```
> #OK to post
>
> #Anne Somalwar, hw17, 11.1.2021
>
>
> read "C:/Users/aks238/OneDrive - Rutgers University/Documents/M17.txt"
>
>
>
>
> #1
> #(iii)
> sys := diff(x(t), t) = 3·x(t) - y(t), diff(y(t), t) = 2·x(t)
```

$$sys := \frac{d}{dt}x(t) = 3x(t) - y(t),\ \frac{d}{dt}y(t) = 2x(t) \tag{1}$$

```
> dsolve([sys, x(0) = 2, y(0) = 3])
```

$$\{x(t) = e^{t} + e^{2t},\ y(t) = 2e^{t} + e^{2t}\} \tag{2}$$

```
>
>
>
> #2
> #(iii)
>
> sys := diff(x(t), t) = x(t) + 9·y(t), diff(y(t), t) = 3·x(t)
```

$$sys := \frac{d}{dt}x(t) = x(t) + 9y(t),\ \frac{d}{dt}y(t) = 3x(t) \tag{3}$$

```
> dsolve([sys, x(0) = 4, y(0) = 0])
```

$$\left\{x(t) = \frac{12\left(\frac{\sqrt{109}}{6} + \frac{1}{6}\right)\sqrt{109}\,e^{\frac{\left(1+\sqrt{109}\right)t}{2}}}{109} - \frac{12\left(-\frac{\sqrt{109}}{6} + \frac{1}{6}\right)\sqrt{109}\,e^{-\frac{\left(-1+\sqrt{109}\right)t}{2}}}{109},\ y(t) = \frac{12\sqrt{109}\,e^{\frac{\left(1+\sqrt{109}\right)t}{2}}}{109} - \frac{12\sqrt{109}\,e^{-\frac{\left(-1+\sqrt{109}\right)t}{2}}}{109}\right\} \tag{4}$$

```
> evalf(%)
```

$$\{x(t) = 2.191565258\,e^{5.720153255t} + 1.808434742\,e^{-4.720153255t},\ y(t) = 1.149391542\,e^{5.720153255t} - 1.149391542\,e^{-4.720153255t}\} \tag{5}$$

```
> 
> 
> 
> 
> 
> #3
> M := Matrix([[1, 1, 1], [1, 1, 0], [1, 0, 0]])
```

$$M := \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{6}$$

```
> Eigenvalues(M)
```

$$\left[\left[\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}+\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}\right],\right. \tag{7}$$

$$\left[-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}\right.$$

$$\left.+\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\right],$$

$$\left[-\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{12}-\frac{7}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}+\frac{2}{3}\right.$$

$$\left.\left.-\frac{I\sqrt{3}\left(\frac{\left(28+84\,I\sqrt{3}\right)^{1/3}}{6}-\frac{14}{3\left(28+84\,I\sqrt{3}\right)^{1/3}}\right)}{2}\right]\right]$$

```
> evalf(Eigenvectors(M))
```

$$\left[\begin{bmatrix} 2.246979605+1.\times10^{-10}\,I \\ -0.8019377358-1.866025404\times10^{-10}\,I \\ 0.5549581322-1.339745960\times10^{-11}\,I \end{bmatrix},\right. \tag{8}$$

$$[[2.246979634+1.514675242\times10^{-9}\,I,$$

$$-0.8019377350+3.686305552\times10^{-10}\,I,\ 0.5549581323-2.254559307\times10^{-11}\,I],$$

$$[1.801937769+1.888769131\times10^{-9}\,I,\ 0.4450418682-5.947994638\times10^{-10}\,I,$$

$$\left.-1.246979604+5.809451696\times10^{-11}\,I]\right],$$

```
  [1., 1., 1.]]
> #This is really messy, I'm not really sure whether I should continue.
>
>
>
>
> #4
>
> (i)
> HW2g(a, b, [[1, 1, 1], [1, 1, 1], [1, 1, 1]])
```

$$\left[a^2 + a\,b + \frac{1}{4}\,b^2,\ -2\,a\,b - 2\,a^2 + 2\,a - \frac{1}{2}\,b^2 + b\right] \tag{9}$$

```
> HW2g(a^2 + a*b + 1/4*b^2, -2*a*b - 2*a^2 + 2*a - 1/2*b^2 + b, [[1, 1, 1], [1, 1, 1], [1, 1, 1]])
```

$$\left[a^2 + a\,b + \frac{1}{4}\,b^2,\ -2\,a\,b - 2\,a^2 + 2\,a - \frac{1}{2}\,b^2 + b\right] \tag{10}$$

```
> #This shows that u and v stabilize after one generation.
>
>
> #(ii)
>
> #Try 1
> HW2g(0.5, 0.5, [[0.1, 0.5, 0.3], [0.7, 1, 1], [1, 0.6, 1]])
```

$$[0.4130434783,\ 0.4782608696] \tag{11}$$

```
> HW2g(0.4130434783, 0.4782608696, [[0.1, 0.5, 0.3], [0.7, 1, 1], [1, 0.6, 1]])
```

$$[0.3030010400,\ 0.5231763484] \tag{12}$$

```
> #This did not stablize in one generation
> HW2g(0.3030010400, 0.5231763484, [[0.1, 0.5, 0.3], [0.7, 1, 1], [1, 0.6, 1]])
```

$$[0.2407931579,\ 0.5202658091] \tag{13}$$

```
> #Try 2
  HW2g(0.5, 0.5, [[0.001, 0.5, 0.3], [0.07, 1, 1], [0.001, 0.6, 1]])
```

$$[0.3411839593,\ 0.4996817314] \tag{14}$$

```
> HW2g(0.3411839593, 0.4996817314, [[0.1, 0.5, 0.3], [0.7, 1, 1], [1, 0.6, 1]])
```

$$[0.2559393146,\ 0.5243949282] \tag{15}$$

```
> #This one didn't either, and the allele corresponding to frequency u seems to be dying out.
>
> #Try 3
> HW2g(0.5, 0.5, [[0.001, 0.5, 0.3], [0.07, 1, 1], [0.001, 0.6, 1]])
```

$$[0.3411839593,\ 0.4996817314] \tag{16}$$

```
> HW2g(0.3411839593, 0.4996817314, [[0.001, 0.5, 0.3], [0.07, 1, 1], [0.001, 0.6, 1]])
```

$$[0.2154135743,\ 0.4911821954] \tag{17}$$

```
> HW2g(0.2154135743, 0.4911821954, [[0.001, 0.5, 0.3], [0.07, 1, 1], [0.001, 0.6, 1]] )
                                  [0.1420288148, 0.4473504268]                                  (18)
> #Same here.
>
> #Try 4
> HW2g(0.5, 0.5, [[0.1, 0.2, 0.3], [0.7, 0.45, 0.1], [0.81, 0.23, 0.87]] )
                                  [0.4568965518, 0.4655172414]                                  (19)
> HW2g(0.4568965518, 0.4655172414, [[0.1, 0.2, 0.3], [0.7, 0.45, 0.1], [0.81, 0.23, 0.87]] )
                                  [0.3848174762, 0.5180657384]                                  (20)
> HW2g(0.3848174762, 0.5180657384, [[0.1, 0.2, 0.3], [0.7, 0.45, 0.1], [0.81, 0.23, 0.87]] )
                                  [0.3533027752, 0.5242186006]                                  (21)
>
> #Try 5
> HW2g(0.5, 0.5, [[0.1, 0.02, 0.3], [0.7, 0.2345, 0.1], [0.62341, 0.24563, 0.8637]] )
                                  [0.4918207681, 0.4525841631]                                  (22)
> HW2g(0.4918207681, 0.4525841631, [[0.1, 0.02, 0.3], [0.7, 0.2345, 0.1], [0.62341, 0.24563,
    0.8637]] )
                                  [0.4322885263, 0.4970091656]                                  (23)
> HW2g(0.4322885263, 0.4970091656, [[0.1, 0.02, 0.3], [0.7, 0.2345, 0.1], [0.62341, 0.24563,
    0.8637]] )
                                  [0.4004295333, 0.5094533624]                                  (24)
>
>
> #Try 6
> HW2g(0.5, 0.5, [[0.8356734, 0.2368, 0.83567], [0.3462, 0.3456, 0.854], [0.1234, 0.7456,
    0.73456]] )
                                  [0.6878601694, 0.2631678288]                                  (25)
> HW2g(0.6878601694, 0.2631678288, [[0.8356734, 0.2368, 0.83567], [0.3462, 0.3456, 0.854],
    [0.1234, 0.7456, 0.73456]] )
                                  [0.7836208875, 0.1852290549]                                  (26)
> HW2g(0.7836208875, 0.1852290549, [[0.8356734, 0.2368, 0.83567], [0.3462, 0.3456,
    0.854], [0.1234, 0.7456, 0.73456]] )
                                  [0.8684933149, 0.1186108030]                                  (27)
> HW2g(0.8684933149, 0.1186108030, [[0.8356734, 0.2368, 0.83567], [0.3462, 0.3456, 0.854],
    [0.1234, 0.7456, 0.73456]] )
                                  [0.9336892495, 0.06269631722]                                 (28)
>
>
> #Try 7
> HW2g(0.5, 0.5, [[0.725, 0.342362, 0.56257], [0.35725, 0.6256, 0.7256], [0.7345, 0.824,
    0.234]] )
                                  [0.6005261894, 0.3231890166]                                  (29)
```

```
> HW2g(0.6005261894, 0.3231890166, [[0.725, 0.342362, 0.56257], [0.35725, 0.6256, 0.7256],
    [0.7345, 0.824, 0.234]] )
                          [0.6155678715, 0.3189015763]                          (30)
> HW2g(0.6155678715, 0.3189015763, [[0.725, 0.342362, 0.56257], [0.35725, 0.6256, 0.7256],
    [0.7345, 0.824, 0.234]] )
                          [0.6400084266, 0.3010270768]                          (31)
>
>
>
> #Try 8
>  HW2g(0.5, 0.5, [[0.7524, 0.1345, 0.34], [0.344, 0.3465, 0.123], [0.2345, 0.23, 0.87]] )
                          [0.6835774057, 0.2615062762]                          (32)
> HW2g(0.6835774057, 0.2615062762, [[0.7524, 0.1345, 0.34], [0.344, 0.3465, 0.123],
    [0.2345, 0.23, 0.87]] )
                          [0.8167623426, 0.1606239217]                          (33)
> HW2g(0.8167623426, 0.1606239217, [[0.7524, 0.1345, 0.34], [0.344, 0.3465, 0.123],
    [0.2345, 0.23, 0.87]] )
                          [0.9139381016, 0.08039458081]                         (34)
>
>
>
> #Try 9
>  HW2g(0.5, 0.5, [[0.968, 0.2, 0.3], [0.7, 0.87098, 0.1], [0.861, 0.75973, 0.87]] )
                          [0.5972095450, 0.3232918824]                          (35)
>  HW2g(0.5972095450, 0.3232918824, [[0.968, 0.2, 0.3], [0.7, 0.87098, 0.1], [0.861, 0.75973,
    0.87]] )
                          [0.6566299686, 0.2866337005]                          (36)
> HW2g(0.6566299686, 0.2866337005, [[0.968, 0.2, 0.3], [0.7, 0.87098, 0.1], [0.861, 0.75973,
    0.87]] )
                          [0.7238125574, 0.2376534203]                          (37)
>
>
>
> #Try 10
>  HW2g(0.5, 0.5, [[0.956, 0.789, 0.3], [0.07, 0.7587, 1], [0.76981, 0.766, 1]] )
                          [0.6120274313, 0.3142751680]                          (38)
> HW2g(0.6120274313, 0.3142751680, [[0.956, 0.789, 0.3], [0.07, 0.7587, 1], [0.76981, 0.766,
    1]] )
                          [0.6631250552, 0.2724785184]                          (39)
> HW2g(0.6631250552, 0.2724785184, [[0.956, 0.789, 0.3], [0.07, 0.7587, 1], [0.76981, 0.766,
    1]] )
                          [0.7184538738, 0.2342319498]                          (40)
>
```