add(add(ra( ) *x^i*y^j, j = 0 ..d-i), i = 0 ..d) / add(add(ra( ) *x^i*y^j, j = 0 ..d-i), i
       = 0 ..d) :
  [f, g] :
   end:

  Orb2 := proc(F, x, y, pt0, K1, K2)  local pt, L, i :
  pt := pt0 :

  for i from 1 to K1-1 do
  pt := subs( {x = pt[1], y = pt[2]}, F) :
  od:

  L := [ ] :
  for i from K1 to K2 do
  L := [op(L), pt] :
  pt := normal(subs( {x = pt[1], y = pt[2]}, F) ) :

  od:
  L :
  end:
> HW3g := proc(u, v, w, M) local tot, LI :
  LI := [

  M[1][1]*u^2 + (M[1][2] + M[2][1]) / 2*u*v + M[2][2]* (1 / 4) *v^2,

  (M[1][2] + M[2][1]) / 2*u*v + (M[1][3] + M[3][1]) *u*w + M[2][2] / 2*v^2
       + (M[2][3] + M[3][2]) / 2*v*w,

  M[2][2]*1 / 4*v^2 + (M[2][3] + M[3][2]) / 2*v*w  + M[3][3]*w^2] :
  tot := LI[1] + LI[2] + LI[3] :
  [LI[1] / tot, LI[2] / tot, LI[3] / tot] :
   end:


  HW2g := proc(u, v, M) local LI, w :
  LI := HW3g(u, v, w, M) :
  normal(subs(w = 1-u-v, [LI[1], LI[2]]) ) :
   end:
```

```
>
```

(8)

```
> HW3g(1, 2, 3, M)
```

$$\left[\frac{1}{9}, \frac{4}{9}, \frac{4}{9}\right] \tag{9}$$

```
> RandomMatrix(3)
```

$$\begin{bmatrix} -45 & -43 & 19 \\ 68 & -85 & 25 \\ 58 & -85 & 17 \end{bmatrix} \tag{10}$$

```
> HW3g(1, 2, 3, RandomMatrix(3))
```

$$\left[-\frac{11}{387}, -\frac{340}{387}, \frac{82}{43}\right] \tag{11}$$

```
> HW3g(1, 2, 3, RandomMatrix(3))
```

$$\left[\frac{53}{200}, \frac{24}{25}, -\frac{9}{40}\right] \tag{12}$$

```
> HW3g(1, 2, 3, RandomMatrix(3))
```

$$\left[\frac{1}{126}, \frac{61}{72}, \frac{73}{504}\right] \tag{13}$$

```
> HW3g(1, 2, 3, RandomMatrix(3))
```

$$\left[\frac{65}{302}, \frac{35}{302}, \frac{101}{151}\right] \tag{14}$$

```
> HW3g(1, 2, 3, RandomMatrix(3))
```

$$\left[-\frac{92}{803}, \frac{166}{803}, \frac{729}{803}\right] \tag{15}$$

```
> HW3g(1, 2, 3, RandomMatrix(3))
```

$$\left[-\frac{1}{105}, \frac{277}{210}, -\frac{13}{42}\right] \tag{16}$$

```
> HW3g(1, 2, 3, RandomMatrix(3))
```

$$\left[\frac{36}{475}, \frac{113}{1425}, \frac{1204}{1425}\right] \tag{17}$$

```
> HW3g(1, 2, 3, RandomMatrix(3))
```

$$\left[\frac{44}{125}, \frac{131}{125}, -\frac{2}{5}\right] \tag{18}$$

```
> HW3g(1, 2, 3, RandomMatrix(3))
```

$$\left[-\frac{5}{194}, -\frac{31}{97}, \frac{261}{194}\right] \tag{19}$$

```
> HW3g(1, 2, 3, RandomMatrix(3))
```

$$\left[\frac{143}{1077}, \frac{797}{1077}, \frac{137}{1077}\right] \tag{20}$$

```
> HW3g(1, 2, 3, RandomMatrix(3))
```

$$\left[\frac{31}{624}, \frac{51}{416}, \frac{1033}{1248}\right] \quad \textbf{(21)}$$

```
>
```

$\lambda = 2$:

$$\begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$a = b$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x(t) = A\begin{bmatrix} 1 \\ 2 \end{bmatrix}e^{t} + B\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{2t}$$

$$x(0) = A\begin{bmatrix} 1 \\ 2 \end{bmatrix} + B\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} A+B \\ 2A+B \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$A + B = 2$$
$$2A + B = 3$$

$A = 1, B = 2$

$$x(t) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}e^{t} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{2t}$$

$$\vec{x}(t) = \begin{bmatrix} e^{t} + e^{2t} \\ 2e^{t} + e^{2t} \end{bmatrix}, \quad x(t) = e^{t} + e^{2t}, \; y(t) = 2e^{t} + e^{2t}$$

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$$x'(t) = x(t) + 8y(t)$$
$$y'(t) = 4x(t)$$
$$x(0) = 3, \quad y(0) = 9$$

$$x'(t) = \begin{bmatrix} 1 & 8 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

$$(1-\lambda)-\lambda - 32 = 0$$
$$\lambda^2 - \lambda - 32 = 0$$
$$\lambda = \frac{1 \pm \sqrt{129}}{2}$$

$$\lambda = \frac{1+\sqrt{129}}{2} : \begin{bmatrix} 1 + \sqrt{129} \\ 8 \end{bmatrix}$$

$$\lambda = \frac{1-\sqrt{129}}{2} : \begin{bmatrix} 1 - \sqrt{129} \\ 8 \end{bmatrix}$$

rest in Maple using dsolve

3) $x'_1(t) = x_1(t) + x_2(t) + x_3(t)$

$x'_2(t) = x_1(t) + x_2(t)$

$x'_3(t) = x_1(t)$

$x_1(0) = 1$, $x_2(0) = 2$, $x_3(0) = -1$

$$x'(t) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$$X(0) = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$