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HW 17

$y(t) = -x'(t) + 3x(t)$

1. a. $x'(t) = 3x(t) - y(t)$ $\quad x(0) = 2$ $\quad x'(0) = 3(2) - 3 = 3$

$y'(t) = 2x(t)$ $\quad y(0) = 3$

$x''(t) = 3x'(t) - y'(t)$

$x''(t) = 3(3x(t) - y(t)) - 2x(t)$

$x''(t) = 7x(t) - 3y(t)$

$x''(t) = 7x(t) - 3(-x'(t) + 3x(t))$

$x''(t) = 7x(t) + 3x'(t) - 9x(t)$

$x''(t) = 3x'(t) - 2x(t)$

$0 = x''(t) - 3x'(t) + 2x(t)$

$0 = r^2 - 3r + 2$

$r = 2, 1$

$x(t) = Ae^{2t} + Be^{t}$ $\quad x'(t) = 2Ae^{2t} + Be^{t}$

$2 = A + B$ $\quad 3 = 2A + B$

$A = 1$

$B = 1$

$x(t) = e^{2t} + e^{t}$ $\quad x'(t) = 2e^{2t} + e^{t}$

$y(t) = -2e^{2t} - e^{t} + 3e^{2t} + 3e^{t}$ $\quad y'(t) = 2e^{2t} + 2e^{t}$

$y(t) = e^{2t} + 2e^{t}$

$$x'(t) = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix} x(t)$$

$$x(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$0 = (3-\lambda)(-\lambda) - (-2)$$

$$0 = \lambda^2 - 3\lambda + 2$$

$$\lambda = 2, 1$$

$$v_1 = \begin{bmatrix} a \\ b \end{bmatrix}$$

$\lambda = 2$:
$$\begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$a - b = 0$
$a = b$
$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda = 1$:
$$\begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$2a = b$
$v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$$x(t) = A\begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + B\begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t}$$

$$x(0) = A\begin{bmatrix} 1 \\ 1 \end{bmatrix} + B\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} A+2B \\ A+2B \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$A + B = 2$
$A + 2B = 3$

$B = 1$
$A = 1$

$$x(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t}$$

$$x(t) = e^{2t} + e^{t}$$

$$y(t) = e^{2t} + 2e^{t}$$

2. $x'(t) = x(t) + 8y(t)$ $\quad x(0) = 4$ $\quad x'(0) = 20$

$y'(t) = 4x(t)$ $\quad y(0) = 2$

$x''(t) = x'(t) + 8y'(t)$

$x''(t) = x'(t) + 32x(t)$

$0 = x''(t) - x'(t) - 32x(t)$

$0 = r^2 - r - 32$

$r = \frac{1}{2} \pm \frac{\sqrt{129}}{2}$

$x(t) = Ae^{(\frac{1}{2} + \frac{\sqrt{129}}{2})t} + Be^{(\frac{1}{2} - \frac{\sqrt{129}}{2})t}$

$x'(t) = (\frac{1}{2} + \frac{\sqrt{129}}{2})Ae^{(\frac{1}{2} + \frac{\sqrt{129}}{2})t} + (\frac{1}{2} - \frac{\sqrt{129}}{2})Be^{(\frac{1}{2} - \frac{\sqrt{129}}{2})t}$

$4 = A + B$ $\quad 20 = (\frac{1}{2} + \frac{\sqrt{129}}{2})A + (\frac{1}{2} - \frac{\sqrt{129}}{2})B$

$A = 4 - \frac{2\sqrt{129} - 18}{\sqrt{129}}$

$B = \frac{2\sqrt{129} - 18}{\sqrt{129}}$

3. $x_1'(t) = x_1(t) + x_2(t) + x_3(t)$ $\quad x_1(0) = 1$

$x_2'(t) = x_1(t) + x_2(t)$ $\quad x_2(0) = 2$

$x_3'(t) = x_1(t)$ $\quad x_3(0) = -1$

$$x_1'(t) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} x_1(t)$$

$$x_1(0) = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$\lambda =$ crazy numbers according to maple

```
> #1 iii
  dsolve( {diff(x(t), t) = 3·x(t) − y(t), diff(y(t), t) = 2·x(t), x(0) = 2, y(0) = 3}, {x(t), y(t)});
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$$\{x(t) = e^{2t} + e^{t}, y(t) = e^{2t} + 2\,e^{t}\} \tag{1}$$

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> #2
  dsolve( {diff(x(t), t) = x(t) + 8· y(t), diff(y(t), t) = 4·x(t), x(0) = 4, y(0) = 2}, {x(t), y(t)});
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$$\left\{x(t) = \left(\frac{\sqrt{129}}{8} + \frac{1}{8}\right)\left(1 + \frac{5\sqrt{129}}{43}\right) e^{\frac{\left(1+\sqrt{129}\right)t}{2}} + \left(-\frac{\sqrt{129}}{8} + \frac{1}{8}\right)\left(1 - \frac{5\sqrt{129}}{43}\right) e^{-\frac{\left(-1+\sqrt{129}\right)t}{2}}, y(t) = \left(1 + \frac{5\sqrt{129}}{43}\right) e^{\frac{\left(1+\sqrt{129}\right)t}{2}} + \left(1 - \frac{5\sqrt{129}}{43}\right) e^{-\frac{\left(-1+\sqrt{129}\right)t}{2}}\right\} \tag{2}$$

```
> #3
  with(LinearAlgebra) :
  Eigenvalues(Matrix([[1, 1, 1], [1, 1, 0], [1, 0, 0]]));
```

$$\left[\left[\frac{\left(28 + 84\,I\sqrt{3}\right)^{1/3}}{6} + \frac{14}{3\left(28 + 84\,I\sqrt{3}\right)^{1/3}} + \frac{2}{3}\right], \left[-\frac{\left(28 + 84\,I\sqrt{3}\right)^{1/3}}{12} - \frac{7}{3\left(28 + 84\,I\sqrt{3}\right)^{1/3}} + \frac{2}{3} + \frac{I\sqrt{3}\left(\frac{\left(28 + 84\,I\sqrt{3}\right)^{1/3}}{6} - \frac{14}{3\left(28 + 84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\right], \left[-\frac{\left(28 + 84\,I\sqrt{3}\right)^{1/3}}{12} - \frac{7}{3\left(28 + 84\,I\sqrt{3}\right)^{1/3}} + \frac{2}{3}\right] \tag{3}$$

$$-\frac{I\sqrt{3}\left(\frac{(28+84I\sqrt{3})^{1/3}}{6}-\frac{14}{3(28+84I\sqrt{3})^{1/3}}\right)}{2}\Bigg]\Bigg]$$

```
> dsolve({diff(x(t),t)=x(t)+y(t)+z(t), diff(y(t),t)=x(t)+y(t), diff(z(t),t)=x(t), x(0)
      =1, y(0)=2, z(0)=-1}, {x(t), y(t), z(t)});
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$$\left\{x(t)=-\left(\left(-105670656+18\,I\left(28+84\,I\sqrt{3}\right)^{8/3}\sqrt{3}+1008\,I\left(28+84\,I\sqrt{3}\right)^{5/3}\sqrt{3}\right.\right.$$

$$+6\left(28+84\,I\sqrt{3}\right)^{8/3}+76608\left(28+84\,I\sqrt{3}\right)^{4/3}-4368\left(28+84\,I\sqrt{3}\right)^{5/3}$$

$$+8015616\left(28+84\,I\sqrt{3}\right)^{1/3}-2585856\left(28+84\,I\sqrt{3}\right)^{2/3}+338688\,I\sqrt{3}\left(28\right.$$

$$\left.+84\,I\sqrt{3}\right)^{1/3}-24192\,I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{4/3}+24385536\,I\sqrt{3}-193536\,I\left(28\right.$$

$$\left.+84\,I\sqrt{3}\right)^{2/3}\sqrt{3}\right)$$

$$e^{-\frac{\left(I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{2/3}-28\,I\sqrt{3}+\left(28+84\,I\sqrt{3}\right)^{2/3}-8\left(28+84\,I\sqrt{3}\right)^{1/3}+28\right)t}{12\left(28+84\,I\sqrt{3}\right)^{1/3}}}\Bigg)\Bigg/$$

$$\left(3\left(I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{4/3}+56\,I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{1/3}-840\,I\sqrt{3}+504\right)\left(\right.\right.$$

$$\left.\left.-18\left(28+84\,I\sqrt{3}\right)^{4/3}+2\,I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{4/3}-1568\,I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{1/3}\right)\right)$$

$$-\left(\left(105670656+18\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{8/3}\sqrt{3}+1008\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{5/3}\sqrt{3}\right.\right.$$

$$+6\left(28+84\,\mathrm{I}\sqrt{3}\right)^{8/3}-20160\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-4368\left(28+84\,\mathrm{I}\sqrt{3}\right)^{5/3}$$

$$+2596608\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}+607488\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}+3048192\,\mathrm{I}\left(28\right.$$

$$\left.+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}+24192\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-24385536\,\mathrm{I}\sqrt{3}$$

$$\left.+1257984\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\sqrt{3}\right)$$

$$\mathrm{e}^{\frac{\left(\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}-28\,\mathrm{I}\sqrt{3}-\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}+8\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}-28\right)t}{12\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}}\Bigg)\Bigg/$$

$$\left(3\left(\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+56\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}-840\,\mathrm{I}\sqrt{3}+504\right)\left(\right.\right.$$

$$\left.\left.-18\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+2\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-1568\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\right)\right)$$

$$+\left(\left(2\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-18\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-16128\,\mathrm{I}\sqrt{3}-288\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\sqrt{3}\right.\right.$$

$$+ 2\,I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{4/3} - 1568\,I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{1/3}\Big)\Big), y(t) =$$

$$-\Bigg(\Big(I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{4/3} - \left(28 + 84\,I\sqrt{3}\right)^{4/3} + 56\,I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{1/3}$$

$$- 1008\,I\sqrt{3} - 48\,\left(28 + 84\,I\sqrt{3}\right)^{2/3} - 56\,\left(28 + 84\,I\sqrt{3}\right)^{1/3} - 336\Big)\,\Big(-105670656$$

$$+ 18\,I\,\left(28 + 84\,I\sqrt{3}\right)^{8/3}\sqrt{3} + 1008\,I\,\left(28 + 84\,I\sqrt{3}\right)^{5/3}\sqrt{3} + 6\,\left(28 + 84\,I\sqrt{3}\right)^{8/3}$$

$$+ 76608\,\left(28 + 84\,I\sqrt{3}\right)^{4/3} - 4368\,\left(28 + 84\,I\sqrt{3}\right)^{5/3} + 8015616\,\left(28 + 84\,I\sqrt{3}\right)^{1/3}$$

$$- 2585856\,\left(28 + 84\,I\sqrt{3}\right)^{2/3} + 338688\,I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{1/3} - 24192\,I\sqrt{3}\,\left(28$$

$$+ 84\,I\sqrt{3}\right)^{4/3} + 24385536\,I\sqrt{3} - 193536\,I\,\left(28 + 84\,I\sqrt{3}\right)^{2/3}\sqrt{3}\Big)$$

$$e^{-\frac{\left(I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{2/3} - 28\,I\sqrt{3} + \left(28 + 84\,I\sqrt{3}\right)^{2/3} - 8\,\left(28 + 84\,I\sqrt{3}\right)^{1/3} + 28\right)\,t}{12\,\left(28 + 84\,I\sqrt{3}\right)^{1/3}}}\Bigg)\Bigg/$$

$$\Big(216\,\left(28 + 84\,I\sqrt{3}\right)^{2/3}\,\left(I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{4/3} + 56\,I\sqrt{3}\,\left(28 + 84\,I\sqrt{3}\right)^{1/3}\right.$$

$$
\begin{aligned}
&-840\,I\sqrt{3}+504\Big)\Big(-18\left(28+84\,I\sqrt{3}\right)^{4/3}+2\,I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{4/3}\\
&-1568\,I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{1/3}\Big)+\Bigg(\Big(I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{4/3}+\left(28+84\,I\sqrt{3}\right)^{4/3}+56\,I\sqrt{3}\,(2\\
&+1344\Big)\Big(105670656+18\,I\left(28+84\,I\sqrt{3}\right)^{8/3}\sqrt{3}+1008\,I\left(28+84\,I\sqrt{3}\right)^{5/3}\sqrt{3}\\
&+6\left(28+84\,I\sqrt{3}\right)^{8/3}-20160\left(28+84\,I\sqrt{3}\right)^{4/3}-4368\left(28+84\,I\sqrt{3}\right)^{5/3}\\
&+2596608\left(28+84\,I\sqrt{3}\right)^{1/3}+607488\left(28+84\,I\sqrt{3}\right)^{2/3}+3048192\,I\,(28\\
&+84\,I\sqrt{3}\Big)^{1/3}\sqrt{3}+24192\,I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{4/3}-24385536\,I\sqrt{3}\\
&+1257984\,I\left(28+84\,I\sqrt{3}\right)^{2/3}\sqrt{3}\Big)\\
&e^{\frac{\left(I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{2/3}-28\,I\sqrt{3}-\left(28+84\,I\sqrt{3}\right)^{2/3}+8\left(28+84\,I\sqrt{3}\right)^{1/3}-28\right)t}{12\left(28+84\,I\sqrt{3}\right)^{1/3}}}\Bigg)\Bigg/\\
&\Big(216\,\left(28+84\,I\sqrt{3}\right)^{2/3}\Big(I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{4/3}+56\,I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{1/3}
\end{aligned}
$$

$$-840\,\mathrm{I}\sqrt{3}+504\Big)\Big(-18\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+2\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}$$

$$-1568\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\Big)\Big)+\Bigg(\Big(\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+168\,\mathrm{I}\sqrt{3}-24\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}+5$$

$$-16128\,\mathrm{I}\sqrt{3}-288\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\sqrt{3}+32256-1568\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}$$

$$-1440\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\Big)\,\mathrm{e}^{\frac{\left(\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}+4\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}+28\right)t}{6\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}}\Bigg)\Bigg/$$

$$\Big(108\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\Big(-18\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+2\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}$$

$$-1568\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\Big)\Big),\,z(t)=\Bigg(\Big(\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-\left(28\right.$$

$$\left.+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-112\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}-336\,\mathrm{I}\sqrt{3}-24\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}$$

$$+112\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}-1680\Big)\Big(-105670656+18\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{8/3}\sqrt{3}$$

$$+1008\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{5/3}\sqrt{3}+6\left(28+84\,\mathrm{I}\sqrt{3}\right)^{8/3}+76608\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}$$

$$-4368\left(28+84\,\mathrm{I}\sqrt{3}\right)^{5/3}+8015616\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}-2585856\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}$$

$$+338688\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}-24192\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+24385536\,\mathrm{I}\sqrt{3}$$

$$-193536\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\sqrt{3}\Bigg)$$

$$\mathrm{e}^{-\frac{\left(\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}-28\,\mathrm{I}\sqrt{3}+\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}-8\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}+28\right)t}{12\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}}\Bigg)\Bigg/$$

$$\left(216\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\left(\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+56\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\right.\right.$$

$$\left.-840\,\mathrm{I}\sqrt{3}+504\right)\left(-18\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+2\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\right.$$

$$\left.-1568\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\right)\Bigg)-\Bigg(\Bigg(\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-112\,\mathrm{I}\sqrt{3}\,($$

$$-336\Big)\left(105670656+18\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{8/3}\sqrt{3}+1008\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{5/3}\sqrt{3}\right.$$

$$\left.+6\left(28+84\,\mathrm{I}\sqrt{3}\right)^{8/3}-20160\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-4368\left(28+84\,\mathrm{I}\sqrt{3}\right)^{5/3}\right.$$

$$+ 2596608 \left(28 + 84\,I\sqrt{3}\right)^{1/3} + 607488 \left(28 + 84\,I\sqrt{3}\right)^{2/3} + 3048192\,I\,(28$$

$$+ 84\,I\sqrt{3}\Big)^{1/3}\sqrt{3} + 24192\,I\sqrt{3}\left(28 + 84\,I\sqrt{3}\right)^{4/3} - 24385536\,I\sqrt{3}$$

$$+ 1257984\,I\left(28 + 84\,I\sqrt{3}\right)^{2/3}\sqrt{3}\Big)$$

$$e^{\frac{\left(I\sqrt{3}\left(28 + 84\,I\sqrt{3}\right)^{2/3} - 28\,I\sqrt{3} - \left(28 + 84\,I\sqrt{3}\right)^{2/3} + 8\left(28 + 84\,I\sqrt{3}\right)^{1/3} - 28\right)t}{12\left(28 + 84\,I\sqrt{3}\right)^{1/3}}}\Bigg) \Bigg/$$

$$\Big(216\left(28 + 84\,I\sqrt{3}\right)^{2/3}\Big(I\sqrt{3}\left(28 + 84\,I\sqrt{3}\right)^{4/3} + 56\,I\sqrt{3}\left(28 + 84\,I\sqrt{3}\right)^{1/3}$$

$$- 840\,I\sqrt{3} + 504\Big)\Big(-18\left(28 + 84\,I\sqrt{3}\right)^{4/3} + 2\,I\sqrt{3}\left(28 + 84\,I\sqrt{3}\right)^{4/3}$$

$$- 1568\,I\sqrt{3}\left(28 + 84\,I\sqrt{3}\right)^{1/3}\Big)\Big) + \Bigg(\Big(-\left(28 + 84\,I\sqrt{3}\right)^{4/3} + 336\,I\sqrt{3} + 12\left(28 + 84\,I\sqrt{3}\right)^{2/3} +$$

$$- 16128\,I\sqrt{3} - 288\,I\left(28 + 84\,I\sqrt{3}\right)^{2/3}\sqrt{3} + 32256 - 1568\,I\sqrt{3}\left(28 + 84\,I\sqrt{3}\right)^{1/3}$$

$$- 1440\left(28 + 84\,I\sqrt{3}\right)^{2/3}\Big)\, e^{\frac{\left(\left(28 + 84\,I\sqrt{3}\right)^{2/3} + 4\left(28 + 84\,I\sqrt{3}\right)^{1/3} + 28\right)t}{6\left(28 + 84\,I\sqrt{3}\right)^{1/3}}}\Bigg) \Bigg/$$

$$\Big(108\left(28 + 84\,I\sqrt{3}\right)^{2/3}\Big(-18\left(28 + 84\,I\sqrt{3}\right)^{4/3} + 2\,I\sqrt{3}\left(28 + 84\,I\sqrt{3}\right)^{4/3}$$

$$- 1568\,I\sqrt{3}\left(28 + 84\,I\sqrt{3}\right)^{1/3}\Big)\Big)\Big\}$$