Hrudai Bottini Hw 17

1) $x'(t) = 3x(t) - y(t)$ $\quad x(0) = 2$ $\quad x'(0) = 3(2) - 3 = 3$

$y'(t) = 2x(t)$ $\quad y(0) = 3$

i) $x''(t) = 3x'(t) - y'(t)$ $\quad (r-2)(r-1)$

$x''(t) = 3x'(t) - 2x(t)$ $\quad r^2 - 3r + 2 = 0 \quad r = 2, 1$

$x(t) = Ae^{2t} + Be^{t} \Rightarrow 2 = A + B \quad A = 1$

$x'(t) = 2Ae^{2t} + Be^{t} \Rightarrow 3 = 2A + B \quad B = 1$

$\boxed{x(t) = e^{2t} + e^{t}}$

$y(t) = 3x(t) - x'(t) = 3e^{2t} + 3e^{t} - 2e^{2t} - e^{t} = e^{2t} + 2e^{t}$

$\boxed{y(t) = e^{2t} + 2e^{t}}$

ii) $X(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ $\quad x'(t) = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix} x(t)$ $\quad x_0 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\begin{bmatrix} 3-\lambda & -1 \\ 2 & 0-\lambda \end{bmatrix} = (3-\lambda)(-\lambda) + 2 = \lambda^2 - 3\lambda + 2 \quad (\lambda-2)(\lambda-1) \quad \lambda = 1, 2$

$\lambda = 1 \quad \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad 2x - y + 2x - y = 4x - 2y = 0 \quad 4x = 2y \quad 2x = y \quad \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\lambda = 2 \quad \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad x - y + 2x - 2y = 3x - 3y = 0 \quad x = y \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$x(t) = A \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{t} + B \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t}$

$x(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} A+B \\ 2A+B \end{bmatrix} \quad A+B=2 \quad 2A+B=3 \quad A=1, \; B=1$

$X(t) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{t} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t}$

$\boxed{\begin{aligned} x(t) &= e^{t} + e^{2t} \\ y(t) &= 2e^{t} + e^{2t} \end{aligned}}$

iii) Maple

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2) $x'(t) = x(t) + 8y(t)$ $x(0) = 5$ $y(0) = 7$ $x'(0) = 5 + 56 = 61$

$y'(t) = 5x(t)$.

i) $x''(t) = x'(t) + 40x(t)$ $r^2 - r - 40$ $r = \frac{1 \pm \sqrt{1-4(40)}}{2}$

$x(t) = Ae^{6.84t} + Be^{-5.84t}$ $A + B = 5$

$x'(t) = 6.84Ae^{6.84t} + -5.84Be^{-5.84t}$ $6.84A - 5.84B = 61$

$A = 5 + \frac{117\sqrt{161} - 805}{322}$ $B = -\frac{117\sqrt{161} - 805}{322}$

$$y(t) = \frac{43\sqrt{161} + 1127}{322} e^{\frac{1+\sqrt{161}}{2}t} + \frac{1127 - 43\sqrt{161}}{322} e^{\frac{1-\sqrt{161}}{2}t}$$

$$x(t) = 5 + \frac{117\sqrt{161} - 805}{322} e^{\frac{1+\sqrt{161}}{2}t} + -\frac{117\sqrt{161} - 805}{322} e^{\frac{1-\sqrt{161}}{2}t}$$

ii) $X(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ $X'(t) = \begin{bmatrix} 1 & 18 \\ 5 & 0 \end{bmatrix}$ $X(0) = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

$\lambda = \frac{1+\sqrt{161}}{2}, \frac{1-\sqrt{161}}{2}$

$\lambda = \frac{1+\sqrt{161}}{2}$ $\begin{bmatrix} 1-\lambda & 8 \\ 5 & -\lambda \end{bmatrix}$ V_1: $\begin{bmatrix} 1+\sqrt{161} \\ 10 \end{bmatrix}$ V_2: $\begin{bmatrix} 1-\sqrt{161} \\ 10 \end{bmatrix}$

$\lambda = \frac{1-\sqrt{161}}{2}$

$X(t) = AV_1e^{\lambda_1 t} + BV_2e^{\lambda_2 t}$ $5 = A(1+\sqrt{161}) + B(1-\sqrt{161})$

$7 = 10A + 10B$

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \frac{1127 + 43\sqrt{161}}{3220} \begin{bmatrix} 1+\sqrt{161} \\ 10 \end{bmatrix} e^{\frac{1+\sqrt{161}}{2}t} + \frac{1127 - 43\sqrt{161}}{3220} e^{\frac{1-\sqrt{161}}{2}t} \quad \#$$

3) $x_1'(t) = x_1(t) + x_2(t) + x_3(t)$ $\quad x_1(0) = 1 \quad x_2(0) = 2 \quad x_3(0) = -1$

$x_2'(t) = x_1(t) + x_2(t)$

$x_3'(t) = x_1(t)$

ii)

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \qquad X'(t) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad X(0) = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$\lambda_1 \approx 0.55495$; $\lambda_2 \approx -0.80193$; $\lambda_3 \approx 2.24697\ldots$

Eigen vectors : $v_1 = \begin{pmatrix} \lambda_1 \\ -1.24697\ldots \\ 1 \end{pmatrix}$; $v_2 = \begin{pmatrix} \lambda_2 \\ 0.44504\ldots \\ 1 \end{pmatrix}$; $v_3 = \begin{pmatrix} \lambda_3 \\ 1.80193\ldots \\ 1 \end{pmatrix}$

$X(t) = A v_1 e^{\lambda_1 t} + B v_2 e^{\lambda_2 t} + C v_3 e^{\lambda_3 t}$

$$X(0) = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

```
#Hrudai Battini hw17
with(LinearAlgebra);
read "/Users/hb334/Documents/M17.txt.";
```

```
#HW1
dsolve({diff(x(t),t)=3*x(t)-y(t), diff(y(t),t)=2*x(t), x(0)=2, y(0)=3}, {x(t),
    y(t)});
```

$$\{x(t) = e^{t} + e^{2t},\ y(t) = 2e^{t} + e^{2t}\} \tag{1}$$

```
#HW2
dsolve({diff(x(t),t)=x(t)+8*y(t), diff(y(t),t)=5*x(t), x(0)=5, y(0)=7}, {x(t),
    y(t)});
```

$$\left\{x(t) = \left(\frac{\sqrt{161}}{10} + \frac{1}{10}\right)\left(\frac{7}{2} + \frac{43\sqrt{161}}{322}\right)e^{\frac{\left(1+\sqrt{161}\right)t}{2}} + \left(-\frac{\sqrt{161}}{10} + \frac{1}{10}\right)\left(\frac{7}{2} - \frac{43\sqrt{161}}{322}\right)e^{-\frac{\left(-1+\sqrt{161}\right)t}{2}},\ y(t) = \left(\frac{7}{2} + \frac{43\sqrt{161}}{322}\right)e^{\frac{\left(1+\sqrt{161}\right)t}{2}} + \left(\frac{7}{2} - \frac{43\sqrt{161}}{322}\right)e^{-\frac{\left(-1+\sqrt{161}\right)t}{2}}\right\} \tag{2}$$

```
#HW3
X := Matrix([[1,1,1],[1,1,0],[1,0,0]]);
L := Eigenvalues(X);
V := Eigenvectors(X);
v := Matrix([[x],[y],[z]]);
o := Matrix([[1],[2],[-1]]);
v1 := V[2][1];
v2 := V[2][2];
v3 := V[2][3];
M := <<v1>,<v2>,<v3>>;
dsolve({diff(x(t),t)=x(t)+y(t)+z(t), diff(y(t),t)=x(t)+y(t), diff(z(t),t)=x(t),
    x(0)=1, y(0)=2, z(0)=-1}, {x(t),y(t),z(t)});
```

$$X := \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$L := \left[\left[\frac{\left(28 + 84\,I\sqrt{3}\right)^{1/3}}{6} + \frac{14}{3\left(28 + 84\,I\sqrt{3}\right)^{1/3}} + \frac{2}{3}\right],\right.$$

$$\left[-\frac{(28+84\,I\sqrt{3})^{1/3}}{12} - \frac{7}{3\,(28+84\,I\sqrt{3})^{1/3}} + \frac{2}{3} + \frac{I\sqrt{3}\left(\frac{(28+84\,I\sqrt{3})^{1/3}}{6} - \frac{14}{3\,(28+84\,I\sqrt{3})^{1/3}}\right)}{2} \right],$$

$$\left[-\frac{(28+84\,I\sqrt{3})^{1/3}}{12} - \frac{7}{3\,(28+84\,I\sqrt{3})^{1/3}} + \frac{2}{3} - \frac{I\sqrt{3}\left(\frac{(28+84\,I\sqrt{3})^{1/3}}{6} - \frac{14}{3\,(28+84\,I\sqrt{3})^{1/3}}\right)}{2} \right] \Bigg]$$

$$V := \left[\left[\frac{(28+84\,I\sqrt{3})^{1/3}}{6} + \frac{14}{3\,(28+84\,I\sqrt{3})^{1/3}} + \frac{2}{3} \right], \right.$$

$$\left[-\frac{(28+84\,I\sqrt{3})^{1/3}}{12} - \frac{7}{3\,(28+84\,I\sqrt{3})^{1/3}} + \frac{2}{3} + \frac{I\sqrt{3}\left(\frac{(28+84\,I\sqrt{3})^{1/3}}{6} - \frac{14}{3\,(28+84\,I\sqrt{3})^{1/3}}\right)}{2} \right],$$

$$\left[-\frac{(28+84\,I\sqrt{3})^{1/3}}{12} - \frac{7}{3\,(28+84\,I\sqrt{3})^{1/3}} + \frac{2}{3} - \frac{I\sqrt{3}\left(\frac{(28+84\,I\sqrt{3})^{1/3}}{6} - \frac{14}{3\,(28+84\,I\sqrt{3})^{1/3}}\right)}{2} \right] \Bigg],$$

$$\left[\left[-1 \right. \right. \Big/$$

$$\left(\left(\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}+\frac{14}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}+\frac{2}{3}\right)^2-\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{3}\right.$$

$$\left.-\frac{28}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}-\frac{7}{3}\right),-1\Bigg/\left(\left(\left(-\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{12}\right.\right.\right.$$

$$\left.-\frac{7}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}+\frac{2}{3}\right.$$

$$\left.+\frac{\mathrm{I}\sqrt{3}\left(\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}\right)}{2}\right)^2+\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}$$

$$+\frac{14}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}-\frac{7}{3}-\mathrm{I}\sqrt{3}\left(\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}\right.$$

$$\left.\left.\left.-\frac{14}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}\right)\right),-1\Bigg/\left(\left(\left(-\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{12}\right.\right.\right.$$

$$-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$

$$-\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}$$

$$+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{7}{3}+I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}\right.$$

$$\left.\left.\left.-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)\right)\right],$$

$$\left[-\left(\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}\right)^2\right.\right.$$

$$\left.\left.-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{8}{3}\right)\Bigg/\right.$$

$$\left(\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}\right)^2-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{3}\right.$$

$$-\frac{28}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}-\frac{7}{3}\Bigg),-\Bigg(\Bigg(-\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{12}$$

$$-\frac{7}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$

$$+\frac{\mathrm{I}\sqrt{3}\left(\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{12}$$

$$+\frac{7}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}-\frac{8}{3}$$

$$-\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)\Bigg/\Bigg(\Bigg($$

$$-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$

$$+\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}$$

$$+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{7}{3}-I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}\right.$$

$$\left.-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)\Bigg)\Bigg),-\Bigg(\Bigg(\Bigg(-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}$$

$$
\begin{bmatrix}
-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}
-\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12} \\
+\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{8}{3}
+\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)\Bigg/\Bigg(\Bigg(\\
-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3} \\
-\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6} \\
+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{7}{3}+I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}\right. \\
\left.-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)\Bigg)\Bigg)\Bigg], \\
\begin{bmatrix}1,\ 1,\ 1\end{bmatrix}\Bigg]
\end{bmatrix}
$$

$$v := \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$o := \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$v1 := \left[-1 \middle/ \left(\left(\frac{\left(28 + 84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6} + \frac{14}{3\left(28 + 84\,\mathrm{I}\sqrt{3}\right)^{1/3}} + \frac{2}{3} \right)^2 \right.\right.$$

$$\left. - \frac{\left(28 + 84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{3} - \frac{28}{3\left(28 + 84\,\mathrm{I}\sqrt{3}\right)^{1/3}} - \frac{7}{3} \right), -1 \Big/ \left(\left(\right.\right.$$

$$\left. - \frac{\left(28 + 84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{12} - \frac{7}{3\left(28 + 84\,\mathrm{I}\sqrt{3}\right)^{1/3}} + \frac{2}{3} \right.$$

$$\left. + \frac{\mathrm{I}\sqrt{3}\left(\frac{\left(28 + 84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6} - \frac{14}{3\left(28 + 84\,\mathrm{I}\sqrt{3}\right)^{1/3}} \right)}{2} \right)^2 + \frac{\left(28 + 84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}$$

$$+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{7}{3}-I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}\right.$$

$$\left.-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)\Bigg),-1\Bigg/\Bigg(\left(\left(-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}\right.\right.$$

$$-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$

$$\left.\left.-\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\right)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}\right.$$

$$+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{7}{3}+I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}\right.$$

$$\left.\left.\left.-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)\right)\right]$$

$$v2:=\left[-\left(\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}\right)^2\right.\right.$$

$$-\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}-\frac{8}{3}\Bigg)\Bigg/$$

$$\left(\left(\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}+\frac{14}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}+\frac{2}{3}\right)^2-\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{3}\right.$$

$$\left.-\frac{28}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}-\frac{7}{3}\right),-\Bigg(\Bigg(-\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{12}$$

$$-\frac{7}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$

$$+\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}$$

$$+\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{8}{3}$$

$$-\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)\Bigg/\Bigg(\Bigg($$

$$-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$

$$+\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}$$

$$+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{7}{3}-I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}\right.$$

$$\left.-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)\Bigg),-\Bigg(\Bigg(-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}$$

$$-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$
$$-\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}$$

$$+\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{8}{3}$$
$$+\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)\Bigg/\Bigg(\Bigg(
$$

$$-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$

$$-\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}$$

$$+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{7}{3}+I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}\right.$$

$$\left.-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)\Bigg)\Bigg]$$

$$v3 := \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

$$M := \left[\left[-1\Big/\left(\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}\right)^2\right.\right.$$

$$\left.\left.-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{3}-\frac{28}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{7}{3}\right),-1\Big/\left(\left(\right.\right.\right.$$

$$-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$

$$+\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}$$

$$+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{7}{3}-I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}\right.$$

$$\left.-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)\Bigg),-1\Bigg/\Bigg(\Bigg(\Bigg(-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}$$

$$-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$

$$-\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}$$

$$+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{7}{3}+I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}\right.$$

$$\left.-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)\Bigg)\Bigg],$$

$$\Bigg[-\left(\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}\right)^2\right.$$

$$-\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}-\frac{8}{3}\Bigg)\Bigg/$$

$$\left(\left(\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{6}+\frac{14}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}+\frac{2}{3}\right)^2-\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{3}\right.$$

$$\left.-\frac{28}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}-\frac{7}{3}\right),-\Bigg(\Bigg(-\frac{\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}{12}$$

$$-\frac{7}{3\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$

$$+\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}$$

$$+\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{8}{3}$$

$$-\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)\Bigg/\Bigg(\Bigg($$

$$-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$

$$+\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}$$

$$+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{7}{3}-I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}\right.$$

$$\left.-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)\Bigg),-\Bigg(\Bigg(-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}$$

$$-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$
$$-\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}$$

$$+\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{8}{3}$$
$$+\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)\Bigg/\Bigg(\Bigg(
$$

$$-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}$$

$$-\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\Bigg)^2+\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}$$

$$+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}-\frac{7}{3}+I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}\right.$$

$$\left.\left.\left.-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)\right)\right],$$

$$\left[\begin{array}{c}1,\,1,\,1\end{array}\right]\Bigg]$$

$$\left\{x(t)=-\left(\left(-105670656+6\left(28+84\,I\sqrt{3}\right)^{8/3}+76608\left(28+84\,I\sqrt{3}\right)^{4/3}-4368\left(28\right.\right.\right.$$

$$+84\,I\sqrt{3}\left.\right)^{5/3}+8015616\left(28+84\,I\sqrt{3}\right)^{1/3}-2585856\left(28+84\,I\sqrt{3}\right)^{2/3}$$

$$+338688\,I\left(28+84\,I\sqrt{3}\right)^{1/3}\sqrt{3}-193536\,I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{2/3}$$

$$+1008\,I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{5/3}+18\,I\sqrt{3}\left(28+84\,I\sqrt{3}\right)^{8/3}-24192\,I\left(28\right.$$

$$+ 84\,I\sqrt{3}\,)^{4/3}\sqrt{3} + 24385536\,I\sqrt{3}\,)$$

$$e^{-\frac{\left(I(28+84\,I\sqrt{3})^{2/3}\sqrt{3}+(28+84\,I\sqrt{3})^{2/3}-28\,I\sqrt{3}-8(28+84\,I\sqrt{3})^{1/3}+28\right)t}{12(28+84\,I\sqrt{3})^{1/3}}}\Bigg)\Bigg/$$

$$\Big(3\,\big(I\,(28+84\,I\sqrt{3})^{4/3}\sqrt{3}+56\,I\,(28+84\,I\sqrt{3})^{1/3}\sqrt{3}-840\,I\sqrt{3}+504\big)\,\Big($$

$$-18\,(28+84\,I\sqrt{3})^{4/3}+2\,I\,(28+84\,I\sqrt{3})^{4/3}\sqrt{3}-1568\,I\,(28+84\,I\sqrt{3})^{1/}$$

$$^{3}\sqrt{3}\,\big)\big)-\Bigg(\Big(105670656+3048192\,I\sqrt{3}\,(28+84\,I\sqrt{3})^{1/3}+1257984\,I\sqrt{3}\,(28+84\,I\sqrt{3})^{2/3}+6$$

$$+84\,I\sqrt{3}\,)^{5/3}+2596608\,(28+84\,I\sqrt{3}\,)^{1/3}+607488\,(28+84\,I\sqrt{3}\,)^{2/3}$$

$$+1008\,I\sqrt{3}\,(28+84\,I\sqrt{3}\,)^{5/3}+18\,I\sqrt{3}\,(28+84\,I\sqrt{3}\,)^{8/3}+24192\,I\,(28$$

$$+84\,I\sqrt{3}\,)^{4/3}\sqrt{3}-24385536\,I\sqrt{3}\,)$$

$$e^{\frac{\left(I(28+84\,I\sqrt{3})^{2/3}\sqrt{3}-(28+84\,I\sqrt{3})^{2/3}-28\,I\sqrt{3}+8(28+84\,I\sqrt{3})^{1/3}-28\right)t}{12(28+84\,I\sqrt{3})^{1/3}}}\Bigg)\Bigg/$$

$$\left(3\left(\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}+56\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}-840\,\mathrm{I}\sqrt{3}+504\right)\left(\right.\right.$$

$$\left.-18\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+2\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}-1568\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/}\right.$$

$$\left.\left.{}^{3}\sqrt{3}\right)\right)-\left(\left(-2\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}+18\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+16128\,\mathrm{I}\sqrt{3}\right.\right.$$

$$\left.+288\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}+1568\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}-32256\right.$$

$$\left.\left.+1440\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\right)\mathrm{e}^{\frac{\left(\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}+4\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}+28\right)t}{6\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}}\right)\Big/\left(3\left(\right.\right.$$

$$\left.-18\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+2\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}-1568\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/}\right.$$

$$\left.\left.{}^{3}\sqrt{3}\right)\right),y(t)=-\left(\left(\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}-\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+56\,\mathrm{I}\left(28\right.\right.\right.$$

$$\left.\left.+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}-48\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}-1008\,\mathrm{I}\sqrt{3}-56\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\right.$$

$$\left.-336\right)\left(-105670656+6\left(28+84\,\mathrm{I}\sqrt{3}\right)^{8/3}+76608\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\right.$$

$$-4368\left(28+84\,\mathrm{I}\sqrt{3}\right)^{5/3}+8015616\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}-2585856\left(28\right.$$

$$\left.+84\,\mathrm{I}\sqrt{3}\right)^{2/3}+338688\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}-193536\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}$$

$$+1008\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{5/3}+18\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{8/3}-24192\,\mathrm{I}\left(28\right.$$

$$\left.+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}+24385536\,\mathrm{I}\sqrt{3}\Bigg)$$

$$e^{-\frac{\left(\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\sqrt{3}+\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}-28\,\mathrm{I}\sqrt{3}-8\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}+28\right)t}{12\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}}\Bigg)\Bigg/$$

$$\left(216\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\left(\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}+56\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}\right.\right.$$

$$\left.-840\,\mathrm{I}\sqrt{3}+504\right)\left(-18\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+2\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}\right.$$

$$\left.-1568\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}\right)\Bigg)+\Bigg(\Bigg(\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}+\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+56\,\mathrm{I}\left(28+\right.$$

$$\left.+1344\right)\left(105670656+3048192\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}+1257984\,\mathrm{I}\sqrt{3}\left(28\right.\right.$$

$$+ 84\,I\sqrt{3}\,\Big)^{2/3} + 6\,\Big(28 + 84\,I\sqrt{3}\,\Big)^{8/3} - 20160\,\Big(28 + 84\,I\sqrt{3}\,\Big)^{4/3} - 4368\,\Big(28$$

$$+ 84\,I\sqrt{3}\,\Big)^{5/3} + 2596608\,\Big(28 + 84\,I\sqrt{3}\,\Big)^{1/3} + 607488\,\Big(28 + 84\,I\sqrt{3}\,\Big)^{2/3}$$

$$+ 1008\,I\sqrt{3}\,\Big(28 + 84\,I\sqrt{3}\,\Big)^{5/3} + 18\,I\sqrt{3}\,\Big(28 + 84\,I\sqrt{3}\,\Big)^{8/3} + 24192\,I\,\Big(28$$

$$+ 84\,I\sqrt{3}\,\Big)^{4/3}\sqrt{3} - 24385536\,I\sqrt{3}\,\Big)$$

$$e^{\frac{\left(I\left(28 + 84\,I\sqrt{3}\right)^{2/3}\sqrt{3} - \left(28 + 84\,I\sqrt{3}\right)^{2/3} - 28\,I\sqrt{3} + 8\left(28 + 84\,I\sqrt{3}\right)^{1/3} - 28\right)t}{12\left(28 + 84\,I\sqrt{3}\right)^{1/3}}}\Bigg) \Bigg/$$

$$\Big(216\,\Big(28 + 84\,I\sqrt{3}\,\Big)^{2/3}\Big(I\,\Big(28 + 84\,I\sqrt{3}\,\Big)^{4/3}\sqrt{3} + 56\,I\,\Big(28 + 84\,I\sqrt{3}\,\Big)^{1/3}\sqrt{3}$$

$$- 840\,I\sqrt{3} + 504\Big)\Big(-18\,\Big(28 + 84\,I\sqrt{3}\,\Big)^{4/3} + 2\,I\,\Big(28 + 84\,I\sqrt{3}\,\Big)^{4/3}\sqrt{3}$$

$$- 1568\,I\,\Big(28 + 84\,I\sqrt{3}\,\Big)^{1/3}\sqrt{3}\Big)\Big) - \Bigg(\Big(\Big(28 + 84\,I\sqrt{3}\,\Big)^{4/3} - 24\,\Big(28 + 84\,I\sqrt{3}\,\Big)^{2/3} + 168\,I\sqrt{3} + 5$$

$$+ 16128\,I\sqrt{3} + 288\,I\sqrt{3}\,\Big(28 + 84\,I\sqrt{3}\,\Big)^{2/3} + 1568\,I\,\Big(28 + 84\,I\sqrt{3}\,\Big)^{1/3}\sqrt{3}$$

$$-32256+1440\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\Bigg)\,\mathrm{e}^{\frac{\left(\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}+4\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}+28\right)t}{6\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}}}\Bigg)\Bigg/$$

$$\Bigg(108\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}\left(-18\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}+2\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}\right.$$

$$\left.-1568\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}\right)\Bigg),z(t)=\Bigg(\Big(\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}-\left(28\right.$$

$$\left.+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-112\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}-24\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}-336\,\mathrm{I}\sqrt{3}$$

$$+112\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}-1680\Big)\left(-105670656+6\left(28+84\,\mathrm{I}\sqrt{3}\right)^{8/3}\right.$$

$$+76608\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}-4368\left(28+84\,\mathrm{I}\sqrt{3}\right)^{5/3}+8015616\left(28\right.$$

$$\left.+84\,\mathrm{I}\sqrt{3}\right)^{1/3}-2585856\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}+338688\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{1/3}\sqrt{3}$$

$$-193536\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{2/3}+1008\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{5/3}$$

$$\left.+18\,\mathrm{I}\sqrt{3}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{8/3}-24192\,\mathrm{I}\left(28+84\,\mathrm{I}\sqrt{3}\right)^{4/3}\sqrt{3}+24385536\,\mathrm{I}\sqrt{3}\right)$$

$$e^{-\frac{\left(I(28+84I\sqrt{3})^{2/3}\sqrt{3}+(28+84I\sqrt{3})^{2/3}-28I\sqrt{3}-8(28+84I\sqrt{3})^{1/3}+28\right)t}{12(28+84I\sqrt{3})^{1/3}}}\Bigg)\Bigg/$$

$$\left(216(28+84I\sqrt{3})^{2/3}\left(I(28+84I\sqrt{3})^{4/3}\sqrt{3}+56I(28+84I\sqrt{3})^{1/3}\sqrt{3}\right.\right.$$

$$\left.-840I\sqrt{3}+504\right)\left(-18(28+84I\sqrt{3})^{4/3}+2I(28+84I\sqrt{3})^{4/3}\sqrt{3}\right.$$

$$\left.-1568I(28+84I\sqrt{3})^{1/3}\sqrt{3}\right)\Bigg)-\Bigg(\Bigg(I(28+84I\sqrt{3})^{4/3}\sqrt{3}+(28+84I\sqrt{3})^{4/3}-112I(28+$$

$$-336\Big)\left(105670656+3048192I\sqrt{3}(28+84I\sqrt{3})^{1/3}+1257984I\sqrt{3}(28\right.$$

$$\left.+84I\sqrt{3})^{2/3}+6(28+84I\sqrt{3})^{8/3}-20160(28+84I\sqrt{3})^{4/3}-4368(28\right.$$

$$\left.+84I\sqrt{3})^{5/3}+2596608(28+84I\sqrt{3})^{1/3}+607488(28+84I\sqrt{3})^{2/3}\right.$$

$$\left.+1008I\sqrt{3}(28+84I\sqrt{3})^{5/3}+18I\sqrt{3}(28+84I\sqrt{3})^{8/3}+24192I(28\right.$$

$$\left.+84I\sqrt{3})^{4/3}\sqrt{3}-24385536I\sqrt{3}\right)$$

$$e^{\frac{\left(I(28+84I\sqrt{3})^{2/3}\sqrt{3}-(28+84I\sqrt{3})^{2/3}-28I\sqrt{3}+8(28+84I\sqrt{3})^{1/3}-28\right)t}{12(28+84I\sqrt{3})^{1/3}}}\Bigg/$$

$$\Big(216(28+84I\sqrt{3})^{2/3}\Big(I(28+84I\sqrt{3})^{4/3}\sqrt{3}+56I(28+84I\sqrt{3})^{1/3}\sqrt{3}$$

$$-840I\sqrt{3}+504\Big)\Big(-18(28+84I\sqrt{3})^{4/3}+2I(28+84I\sqrt{3})^{4/3}\sqrt{3}$$

$$-1568I(28+84I\sqrt{3})^{1/3}\sqrt{3}\Big)-\Bigg(\Big(-(28+84I\sqrt{3})^{4/3}+336I\sqrt{3}$$

$$+12(28+84I\sqrt{3})^{2/3}+112(28+84I\sqrt{3})^{1/3}-672\Big)\Big(-2I(28$$

$$+84I\sqrt{3})^{4/3}\sqrt{3}+18(28+84I\sqrt{3})^{4/3}+16128I\sqrt{3}+288I\sqrt{3}(28$$

$$+84I\sqrt{3})^{2/3}+1568I(28+84I\sqrt{3})^{1/3}\sqrt{3}-32256+1440(28+84I\sqrt{3})^{2/3}\Big)$$

$$e^{\frac{\left((28+84I\sqrt{3})^{2/3}+4(28+84I\sqrt{3})^{1/3}+28\right)t}{6(28+84I\sqrt{3})^{1/3}}}\Bigg)\Bigg/\Big(108(28+84I\sqrt{3})^{2/3}\Big($$

$$-18(28+84I\sqrt{3})^{4/3}+2I(28+84I\sqrt{3})^{4/3}\sqrt{3}-1568I(28+84I\sqrt{3})^{1/3}\sqrt{3}\Big)\Big)\Big\}$$

```
> #HW4
M := [[1, 1, 1], [1, 1, 1], [1, 1, 1]];
 Orb2(HW2g(x, y, M), x, y, [0.5, 0.5], 1, 5);
     #With any values for x,y for the values altering the Hardy-Weingberg Transformation, the
     system stabilizes after 1 generation.
Orb2(HW2g(x, y, M), x, y, [1, 1], 1, 5);
 Orb2(HW2g(x, y, M), x, y, [0.1, 0.1], 1, 5);
 Orb2(HW2g(x, y, M), x, y, [0.3, 0.7], 1, 5);
 R1 := RandomMatrix(3, 3);
Orb2(HW2g(x, y, R1), x, y, [0.7, 0.2], 1000, 1005);
 R2 := RandomMatrix(3, 3);
Orb2(HW2g(x, y, R2), x, y, [0.1, 0.5], 1000, 1005);
 R3 := RandomMatrix(3, 3);
```

```
Orb2(HW2g(x, y, R3), x, y, [0.6, 0.4], 1000, 1005);
R4 := RandomMatrix(3, 3);
Orb2(HW2g(x, y, R4), x, y, [0.3, 0.3], 1000, 1005);
R5 := RandomMatrix(3, 3);
Orb2(HW2g(x, y, R5), x, y, [0.1, 0.9], 1000, 1005);
R6 := RandomMatrix(3, 3);
Orb2(HW2g(x, y, R6), x, y, [0.8, 0.2], 1000, 1005);
R7 := RandomMatrix(3, 3);
Orb2(HW2g(x, y, R7), x, y, [0.2, 0.2], 1000, 1005);
R8 := RandomMatrix(3, 3);
Orb2(HW2g(x, y, R8), x, y, [0.6, 0.2], 1000, 1005);
R9 := RandomMatrix(3, 3);
Orb2(HW2g(x, y, R9), x, y, [0.3, 0.7], 1000, 1005);
R10 := RandomMatrix(3, 3);
Orb2(HW2g(x, y, R10), x, y, [0.5, 0.5], 1000, 1005);
```

$$M := [[1, 1, 1], [1, 1, 1], [1, 1, 1]]$$

[[0.5, 0.5], [0.5625000000, 0.3750000000], [0.5625000000, 0.3750000000], [0.5625000000, 0.3750000000], [0.5625000000, 0.3750000000]]

$$\left[[1, 1], \left[\frac{9}{4}, -\frac{3}{2}\right], \left[\frac{9}{4}, -\frac{3}{2}\right], \left[\frac{9}{4}, -\frac{3}{2}\right], \left[\frac{9}{4}, -\frac{3}{2}\right]\right]$$

[[0.1, 0.1], [0.02250000000, 0.2550000000], [0.02250000000, 0.2550000000], [0.02250000000, 0.2550000000], [0.02250000000, 0.2550000000]]

[[0.3, 0.7], [0.4225000000, 0.4550000000], [0.4225000000, 0.4550000000], [0.4225000000, 0.4550000000], [0.4225000000, 0.4550000000]]

$$R1 := \begin{bmatrix} 70 & 96 & 34 \\ 91 & 71 & 23 \\ 85 & -20 & -53 \end{bmatrix}$$

[[0.6599811675, 0.3174634204], [0.6599811675, 0.3174634204], [0.6599811675, 0.3174634204], [0.6599811675, 0.3174634204], [0.6599811675, 0.3174634204], [0.6599811675, 0.3174634204]]

$$R2 := \begin{bmatrix} 10 & -6 & -61 \\ -1 & -49 & -11 \\ 79 & -33 & -85 \end{bmatrix}$$

$$[[1.134698462 \times 10^{-1172}, 7.262487645 \times 10^{-587}], [7.601301802 \times 10^{-1174}, 1.879702684 \times 10^{-587}], [5.092083140 \times 10^{-1175}, 4.865112830 \times 10^{-588}], [3.411167118 \times 10^{-1176}, 1.259205674 \times 10^{-588}], [2.285127868 \times 10^{-1177}, 3.259120568 \times 10^{-589}], [1.530798462 \times 10^{-1178}, 8.435370880 \times 10^{-590}]]$$

$$R3 := \begin{bmatrix} 51 & 66 & -48 \\ -61 & 29 & -79 \\ -60 & 63 & 48 \end{bmatrix}$$

$$[[1.000000000, -4.869786411 \times 10^{-803}], [1.000000000, 7.638880645 \times 10^{-804}], [1.000000000, -1.198255787 \times 10^{-804}], [1.000000000, 1.879616920 \times 10^{-805}], [1.000000000, -2.948418698 \times 10^{-806}], [1.000000000, 4.624970507 \times 10^{-807}]]$$

$$R4 := \begin{bmatrix} -98 & 27 & -69 \\ 80 & 23 & 20 \\ 85 & 70 & -7 \end{bmatrix}$$

$$[[1.000000000, -3.097620810 \times 10^{-335}], [1.000000000, 1.422376902 \times 10^{-335}], [1.000000000, -6.531322510 \times 10^{-336}], [1.000000000, 2.999076663 \times 10^{-336}], [1.000000000, -1.377127040 \times 10^{-336}], [1.000000000, 6.323542530 \times 10^{-337}]]$$

$$R5 := \begin{bmatrix} 37 & -46 & 73 \\ 31 & 97 & 90 \\ -15 & -22 & 33 \end{bmatrix}$$

$$[[0.1277717379, 0.4631118988], [0.1277717379, 0.4631118988], [0.1277717379, 0.4631118988], [0.1277717379, 0.4631118988], [0.1277717379, 0.4631118988], [0.1277717379, 0.4631118988]]$$

$$R6 := \begin{bmatrix} -27 & 10 & -20 \\ -48 & 88 & 62 \\ 28 & -53 & -70 \end{bmatrix}$$

$$[[1.000000001, 4.557091656 \times 10^{-10}], [0.9999999993, 8.499743960 \times 10^{-10}], [0.9999999993, 5.990783400 \times 10^{-10}], [1.000000001, 4.557091656 \times 10^{-10}], [0.9999999993, 8.499743960 \times 10^{-10}], [0.9999999993, 5.990783400 \times 10^{-10}]]$$

$$R7 := \begin{bmatrix} -97 & -65 & 25 \\ -13 & -80 & 33 \\ 10 & -39 & -65 \end{bmatrix}$$

$$[[2.838748011 \times 10^{-2660}, -1.401886973 \times 10^{-1331}], [6.047037185 \times 10^{-2663}, -6.470247568 \times 10^{-1333}], [1.288126264 \times 10^{-2665}, -2.986268108 \times 10^{-1334}], [2.743937605 \times 10^{-2668}, -1.378277588 \times 10^{-1335}], [5.845074185 \times 10^{-2671}, -6.361281175 \times 10^{-1337}], [1.245104560 \times 10^{-2673}, -2.935975926 \times 10^{-1338}]]$$

$$R8 := \begin{bmatrix} 7 & -58 & -8 \\ 59 & 82 & -43 \\ -22 & -18 & 21 \end{bmatrix}$$

$$[[0.2153376601, 0.4731032305], [0.5057739055, 0.2770796869], [1.525523202, -0.8493051660], [0.3886261845, 0.2872936968], [4.096891435, -4.641949512], [0.2541291293, 0.4173597020]]$$

$$R9 := \begin{bmatrix} 6 & 37 & 72 \\ -85 & 3 & 30 \\ 54 & 34 & -67 \end{bmatrix}$$

$$[[-8.632612058 \times 10^{-638}, -1.326334472 \times 10^{-318}], [-1.969212461 \times 10^{-638}, 6.334731805 \times 10^{-319}], [-4.492032878 \times 10^{-639}, -3.025543548 \times 10^{-319}], [-1.024691839 \times 10^{-639}, 1.445035724 \times 10^{-319}], [-2.337456989 \times 10^{-640}, -6.901663160 \times 10^{-320}], [-5.332047131 \times 10^{-641}, 3.296316733 \times 10^{-320}]]$$

$$R10 := \begin{bmatrix} -48 & -29 & 33 \\ 88 & 95 & 27 \\ -35 & -28 & -53 \end{bmatrix}$$

$$[[0.2567522578, 0.5683862475], [0.2567522578, 0.5683862475], [0.2567522578, 0.5683862475], [0.2567522578, 0.5683862475], [0.2567522578, 0.5683862475], [0.2567522578, 0.5683862475]] \quad \textbf{(4)}$$

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