

NOT OK to post

Anusha Nagar, Homework 17, 10.30.2021

$$① \quad x'(t) = 3x(t) - y(t)$$

$$y'(t) = 2x(t)$$

$$x(0) = 2, y(0) = 3$$

$$(i) \quad x''(t) = 3x'(t) - y'(t) \quad x(0) = 2$$

$$y'(t) = 2x(t) \quad x'(0) = 3x(0) - y(0)$$

$$x''(t) = 3x'(t) - 2x(t) \quad = 6 - 3 = 3$$

$$x''(t) - 3x'(t) + 2x(t) = 0$$

$$r^2 - 3r + 2 = 0$$

$$(r-2)(r-1)$$

$$r=1, 2$$

$$x(t) = Ae^t + Be^{2t} \Rightarrow 2 = A+B$$

$$x'(t) = Ae^t + 2Be^{2t} \Rightarrow 3 = A+2B$$

$$\hookrightarrow B=1$$

$$A=1$$

$$x(t) = e^t + e^{2t}$$

$$y(t) = 3x(t) - x'(t)$$

$$= 3e^t + 3e^{2t} - e^t - 2e^{2t}$$

$$\boxed{\begin{cases} y(t) = 2e^t + e^{2t} \\ x(t) = e^t + e^{2t} \end{cases}}$$

$$(ii) \quad x'(t) = 3x(t) - y(t)$$

$$y'(t) = 2x(t) + 0; y(t)$$

$$x(0) = 2, y(0) = 3$$

$$\vec{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

$$\vec{x}'(t) = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix} \vec{x}(t)$$

$$\vec{x}(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \xrightarrow{\hspace{1cm}} A$$

$$\det(A - \lambda I) = \det \begin{bmatrix} 3-\lambda & -1 \\ 2 & -\lambda \end{bmatrix} = (3-\lambda)(-\lambda) + 2 = \lambda^2 - 3\lambda + 2 = (\lambda-1)(\lambda-2)$$

$\lambda=1, 2$

$$v_1 = \begin{bmatrix} a \\ b \end{bmatrix} \Rightarrow \lambda=1 \Rightarrow \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2a - b = 0$$

$$v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} a \\ b \end{bmatrix} \Rightarrow \lambda=2 \Rightarrow \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$a - b = 0$$

$$v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\vec{x}(t) = A \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^t + B \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t}$$

$$\vec{x}(0) = A \begin{bmatrix} 1 \\ 2 \end{bmatrix} + B \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} A+B \\ 2A+B \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \begin{array}{l} A+B=2 \\ 2A+B=3 \end{array}$$

$$A=B=1$$

$$\boxed{\vec{x}(t) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^t + \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t}}$$

OR

$$x(t) = e^t + e^{2t}$$

$$y(t) = 2e^t + e^{2t}$$

$$\text{(iii) } \text{dsolve}(\{ \text{diff}(x(t), t) = 3*x(t) - y(t), \\ \text{diff}(y(t), t) = 2*x(t), x(0)=2, y(0)=3 \}, \\ \{x(t), y(t)\});$$

$$x'(t) = 3x(t) - y(t)$$

$$y'(t) = 2x(t) + 0 \cdot y(t)$$

$$x(0) = 2, y(0) = 3$$

(2) RUID: 185007365

$$\begin{cases} x'(t) = x(t) + 8y(t) \\ y'(t) = 5x(t) \end{cases}$$

$$x(0) = 3, y(0) = 6$$

$$x''(t) = x'(t) + 8y'(t)$$

$$6x''(t) = x'(t) + 40x(t)$$

$$x''(t) - x'(t) - 40x(t) = 0$$

$$r^2 - r - 40 = 0$$

$$\frac{1 \pm \sqrt{1-4(1)(-40)}}{2} = \frac{1 \pm \sqrt{161}}{2}$$

$$x(t) = A e^{\frac{1+\sqrt{161}}{2}t} + B e^{\frac{1-\sqrt{161}}{2}t} \Rightarrow x'(t) = \frac{1+\sqrt{161}}{2} A e^{\frac{1+\sqrt{161}}{2}t} + \frac{1-\sqrt{161}}{2} B e^{\frac{1-\sqrt{161}}{2}t}$$

$$x(0) = 3 = A + B$$

$$x'(0) = -4S = \frac{1+\sqrt{161}}{2} A + \frac{1-\sqrt{161}}{2} B \quad \left. \begin{array}{l} A = -2.165 \\ B = 5.165 \end{array} \right.$$

$$x(t) = -2.165 e^{\frac{1+\sqrt{161}}{2}t} + 5.165 e^{\frac{1-\sqrt{161}}{2}t}$$

$$y(t) = \frac{1+\sqrt{161}}{2} (-2.165) e^{\frac{1+\sqrt{161}}{2}t} + \left(\frac{1-\sqrt{161}}{2} \right) (5.165) e^{\frac{1-\sqrt{161}}{2}t} - 5.165 e^{\frac{1-\sqrt{161}}{2}t}$$

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(ii)

$$\begin{cases} x'(t) = x(t) + 8y(t) \\ y'(t) = 5x(t) \end{cases}$$

$$x(0) = 3, y(0) = 6$$

$$\vec{x}'(t) = \begin{bmatrix} 1 & 8 \\ 5 & 8 \end{bmatrix} \vec{x}(t)$$

$$\vec{x}(0) = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$\lambda = \frac{1}{2} + \frac{\sqrt{161}}{2}, \frac{1}{2} - \frac{\sqrt{161}}{2}$$

$$\vec{V}_1 = \begin{bmatrix} \frac{1}{2} + \frac{\sqrt{161}}{2} \\ 1 \end{bmatrix}, \vec{V}_2 = \begin{bmatrix} \frac{1}{2} - \frac{\sqrt{161}}{2} \\ 1 \end{bmatrix}$$

$$\vec{x}(t) = A \begin{bmatrix} \frac{1}{2} + \frac{\sqrt{161}}{2} \\ 1 \end{bmatrix} e^{(\frac{1}{2} + \frac{\sqrt{161}}{2})t} + B \begin{bmatrix} \frac{1}{2} - \frac{\sqrt{161}}{2} \\ 1 \end{bmatrix} e^{(\frac{1}{2} - \frac{\sqrt{161}}{2})t}$$

$$\vec{x}(0) = \begin{bmatrix} 3 \\ 6 \end{bmatrix} = A \begin{bmatrix} 8 \\ -\frac{1}{2} + \frac{\sqrt{161}}{2} \\ 1 \end{bmatrix} + B \begin{bmatrix} 8 \\ -\frac{1}{2} - \frac{\sqrt{161}}{2} \\ 1 \end{bmatrix}$$

$$\begin{cases} A+B=6 \\ 3 = \frac{8}{-\frac{1}{2} + \frac{\sqrt{161}}{2}} A + \frac{8}{-\frac{1}{2} - \frac{\sqrt{161}}{2}} B \end{cases}$$

$$A = 3 + \frac{12\sqrt{161}}{161}, \quad B = 3 - \frac{12\sqrt{161}}{161}$$

$$\boxed{\vec{x}(t) = \begin{bmatrix} 8 \\ -\frac{1}{2} + \frac{\sqrt{161}}{2} \\ 1 \end{bmatrix} \left[3 + \frac{12\sqrt{161}}{161} \right] e^{\left(\frac{1}{2} + \frac{\sqrt{161}}{2} \right)t} + \begin{bmatrix} 8 \\ -\frac{1}{2} - \frac{\sqrt{161}}{2} \\ 1 \end{bmatrix} \left[3 - \frac{12\sqrt{161}}{161} \right] e^{\left(\frac{1}{2} - \frac{\sqrt{161}}{2} \right)t}}$$

$$\textcircled{3} \quad x_1'(t) = x_1(t) + x_2(t) + x_3(t)$$

$$x_2'(t) = x_1(t) + x_2(t)$$

$$x_3'(t) = x_1(t)$$

$$x_1(0) = 1, \quad x_2(0) = 2, \quad x_3(0) = -1$$

$$\vec{x}'(t) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \vec{x}(t), \quad \vec{x}(0) = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$\vec{x}(0) = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = A \begin{bmatrix} v_2[1] \\ v_2[2] \\ v_2[3] \end{bmatrix} + B \begin{bmatrix} v_2[1] \\ v_2[2] \\ v_2[3] \end{bmatrix} + C \begin{bmatrix} v_2[1] \\ v_2[2] \\ v_2[3] \end{bmatrix}$$

Column(E,1) Column(E,2)

$$1 = v_2[1,1] \cdot F + v_2[1,2] \cdot G + v_2[1,3] \cdot H$$

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> #Ok to post
> #Anusha Nagar, Homework 17, 10.30.2021
>
> #PRoblem 1 part iii
> dsolve( {diff(x(t), t) = 3·x(t) - y(t), diff(y(t), t) = 2·x(t), x(0) = 2, y(0) = 3}, {x(t),y(t)}) 
    {x(t) = e^2t + e^t, y(t) = e^2t + 2 e^t}                                         (1)

> #Problem 2 part iii
> dsolve( {diff(x(t), t) = x(t) + 8·y(t), diff(y(t), t) = 5·x(t), x(0) = 3, y(0) = 6}, {x(t),y(t)}) 
    {x(t) = (sqrt(161)/10 + 1/10) * (3 + 12*sqrt(161)/161) * e^( (1+sqrt(161))/2*t) + (-sqrt(161)/10 + 1/10) * (3 - 12*sqrt(161)/161) * e^(- (1+sqrt(161))/2*t), y(t) = (3 + 12*sqrt(161)/161) * e^( (1+sqrt(161))/2*t) + (3 - 12*sqrt(161)/161) * e^(- (1+sqrt(161))/2*t)}                                (2)

> with(LinearAlgebra) :
>

A := Matrix( [[1, 8], [5, 0]])
A := 
$$\begin{bmatrix} 1 & 8 \\ 5 & 0 \end{bmatrix} \quad (3)$$


> e, v := Eigenvectors(A)
e, v := 
$$\begin{bmatrix} \frac{1}{2} + \frac{\sqrt{161}}{2} \\ \frac{1}{2} - \frac{\sqrt{161}}{2} \end{bmatrix}, \begin{bmatrix} \frac{8}{-\frac{1}{2} + \frac{\sqrt{161}}{2}} & \frac{8}{-\frac{1}{2} - \frac{\sqrt{161}}{2}} \\ 1 & 1 \end{bmatrix} \quad (4)$$


> sys := 
$$\left\{ \left( \frac{8}{-\frac{1}{2} + \frac{\sqrt{161}}{2}} \right) \cdot C + \left( \frac{8}{-\frac{1}{2} - \frac{\sqrt{161}}{2}} \right) \cdot D = 3, C + D = 6 \right\}$$

    sys := 
$$\left\{ C + D = 6, \frac{8C}{-\frac{1}{2} + \frac{\sqrt{161}}{2}} + \frac{8D}{-\frac{1}{2} - \frac{\sqrt{161}}{2}} = 3 \right\} \quad (5)$$


> solve(sys, {C, D})
    {C = 3 +  $\frac{12\sqrt{161}}{161}$ , D =  $3 - \frac{12\sqrt{161}}{161}$ }                                         (6)

> #Problem3
> E := Matrix( [[1, 1, 1], [1, 1, 0], [1, 0, 0]])
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(7)

$$E := \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad (7)$$

> $e2, v2 := \text{Eigenvectors}(E)$

$$e2, v2 := \left[\begin{array}{l} \left[\frac{\left(28 + 84 I\sqrt{3}\right)^{1/3}}{6} + \frac{14}{3 \left(28 + 84 I\sqrt{3}\right)^{1/3}} + \frac{2}{3}, \right. \\ \left. - \frac{\left(28 + 84 I\sqrt{3}\right)^{1/3}}{12} - \frac{7}{3 \left(28 + 84 I\sqrt{3}\right)^{1/3}} + \frac{2}{3} \right. \\ \left. + \frac{I\sqrt{3} \left(\frac{\left(28 + 84 I\sqrt{3}\right)^{1/3}}{6} - \frac{14}{3 \left(28 + 84 I\sqrt{3}\right)^{1/3}} \right)}{2} \right], \\ \left[\begin{array}{l} \left[- \frac{\left(28 + 84 I\sqrt{3}\right)^{1/3}}{12} - \frac{7}{3 \left(28 + 84 I\sqrt{3}\right)^{1/3}} + \frac{2}{3} \right. \\ \left. - \frac{I\sqrt{3} \left(\frac{\left(28 + 84 I\sqrt{3}\right)^{1/3}}{6} - \frac{14}{3 \left(28 + 84 I\sqrt{3}\right)^{1/3}} \right)}{2} \right], \\ \left[\begin{array}{l} -1 \\ 1 \end{array} \right] \end{array} \right], \\ \left(\left(\frac{\left(28 + 84 I\sqrt{3}\right)^{1/3}}{6} + \frac{14}{3 \left(28 + 84 I\sqrt{3}\right)^{1/3}} + \frac{2}{3} \right)^2 - \frac{\left(28 + 84 I\sqrt{3}\right)^{1/3}}{3} \right)$$

$$\begin{aligned}
& - \frac{28}{3(28 + 84I\sqrt{3})^{1/3}} - \frac{7}{3} \Bigg), -1 \Bigg/ \left(\left(- \frac{(28 + 84I\sqrt{3})^{1/3}}{12} \right. \right. \\
& - \frac{7}{3(28 + 84I\sqrt{3})^{1/3}} + \frac{2}{3} \\
& + \frac{I\sqrt{3} \left(\frac{(28 + 84I\sqrt{3})^{1/3}}{6} - \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} \right)^2}{2} + \frac{(28 + 84I\sqrt{3})^{1/3}}{6} \\
& + \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} - \frac{7}{3} - I\sqrt{3} \left(\frac{(28 + 84I\sqrt{3})^{1/3}}{6} \right. \\
& - \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} \Bigg) \Bigg), -1 \Bigg/ \left(\left(- \frac{(28 + 84I\sqrt{3})^{1/3}}{12} \right. \right. \\
& - \frac{7}{3(28 + 84I\sqrt{3})^{1/3}} + \frac{2}{3} \\
& - \frac{I\sqrt{3} \left(\frac{(28 + 84I\sqrt{3})^{1/3}}{6} - \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} \right)^2}{2} + \frac{(28 + 84I\sqrt{3})^{1/3}}{6}
\end{aligned}$$

$$+ \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} - \frac{7}{3} + I\sqrt{3} \left(\frac{(28 + 84I\sqrt{3})^{1/3}}{6} \right. \\ \left. - \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} \right) \Bigg),$$

$$\left[- \left(\left(\frac{(28 + 84I\sqrt{3})^{1/3}}{6} + \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} + \frac{2}{3} \right)^2 \right. \right.$$

$$\left. \left. - \frac{(28 + 84I\sqrt{3})^{1/3}}{6} - \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} - \frac{8}{3} \right) \right]$$

$$\left(\left(\frac{(28 + 84I\sqrt{3})^{1/3}}{6} + \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} + \frac{2}{3} \right)^2 - \frac{(28 + 84I\sqrt{3})^{1/3}}{3} \right.$$

$$\left. - \frac{28}{3(28 + 84I\sqrt{3})^{1/3}} - \frac{7}{3} \right), - \left(\left(- \frac{(28 + 84I\sqrt{3})^{1/3}}{12} \right. \right.$$

$$-\frac{7}{3(28 + 84I\sqrt{3})^{1/3}} + \frac{2}{3}$$

$$+ \frac{I\sqrt{3} \left(\frac{(28 + 84I\sqrt{3})^{1/3}}{6} - \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} \right)^2}{2} + \frac{(28 + 84I\sqrt{3})^{1/3}}{12}$$

$$+ \frac{7}{3(28 + 84I\sqrt{3})^{1/3}} - \frac{8}{3}$$

$$- \frac{I\sqrt{3} \left(\frac{(28 + 84I\sqrt{3})^{1/3}}{6} - \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} \right)^2}{2}$$

$$\begin{aligned}
& - \frac{(28 + 84I\sqrt{3})^{1/3}}{12} - \frac{7}{3(28 + 84I\sqrt{3})^{1/3}} + \frac{2}{3} \\
& + \frac{I\sqrt{3} \left(\frac{(28 + 84I\sqrt{3})^{1/3}}{6} - \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} \right)^2}{2} + \frac{(28 + 84I\sqrt{3})^{1/3}}{6} \\
& + \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} - \frac{7}{3} - I\sqrt{3} \left(\frac{(28 + 84I\sqrt{3})^{1/3}}{6} \right. \\
& \quad \left. - \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} \right), - \left(\left(- \frac{(28 + 84I\sqrt{3})^{1/3}}{12} \right. \right. \\
& \quad \left. \left. - \frac{7}{3(28 + 84I\sqrt{3})^{1/3}} + \frac{2}{3} \right. \right. \\
& \quad \left. \left. - \frac{I\sqrt{3} \left(\frac{(28 + 84I\sqrt{3})^{1/3}}{6} - \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} \right)^2}{2} + \frac{(28 + 84I\sqrt{3})^{1/3}}{12} \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{7}{3(28 + 84I\sqrt{3})^{1/3}} - \frac{8}{3} \\
& + \frac{I\sqrt{3} \left(\frac{(28 + 84I\sqrt{3})^{1/3}}{6} - \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} \right)}{2} \Bigg) \Bigg) \\
& - \frac{(28 + 84I\sqrt{3})^{1/3}}{12} - \frac{7}{3(28 + 84I\sqrt{3})^{1/3}} + \frac{2}{3} \\
& - \frac{I\sqrt{3} \left(\frac{(28 + 84I\sqrt{3})^{1/3}}{6} - \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} \right)^2}{2} + \frac{(28 + 84I\sqrt{3})^{1/3}}{6} \\
& + \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} - \frac{7}{3} + I\sqrt{3} \left(\frac{(28 + 84I\sqrt{3})^{1/3}}{6} \right. \\
& \left. - \frac{14}{3(28 + 84I\sqrt{3})^{1/3}} \right) \Bigg),
\end{aligned}$$

$$\begin{bmatrix} 1, 1, 1 \end{bmatrix}$$

> $\text{sys2} := \{1 = v2[1, 1] \cdot F + v2[1, 2] \cdot G + v2[1, 3] \cdot H, 2 = v2[2, 1] \cdot F + v2[2, 2] \cdot G + v2[2, 3] \cdot H, -1 = v2[3, 1] \cdot F + v2[3, 2] \cdot G + v2[3, 3] \cdot H\} :$

> $\text{solve}(\text{sys2}, \{F, G, H\})$

$$\begin{cases} F = \left(261996044077881724030156800 \sqrt{3} (28 + 84I\sqrt{3})^{2/3} \right. \\ \left. - 16047631979833223999310004224 \sqrt{3} + 129024 (28 + 84I\sqrt{3})^{31/3} \sqrt{3} \right) \end{cases} \quad (9)$$

$$- 16047631979833223999310004224 \sqrt{3} + 129024 (28 + 84I\sqrt{3})^{31/3} \sqrt{3}$$

$$\begin{aligned}
& + 343348346880 (28 + 84 I \sqrt{3})^{23/3} \sqrt{3} - 98058240 (28 + 84 I \sqrt{3})^{28/3} \sqrt{3} \\
& - 182236817129472 (28 + 84 I \sqrt{3})^{20/3} \sqrt{3} + 31238258688 (28 + 84 I \sqrt{3})^{25/3} \sqrt{3} \\
& + 38004342053142528 (28 + 84 I \sqrt{3})^{17/3} \sqrt{3} - 5139292225536 (28 \\
& + 84 I \sqrt{3})^{22/3} \sqrt{3} - 3564463846343049216 (28 + 84 I \sqrt{3})^{14/3} \sqrt{3} \\
& + 6127765235215368192 (28 + 84 I \sqrt{3})^{13/3} \sqrt{3} \\
& + 642276335245043372654592 (28 + 84 I \sqrt{3})^{7/3} \sqrt{3} \\
& - 3319028998750768988160 (28 + 84 I \sqrt{3})^{10/3} \sqrt{3} \\
& + 14124989936209118625792 (28 + 84 I \sqrt{3})^{8/3} \sqrt{3} \\
& - 3246306668603490749644800 (28 + 84 I \sqrt{3})^{5/3} \sqrt{3} \\
& - 45099592402312964673110016 (28 + 84 I \sqrt{3})^{4/3} \sqrt{3} + 410915602169856 (28 \\
& + 84 I \sqrt{3})^{19/3} \sqrt{3} - 19063743442845696 (28 + 84 I \sqrt{3})^{16/3} \sqrt{3} \\
& + 111184137028733239296 (28 + 84 I \sqrt{3})^{11/3} \sqrt{3} \\
& - 39169157149769254547434242048 I - 4684489268112525225659203584 I (28 \\
& + 84 I \sqrt{3})^{2/3} + 294174720 I (28 + 84 I \sqrt{3})^{28/3} - 93714776064 I (28
\end{aligned}$$

$$\begin{aligned}
& + 84 \text{I} \sqrt{3})^{25/3} - 54917357000196096 \text{I} (28 + 84 \text{I} \sqrt{3})^{17/3} \\
& + 15417876676608 \text{I} (28 + 84 \text{I} \sqrt{3})^{22/3} - 52288831881216 \text{I} (28 + 84 \text{I} \sqrt{3})^{20/3} \\
& + 62124659142819840 \text{I} (28 + 84 \text{I} \sqrt{3})^{16/3} + 468202291200 \text{I} (28 + 84 \text{I} \sqrt{3})^{23/3} \\
& - 1232746806509568 \text{I} (28 + 84 \text{I} \sqrt{3})^{19/3} - 387072 \text{I} (28 + 84 \text{I} \sqrt{3})^{31/3} \\
& + 18270869711845588992 \text{I} (28 + 84 \text{I} \sqrt{3})^{14/3} - 21067080980615921664 \text{I} (28 \\
& + 84 \text{I} \sqrt{3})^{13/3} - 2191573793216284065792 \text{I} (28 + 84 \text{I} \sqrt{3})^{11/3} \\
& + 7887414940084406845440 \text{I} (28 + 84 \text{I} \sqrt{3})^{10/3} \\
& - 115855958172487234093056 \text{I} (28 + 84 \text{I} \sqrt{3})^{8/3} \\
& - 1264599621565777811865600 \text{I} (28 + 84 \text{I} \sqrt{3})^{7/3} \\
& + 58043963234630414603649024 \text{I} (28 + 84 \text{I} \sqrt{3})^{5/3} \\
& + 79475598926429385941581824 \text{I} (28 + 84 \text{I} \sqrt{3})^{4/3} \Big) / \Big(9 (-526848 (28 \\
& + 84 \text{I} \sqrt{3})^4)^{1/3} - 14751744 (28 + 84 \text{I} \sqrt{3})^{2/3} - 672 (28 + 84 \text{I} \sqrt{3})^{8/3} \\
& - 24 (28 + 84 \text{I} \sqrt{3})^{10/3} + 60928 \text{I} (28 + 84 \text{I} \sqrt{3})^{5/3} \sqrt{3} - 224 \text{I} (28 \\
& + 84 \text{I} \sqrt{3})^{8/3} \sqrt{3} - 2176 \text{I} (28 + 84 \text{I} \sqrt{3})^{7/3} \sqrt{3} + 6528 (28 + 84 \text{I} \sqrt{3})^{7/3}
\end{aligned}$$

$$\begin{aligned}
& + 182784 (28 + 84 I \sqrt{3})^{5/3} + 175616 I (28 + 84 I \sqrt{3})^{4/3} \sqrt{3} - 4917248 I (28 \\
& + 84 I \sqrt{3})^{2/3} \sqrt{3} + 8 I (28 + 84 I \sqrt{3})^{10/3} \sqrt{3}) (13429889549991936 \sqrt{3} (28 \\
& + 84 I \sqrt{3})^{2/3} - 16785408 I (28 + 84 I \sqrt{3})^{17/3} + 1025310720 I (28 + 84 I \sqrt{3})^{14/3} \\
& + 35876241408 I (28 + 84 I \sqrt{3})^{11/3} + 17454735556608 I (28 + 84 I \sqrt{3})^{8/3} \\
& - 6156997468618752 I (28 + 84 I \sqrt{3})^{5/3} + 496905913349701632 I (28 \\
& + 84 I \sqrt{3})^{2/3} - 192 I (28 + 84 I \sqrt{3})^{23/3} + 93696 I (28 + 84 I \sqrt{3})^{20/3} \\
& + 611784327168 (28 + 84 I \sqrt{3})^{8/3} \sqrt{3} - 166405336989696 (28 + 84 I \sqrt{3})^{5/3} \\
& \sqrt{3}) \Big), G = - \Big((-255451170078720 I (28 + 84 I \sqrt{3})^{20/3} \sqrt{3} + 387072 I (28 \\
& + 84 I \sqrt{3})^{31/3} \sqrt{3} + 93714776064 I (28 + 84 I \sqrt{3})^{25/3} \sqrt{3} \\
& + 749123665920 I (28 + 84 I \sqrt{3})^{23/3} \sqrt{3} + 1232746806509568 I (28 \\
& + 84 I \sqrt{3})^{19/3} \sqrt{3} - 294174720 I (28 + 84 I \sqrt{3})^{28/3} \sqrt{3} \\
& - 15417876676608 I (28 + 84 I \sqrt{3})^{22/3} \sqrt{3} + 5585466053099520 I (28 \\
& + 84 I \sqrt{3})^{17/3} \sqrt{3} - 62124659142819840 I (28 + 84 I \sqrt{3})^{16/3} \sqrt{3} \\
& + 8981525301213265920 I (28 + 84 I \sqrt{3})^{14/3} \sqrt{3}
\end{aligned}$$

$$\begin{aligned}
& + 21067080980615921664 I \left(28 + 84 I \sqrt{3} \right)^{13/3} \sqrt{3} \\
& - 1455032604959126323200 I \left(28 + 84 I \sqrt{3} \right)^{11/3} \sqrt{3} \\
& - 1378604169394372214784 I \left(28 + 84 I \sqrt{3} \right)^{10/3} \sqrt{3} \\
& - 30432651567418526662656 I \left(28 + 84 I \sqrt{3} \right)^{8/3} \sqrt{3} \\
& - 505796908061911607672832 I \left(28 + 84 I \sqrt{3} \right)^{7/3} \sqrt{3} \\
& + 28210404950164334614413312 I \left(28 + 84 I \sqrt{3} \right)^{5/3} \sqrt{3} \\
& + 63405815111758254270578688 I \left(28 + 84 I \sqrt{3} \right)^{4/3} \sqrt{3} \\
& - 179110975573403061815976394752 - 387072 \left(28 + 84 I \sqrt{3} \right)^{31/3} \\
& - 187280916480 \left(28 + 84 I \sqrt{3} \right)^{23/3} + 294174720 \left(28 + 84 I \sqrt{3} \right)^{28/3} \\
& - 93714776064 \left(28 + 84 I \sqrt{3} \right)^{25/3} - 73851444904673300369620598784 I \sqrt{3} \\
& + 1564116383144953892460036096 \left(28 + 84 I \sqrt{3} \right)^{2/3} \\
& - 845113247946895521742848 \left(28 + 84 I \sqrt{3} \right)^{7/3} \\
& + 5980190827913209184256 \left(28 + 84 I \sqrt{3} \right)^{10/3} \\
& + 182943643778962151178240 \left(28 + 84 I \sqrt{3} \right)^{8/3}
\end{aligned}$$

$$\begin{aligned}
& - 19380450811562839775379456 \left(28 + 84 I\sqrt{3} \right)^{5|3} \\
& + 47997952519559019547852800 \left(28 + 84 I\sqrt{3} \right)^{4|3} - 1232746806509568 \left(28 \right. \\
& \left. + 84 I\sqrt{3} \right)^{19|3} + 57191230328537088 \left(28 + 84 I\sqrt{3} \right)^{16|3} \\
& - 2276745623036792181822062592 I \left(28 + 84 I\sqrt{3} \right)^{2|3} \sqrt{3} \\
& - 148165873738186752 \left(28 + 84 I\sqrt{3} \right)^{17|3} - 18383295705646104576 \left(28 \right. \\
& \left. + 84 I\sqrt{3} \right)^{13|3} + 15417876676608 \left(28 + 84 I\sqrt{3} \right)^{22|3} + 333659680800768 \left(28 \right. \\
& \left. + 84 I\sqrt{3} \right)^{20|3} + 29560434240377585664 \left(28 + 84 I\sqrt{3} \right)^{14|3} \\
& - 2858126071361483833344 \left(28 + 84 I\sqrt{3} \right)^{11|3} \sqrt{3}) / (27 \left(-526848 \left(28 \right. \right. \\
& \left. \left. + 84 I\sqrt{3} \right)^{4|3} - 14751744 \left(28 + 84 I\sqrt{3} \right)^{2|3} - 672 \left(28 + 84 I\sqrt{3} \right)^{8|3} \right. \\
& \left. - 24 \left(28 + 84 I\sqrt{3} \right)^{10|3} + 60928 I \left(28 + 84 I\sqrt{3} \right)^{5|3} \sqrt{3} - 224 I \left(28 \right. \right. \\
& \left. \left. + 84 I\sqrt{3} \right)^{8|3} \sqrt{3} - 2176 I \left(28 + 84 I\sqrt{3} \right)^{7|3} \sqrt{3} + 6528 \left(28 + 84 I\sqrt{3} \right)^{7|3} \right. \\
& \left. + 182784 \left(28 + 84 I\sqrt{3} \right)^{5|3} + 175616 I \left(28 + 84 I\sqrt{3} \right)^{4|3} \sqrt{3} - 4917248 I \left(28 \right. \right. \\
& \left. \left. + 84 I\sqrt{3} \right)^{2|3} \sqrt{3} + 8 I \left(28 + 84 I\sqrt{3} \right)^{10|3} \sqrt{3} \right) \left(13429889549991936 \sqrt{3} \right. \\
& \left. + 84 I\sqrt{3} \right)^{2|3} - 16785408 I \left(28 + 84 I\sqrt{3} \right)^{17|3} + 1025310720 I \left(28 + 84 I\sqrt{3} \right)^{14|3}
\end{aligned}$$

- #Final answer is of form : $X(t) = F \cdot \text{Column}(v2, 1) \cdot e^{\text{Row}(e2, 1) \cdot t} + G \cdot \text{Column}(v2, 2) \cdot e^{\text{Row}(e2, 2) \cdot t} + H \cdot \text{Column}(v2, 3) \cdot e^{\text{Row}(e2, 3) \cdot t}$

> $dsolve(\{diff(x1(t), t) = x1(t) + x2(t) + x3(t), diff(x2(t), t) = x1(t) + x2(t), diff(x3(t), t) = x1(t), x1(0) = 1, x2(0) = 2, x3(0) = -1\}, \{x1(t), x2(t), x3(t)\})$

$$\left\{ x1(t) = - \left(\left(1008 I (28 + 84 I \sqrt{3})^{5/3} \sqrt{3} + 18 I (28 + 84 I \sqrt{3})^{8/3} \sqrt{3} + 6 (28 + 84 I \sqrt{3})^{8/3} - 24192 I (28 + 84 I \sqrt{3})^{4/3} \sqrt{3} + 338688 I (28 + 84 I \sqrt{3})^{1/3} \sqrt{3} + 76608 (28 + 84 I \sqrt{3})^{4/3} - 193536 I (28 + 84 I \sqrt{3})^{2/3} \sqrt{3} - 4368 (28 + 84 I \sqrt{3})^{5/3} + 24385536 I \sqrt{3} - 105670656 + 8015616 (28 + 84 I \sqrt{3})^{1/3} - 2585856 (28 + 84 I \sqrt{3})^{2/3} \right) \frac{(I (28 + 84 I \sqrt{3})^{2/3} \sqrt{3} + (28 + 84 I \sqrt{3})^{2/3} - 28 I \sqrt{3} - 8 (28 + 84 I \sqrt{3})^{1/3} + 28)_t}{12 (28 + 84 I \sqrt{3})^{1/3}} \right) \right\}$$

$$+ 76608 (28 + 84 I \sqrt{3})^{4/3} - 193536 I (28 + 84 I \sqrt{3})^{2/3} \sqrt{3} - 4368 (28 + 84 I \sqrt{3})^{5/3} + 24385536 I \sqrt{3} - 105670656 + 8015616 (28 + 84 I \sqrt{3})^{1/3}$$

$$- 2585856 (28 + 84 I \sqrt{3})^{2/3})$$

$$e^{- \frac{(I (28 + 84 I \sqrt{3})^{2/3} \sqrt{3} + (28 + 84 I \sqrt{3})^{2/3} - 28 I \sqrt{3} - 8 (28 + 84 I \sqrt{3})^{1/3} + 28)_t}{12 (28 + 84 I \sqrt{3})^{1/3}}}$$

$$(3 (I (28 + 84 I \sqrt{3})^{4/3} \sqrt{3} + 56 I (28 + 84 I \sqrt{3})^{1/3} \sqrt{3} - 840 I \sqrt{3} + 504) ($$

$$- 18 (28 + 84 I \sqrt{3})^{4/3} + 2 I (28 + 84 I \sqrt{3})^{4/3} \sqrt{3} - 1568 I (28 + 84 I \sqrt{3})^{1/3}$$

$$^3 \sqrt{3})) - \left((105670656 + 1008 I (28 + 84 I \sqrt{3})^{5/3} \sqrt{3} + 18 I (28 + 84 I \sqrt{3})^{8/3} \sqrt{3} - 20160 (28 + 84 I \sqrt{3})^{5/3} \sqrt{3} + 24385536 I \sqrt{3} - 105670656 + 8015616 (28 + 84 I \sqrt{3})^{1/3} - 2585856 (28 + 84 I \sqrt{3})^{2/3} \right)$$

$$+ 84 \text{I} \sqrt{3})^{1/3} + 6 (28 + 84 \text{I} \sqrt{3})^{8/3} - 4368 (28 + 84 \text{I} \sqrt{3})^{5/3} + 24192 \text{I} (28$$

$$+ 84 \text{I} \sqrt{3})^{4/3} \sqrt{3} + 3048192 \text{I} (28 + 84 \text{I} \sqrt{3})^{1/3} \sqrt{3} + 1257984 \text{I} (28$$

$$+ 84 \text{I} \sqrt{3})^{2/3} \sqrt{3} - 24385536 \text{I} \sqrt{3})$$

$$\left. \frac{\left(\text{I} (28 + 84 \text{I} \sqrt{3})^{2/3} \sqrt{3} - (28 + 84 \text{I} \sqrt{3})^{2/3} - 28 \text{I} \sqrt{3} + 8 (28 + 84 \text{I} \sqrt{3})^{1/3} - 28 \right)_t}{12 (28 + 84 \text{I} \sqrt{3})^{1/3}} \right\}$$

$$(3 (\text{I} (28 + 84 \text{I} \sqrt{3})^{4/3} \sqrt{3} + 56 \text{I} (28 + 84 \text{I} \sqrt{3})^{1/3} \sqrt{3} - 840 \text{I} \sqrt{3} + 504) ($$

$$- 18 (28 + 84 \text{I} \sqrt{3})^{4/3} + 2 \text{I} (28 + 84 \text{I} \sqrt{3})^{4/3} \sqrt{3} - 1568 \text{I} (28 + 84 \text{I} \sqrt{3})^{1/3}$$

$$^3 \sqrt{3})) + \left((- 16128 \text{I} \sqrt{3} + 2 \text{I} (28 + 84 \text{I} \sqrt{3})^{4/3} \sqrt{3} - 18 (28 + 84 \text{I} \sqrt{3})^{4/3}$$

$$+ 32256 - 288 \text{I} (28 + 84 \text{I} \sqrt{3})^{2/3} \sqrt{3} - 1568 \text{I} (28 + 84 \text{I} \sqrt{3})^{1/3} \sqrt{3}$$

$$- 1440 (28 + 84 \text{I} \sqrt{3})^{2/3}) \text{e}^{\frac{\left((28 + 84 \text{I} \sqrt{3})^{2/3} + 4 (28 + 84 \text{I} \sqrt{3})^{1/3} + 28 \right)_t}{6 (28 + 84 \text{I} \sqrt{3})^{1/3}}} \right\} (3 ($$

$$\begin{aligned}
& -18 \left(28 + 84 I \sqrt{3}\right)^{4/3} + 2 I \left(28 + 84 I \sqrt{3}\right)^{4/3} \sqrt{3} - 1568 I \left(28 + 84 I \sqrt{3}\right)^{1/3} \\
& \left(\left(I \left(28 + 84 I \sqrt{3}\right)^{4/3} \sqrt{3} - \left(28 + 84 I \sqrt{3}\right)^{4/3} + 56 I \left(28 + 84 I \sqrt{3}\right)^{1/3}\right)\right), x2(t) = -\left(\left(I \left(28 + 84 I \sqrt{3}\right)^{4/3} \sqrt{3} - \left(28 + 84 I \sqrt{3}\right)^{4/3} + 56 I \left(28 + 84 I \sqrt{3}\right)^{1/3}\right)\right), \\
& + 84 I \sqrt{3}\right)^{1/3} \sqrt{3} - 48 \left(28 + 84 I \sqrt{3}\right)^{2/3} - 1008 I \sqrt{3} - 56 \left(28 + 84 I \sqrt{3}\right)^{1/3} \\
& - 336 \left(1008 I \left(28 + 84 I \sqrt{3}\right)^{5/3} \sqrt{3} + 18 I \left(28 + 84 I \sqrt{3}\right)^{8/3} \sqrt{3} + 6 \left(28 + 84 I \sqrt{3}\right)^{8/3} - 24192 I \left(28 + 84 I \sqrt{3}\right)^{4/3} \sqrt{3} + 338688 I \left(28 + 84 I \sqrt{3}\right)^{1/3} \sqrt{3}\right. \\
& + 76608 \left(28 + 84 I \sqrt{3}\right)^{4/3} - 193536 I \left(28 + 84 I \sqrt{3}\right)^{2/3} \sqrt{3} - 4368 \left(28 + 84 I \sqrt{3}\right)^{5/3} + 24385536 I \sqrt{3} - 105670656 + 8015616 \left(28 + 84 I \sqrt{3}\right)^{1/3} \\
& - 2585856 \left(28 + 84 I \sqrt{3}\right)^{2/3}) \\
& \left. e^{-\frac{\left(I \left(28 + 84 I \sqrt{3}\right)^{2/3} \sqrt{3} + \left(28 + 84 I \sqrt{3}\right)^{2/3} - 28 I \sqrt{3} - 8 \left(28 + 84 I \sqrt{3}\right)^{1/3} + 28\right)_t}{12 \left(28 + 84 I \sqrt{3}\right)^{1/3}}}\right)
\end{aligned}$$

$$\left(216 \left(28 + 84 I \sqrt{3}\right)^2\right)^{1/3} \left(I \left(28 + 84 I \sqrt{3}\right)^4\right)^{1/3} \sqrt{3} + 56 I \left(28 + 84 I \sqrt{3}\right)^1\right)^{1/3} \sqrt{3}$$

$$- 840 I \sqrt{3} + 504\right) \left(-18 \left(28 + 84 I \sqrt{3}\right)^4\right)^{1/3} + 2 I \left(28 + 84 I \sqrt{3}\right)^4\right)^{1/3} \sqrt{3}$$

$$- 1568 I \left(28 + 84 I \sqrt{3}\right)^1\right)^{1/3} \sqrt{3})\Big) + \left(\left(I \left(28 + 84 I \sqrt{3}\right)^4\right)^{1/3} \sqrt{3} + \left(28 + 84 I \sqrt{3}\right)^4\right)^{1/3} + 56 I \left(28 + 84 I \sqrt{3}\right)^1\right)^{1/3} \sqrt{3}$$

$$+ 1344\right) \left(105670656 + 1008 I \left(28 + 84 I \sqrt{3}\right)^5\right)^{1/3} \sqrt{3} + 18 I \left(28 + 84 I \sqrt{3}\right)^8\right)^{1/3} \sqrt{3}$$

$$- 20160 \left(28 + 84 I \sqrt{3}\right)^4\right)^{1/3} + 607488 \left(28 + 84 I \sqrt{3}\right)^2\right)^{1/3} + 2596608 \left(28 + 84 I \sqrt{3}\right)^1\right)^{1/3} + 6 \left(28 + 84 I \sqrt{3}\right)^8 - 4368 \left(28 + 84 I \sqrt{3}\right)^5\right)^{1/3} + 24192 I \left(28 + 84 I \sqrt{3}\right)^4\right)^{1/3} \sqrt{3} + 3048192 I \left(28 + 84 I \sqrt{3}\right)^1\right)^{1/3} \sqrt{3} + 1257984 I \left(28 + 84 I \sqrt{3}\right)^2\right)^{1/3} \sqrt{3} - 24385536 I \sqrt{3}\Big)$$

$$e^{\frac{\left(I(28 + 84 I \sqrt{3})^2\right)^{1/3} \sqrt{3} - \left(28 + 84 I \sqrt{3}\right)^2\right)^{1/3} - 28 I \sqrt{3} + 8 \left(28 + 84 I \sqrt{3}\right)^1\right)^{1/3} - 28}{12 \left(28 + 84 I \sqrt{3}\right)^1}}\Bigg)$$

$$(216 (28 + 84 I \sqrt{3})^{2/3} (I (28 + 84 I \sqrt{3})^{4/3} \sqrt{3} + 56 I (28 + 84 I \sqrt{3})^{1/3} \sqrt{3}$$

$$- 840 I \sqrt{3} + 504) (-18 (28 + 84 I \sqrt{3})^{4/3} + 2 I (28 + 84 I \sqrt{3})^{4/3} \sqrt{3}$$

$$- 1568 I (28 + 84 I \sqrt{3})^{1/3} \sqrt{3})) + \left(((28 + 84 I \sqrt{3})^{4/3} - 24 (28 + 84 I \sqrt{3})^{2/3} + 168 I \sqrt{3} + 5$$

$$+ 84 I \sqrt{3})^{4/3} + 32256 - 288 I (28 + 84 I \sqrt{3})^{2/3} \sqrt{3} - 1568 I (28$$

$$+ 84 I \sqrt{3})^{1/3} \sqrt{3} - 1440 (28 + 84 I \sqrt{3})^{2/3})$$

$$\frac{\left((28 + 84 I \sqrt{3})^{2/3} + 4 (28 + 84 I \sqrt{3})^{1/3} + 28\right)_t}{6 (28 + 84 I \sqrt{3})^{1/3}} \Bigg) \Bigg/ (108 (28 + 84 I \sqrt{3})^{2/3} ($$

$$- 18 (28 + 84 I \sqrt{3})^{4/3} + 2 I (28 + 84 I \sqrt{3})^{4/3} \sqrt{3} - 1568 I (28 + 84 I \sqrt{3})^{1/3}$$

$$^3 \sqrt{3}))_{x3(t)} = \left(I (28 + 84 I \sqrt{3})^{4/3} \sqrt{3} - (28 + 84 I \sqrt{3})^{4/3}\right.$$

$$- 112 I (28 + 84 I \sqrt{3})^{1/3} \sqrt{3} - 24 (28 + 84 I \sqrt{3})^{2/3} - 336 I \sqrt{3} + 112 (28$$

$$+ 84 \text{I} \sqrt{3})^{1/3} - 1680) (1008 \text{I} (28 + 84 \text{I} \sqrt{3})^{5/3} \sqrt{3} + 18 \text{I} (28 + 84 \text{I} \sqrt{3})^{8/3}$$

$$^3 \sqrt{3} + 6 (28 + 84 \text{I} \sqrt{3})^{8/3} - 24192 \text{I} (28 + 84 \text{I} \sqrt{3})^{4/3} \sqrt{3} + 338688 \text{I} (28$$

$$+ 84 \text{I} \sqrt{3})^{1/3} \sqrt{3} + 76608 (28 + 84 \text{I} \sqrt{3})^{4/3} - 193536 \text{I} (28 + 84 \text{I} \sqrt{3})^{2/3} \sqrt{3}$$

$$- 4368 (28 + 84 \text{I} \sqrt{3})^{5/3} + 24385536 \text{I} \sqrt{3} - 105670656 + 8015616 (28$$

$$+ 84 \text{I} \sqrt{3})^{1/3} - 2585856 (28 + 84 \text{I} \sqrt{3})^{2/3})$$

$$\left. \left. - \frac{(\text{I}(28 + 84 \text{I} \sqrt{3})^{2/3} \sqrt{3} + (28 + 84 \text{I} \sqrt{3})^{2/3} - 28 \text{I} \sqrt{3} - 8 (28 + 84 \text{I} \sqrt{3})^{1/3} + 28) t}{12 (28 + 84 \text{I} \sqrt{3})^{1/3}} \right) \right\}$$

$$(216 (28 + 84 \text{I} \sqrt{3})^{2/3} (\text{I} (28 + 84 \text{I} \sqrt{3})^{4/3} \sqrt{3} + 56 \text{I} (28 + 84 \text{I} \sqrt{3})^{1/3} \sqrt{3}$$

$$- 840 \text{I} \sqrt{3} + 504) (-18 (28 + 84 \text{I} \sqrt{3})^{4/3} + 2 \text{I} (28 + 84 \text{I} \sqrt{3})^{4/3} \sqrt{3}$$

$$- 1568 \text{I} (28 + 84 \text{I} \sqrt{3})^{1/3} \sqrt{3})) - \left((\text{I} (28 + 84 \text{I} \sqrt{3})^{4/3} \sqrt{3} + (28 + 84 \text{I} \sqrt{3})^{4/3} - 112 \text{I} (28 + 84 \text{I} \sqrt{3})^{1/3} \sqrt{3}) t \right)$$

$$- 336 \Big) \left(105670656 + 1008 I \left(28 + 84 I \sqrt{3} \right)^5 \sqrt{3} + 18 I \left(28 + 84 I \sqrt{3} \right)^8 \sqrt{3} \right.$$

$$- 20160 \left(28 + 84 I \sqrt{3} \right)^4 \sqrt{3} + 607488 \left(28 + 84 I \sqrt{3} \right)^2 \sqrt{3} + 2596608 \left(28 \right.$$

$$+ 84 I \sqrt{3} \Big)^1 \sqrt{3} + 6 \left(28 + 84 I \sqrt{3} \right)^8 \sqrt{3} - 4368 \left(28 + 84 I \sqrt{3} \right)^5 \sqrt{3} + 24192 I \left(28 \right.$$

$$+ 84 I \sqrt{3} \Big)^4 \sqrt{3} + 3048192 I \left(28 + 84 I \sqrt{3} \right)^1 \sqrt{3} + 1257984 I \left(28 \right.$$

$$+ 84 I \sqrt{3} \Big)^2 \sqrt{3} - 24385536 I \sqrt{3} \Big)$$

$$\left. e^{\frac{\left(I \left(28 + 84 I \sqrt{3} \right)^2 \sqrt{3} - \left(28 + 84 I \sqrt{3} \right)^2 \sqrt{3} - 28 I \sqrt{3} + 8 \left(28 + 84 I \sqrt{3} \right)^1 \sqrt{3} - 28 \right) t}{12 \left(28 + 84 I \sqrt{3} \right)^1 \sqrt{3}}} \right)$$

$$\left(216 \left(28 + 84 I \sqrt{3} \right)^2 \sqrt{3} \left(I \left(28 + 84 I \sqrt{3} \right)^4 \sqrt{3} + 56 I \left(28 + 84 I \sqrt{3} \right)^1 \sqrt{3} \right. \right.$$

$$\left. \left. - 840 I \sqrt{3} + 504 \right) \left(- 18 \left(28 + 84 I \sqrt{3} \right)^4 \sqrt{3} + 2 I \left(28 + 84 I \sqrt{3} \right)^4 \sqrt{3} \right. \right.$$

$$\left. \left. - 1568 I \left(28 + 84 I \sqrt{3} \right)^1 \sqrt{3} \right) \right) - \left(\left(\left(28 + 84 I \sqrt{3} \right)^4 \sqrt{3} - 12 \left(28 + 84 I \sqrt{3} \right)^2 \sqrt{3} - 336 I \sqrt{3} - 1 \right. \right.$$

$$\left. \left. + 84 I \sqrt{3} \right)^4 \sqrt{3} + 32256 - 288 I \left(28 + 84 I \sqrt{3} \right)^2 \sqrt{3} - 1568 I \left(28 \right. \right.$$

$$\begin{aligned}
 & + 84 I \sqrt{3})^{1/3} \sqrt{3} - 1440 (28 + 84 I \sqrt{3})^{2/3}) \\
 & \frac{\left((28 + 84 I \sqrt{3})^{2/3} + 4 (28 + 84 I \sqrt{3})^{1/3} + 28 \right)_t}{6 (28 + 84 I \sqrt{3})^{1/3}} \Bigg) \Bigg/ \left(108 (28 + 84 I \sqrt{3})^{2/3} \left(\right. \right. \\
 & \left. \left. - 18 (28 + 84 I \sqrt{3})^{4/3} + 2 I (28 + 84 I \sqrt{3})^{4/3} \sqrt{3} - 1568 I (28 + 84 I \sqrt{3})^{1/3} \right. \right. \\
 & \left. \left. \sqrt{3} \right) \right) \Bigg\}
 \end{aligned}$$

>

>

> **read** "C://Users/an646/Documents/M17.txt"

> #Problem 4 part i

> $M := \text{Matrix}([[1, 1, 1], [1, 1, 1], [1, 1, 1]])$

$$M := \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad (11)$$

> $L := HW2g(.5, .5, M) :$

>

>

> $Orb2(L, u, v, [5, 5], 1, 5)$

$$[[5, 5], [0.5625000000, 0.3750000000], [0.5625000000, 0.3750000000], [0.5625000000, 0.3750000000], [0.5625000000, 0.3750000000]] \quad (12)$$

> $Orb2(L, u, v, [8, 12], 1, 5)$

$$[[8, 12], [0.5625000000, 0.3750000000], [0.5625000000, 0.3750000000], [0.5625000000, 0.3750000000], [0.5625000000, 0.3750000000]] \quad (13)$$

> $Orb2(L, u, v, [1000, 16], 1, 5)$

$$[[1000, 16], [0.5625000000, 0.3750000000], [0.5625000000, 0.3750000000], [0.5625000000, 0.3750000000], [0.5625000000, 0.3750000000]] \quad (14)$$

> $M2 := \text{Matrix}([[1, 0.5, 0.7], [.42, .13, 0.6], [0.04, 0.92, 0.83]])$

$$M2 := \begin{bmatrix} 1 & 0.5 & 0.7 \\ 0.42 & 0.13 & 0.6 \\ 0.04 & 0.92 & 0.83 \end{bmatrix} \quad (15)$$

> $N := HW2g(.5, .5, M2)$

$$N := [0.7280487804, 0.2560975610] \quad (16)$$

> $Orb2(N, u, v, [5, 5], 1, 5)$

$$[[5, 5], [0.7280487804, 0.2560975610], [0.7280487804, 0.2560975610], [0.7280487804, 0.2560975610], [0.7280487804, 0.2560975610]] \quad (17)$$

>

```

M3 := RandomMatrix(3, generator = 0 .. 1.00)
M3 := [ 0.959743958516081075  0.162611735194630569  0.276025076998578367
        0.498364051982142953  0.655098003973840659  0.754686681982360885
        0.118997681558376645  0.679702676853674803  0.709364830858072559 ] (18)

> M4 := RandomMatrix(3, generator = 0 .. 1.00)
M4 := [ 0.959291425205444304  0.505957051665142377  0.223811939491136980
        0.890903252535798496  0.255095115459269106  0.585267750979777346
        0.699076722656685967  0.751267059305652851  0.340385726666133204 ] (19)

> N3 := HW2g(.5, .5, M3)
N3 := [ 0.6388940210, 0.2891430545 ] (20)

> Orb2(N3, u, v, [5, 5], 1, 5)
[[5, 5], [0.6388940210, 0.2891430545], [0.6388940210, 0.2891430545], [0.6388940210,
 0.2891430545], [0.6388940210, 0.2891430545]] (21)

> N4 := HW2g(.5, .5, M4)
N4 := [ 0.6592618233, 0.3163154458 ] (22)

> Orb2(N4, u, v, [5, 5], 1, 5)
[[5, 5], [0.6592618233, 0.3163154458], [0.6592618233, 0.3163154458], [0.6592618233,
 0.3163154458], [0.6592618233, 0.3163154458]] (23)

> M5 := RandomMatrix(3, generator = 0 .. 1.00)
M5 := [ 0.929263623187227772  0.254282178971531048  0.149294005559057474
        0.243524968724989299  0.840717255983662537  0.138624442828679140
        0.814284826068816359  0.257508254123736458  0.547215529963803071 ] (24)

> M6 := RandomMatrix(3, generator = 0 .. 1.00)
M6 := [ 0.549723608291139532  0.351659507062996757  0.251083857976031055
        0.585264091152724264  0.473288848902729264  0.196595250431208179
        0.830828627896290861  0.616044676146639159  0.349983765984808737 ] (25)

> M7 := RandomMatrix(3, generator = 0 .. 1.00)
M7 := [ 0.530797553008972689  0.567821640725221122  0.757200229110721268
        0.0539501186666071497  0.380445846975356661  0.285839018820373547
        0.0758542895630636149  0.753729094278495326  0.917193663829810046 ] (26)

> M8 := RandomMatrix(3, generator = 0 .. 1.00)
M8 := [ 0.794284540683906970  0.0119020695012413968  0.129906208473730134
        0.162182308193242752  0.469390641058205826  0.934010684229182986
        0.337122644398881510  0.568823660872192716  0.779167230102011166 ] (27)

> M9 := RandomMatrix(3, generator = 0 .. 1.00)

```

$$M9 := \begin{bmatrix} 0.450541598502497753 & 0.654079098476782250 & 0.165648729499780933 \\ 0.748151592823709466 & 0.262971284540144290 & 0.528533135506212726 \\ 0.689214503140007761 & 0.601981941401636522 & 0.311215042044804879 \end{bmatrix} \quad (28)$$

> $M10 := \text{RandomMatrix}(3, \text{generator} = 0 .. 1.00)$

$$M10 := \begin{bmatrix} 0.442678269775446331 & 0.538342435260057095 & 0.913337361501669553 \\ 0.0781755287531836762 & 0.825816977489547410 & 0.228976968716818829 \\ 0.996134716626885464 & 0.152378018969222984 & 0.0838213779969325667 \end{bmatrix} \quad (29)$$

> $M11 := \text{RandomMatrix}(3, \text{generator} = 0 .. 1.00)$

$$M11 := \begin{bmatrix} 0.259870402850654214 & 0.868694705363509678 & 0.00463422413406744393 \\ 0.399782649098896492 & 0.817303220653432971 & 0.961898080855053683 \\ 0.0844358455109103234 & 0.774910464711502378 & 0.106652770180584389 \end{bmatrix} \quad (30)$$

> $N5 := HW2g(.5, .5, M5)$

$$N5 := [0.6122029460, 0.2951167366] \quad (31)$$

> $N6 := HW2g(.5, .5, M6)$

$$N6 := [0.5798697444, 0.3597598102] \quad (32)$$

> $N7 := HW2g(.5, .5, M7)$

$$N7 := [0.6110799939, 0.3268779179] \quad (33)$$

> $N8 := HW2g(.5, .5, M8)$

$$N8 := [0.6946045903, 0.2237769919] \quad (34)$$

> $N9 := HW2g(.5, .5, M9)$

$$N9 := [0.5754004314, 0.3935264169] \quad (35)$$

> $N10 := HW2g(.5, .5, M10)$

$$N10 := [0.5078964383, 0.3825795313] \quad (36)$$

> $N11 := HW2g(.5, .5, M11)$

$$N11 := [0.4682857319, 0.4446059100] \quad (37)$$

> $Orb2(N5, u, v, [5, 5], 1, 5)$

$$[[5, 5], [0.6122029460, 0.2951167366], [0.6122029460, 0.2951167366], [0.6122029460, 0.2951167366], [0.6122029460, 0.2951167366]] \quad (38)$$

> $Orb2(N6, u, v, [5, 5], 1, 5)$

$$[[5, 5], [0.5798697444, 0.3597598102], [0.5798697444, 0.3597598102], [0.5798697444, 0.3597598102], [0.5798697444, 0.3597598102], [0.5798697444, 0.3597598102]] \quad (39)$$

> $Orb2(N7, u, v, [5, 5], 1, 5)$

$$[[5, 5], [0.6110799939, 0.3268779179], [0.6110799939, 0.3268779179], [0.6110799939, 0.3268779179], [0.6110799939, 0.3268779179], [0.6110799939, 0.3268779179]] \quad (40)$$

> $Orb2(N8, u, v, [5, 5], 1, 5)$

$$[[5, 5], [0.6946045903, 0.2237769919], [0.6946045903, 0.2237769919], [0.6946045903, 0.2237769919], [0.6946045903, 0.2237769919], [0.6946045903, 0.2237769919]] \quad (41)$$

> $Orb2(N9, u, v, [5, 5], 1, 5)$
[[5, 5], [0.5754004314, 0.3935264169], [0.5754004314, 0.3935264169], [0.5754004314, 0.3935264169], [0.5754004314, 0.3935264169]]

(42)

=> $Orb2(N10, u, v, [5, 5], 1, 5)$
[[5, 5], [0.5078964383, 0.3825795313], [0.5078964383, 0.3825795313], [0.5078964383, 0.3825795313], [0.5078964383, 0.3825795313]]

(43)

=> $Orb2(N11, u, v, [5, 5], 1, 5)$
[[5, 5], [0.4682857319, 0.4446059100], [0.4682857319, 0.4446059100], [0.4682857319, 0.4446059100], [0.4682857319, 0.4446059100]]

(44)

[>