

$$1) x'(t) = 3x(t) - y(t), y'(t) = 2x(t); x(0) = 2, y(0) = 3$$

$$i) y''(t) = 2x'(t)$$

$$\frac{1}{2} y''(t) = 3x(t) - y(t)$$

$$\frac{1}{2} y''(t) = 3(2) - y(t)$$

$$\frac{1}{2} y''(t) + y(t) - 6 = 0$$

$$\frac{1}{2} r^2 - 5 = 0$$

$$Ae^{\sqrt{10}t} + Be^{-\sqrt{10}t}$$

$$y(t) = Ae^{\sqrt{10}t} + Be^{-\sqrt{10}t}$$

$$\text{ii) } x'(t) = 3x(t) - y(t)$$

$$y'(t) = 2x(t) + 0 \cdot y(t)$$

$$x'(t) = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix} x(t)$$

$$x(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\det \begin{bmatrix} 3-\lambda & -1 \\ 2 & -\lambda \end{bmatrix}$$

$$(3-\lambda)(-\lambda) - (-2) = 0$$

$$-3\lambda + \lambda^2 + 2 = 0$$

$$\lambda = 1, 2$$

$$\lambda = 2, \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$a - b = 0$$

$$a = b$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}_{\lambda=2}$$

$$\begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2a - b = 0$$

$$2a = b$$

$$\star \begin{bmatrix} 1 \\ 2 \end{bmatrix}_{\lambda=1}$$

$$X(0) = A \begin{bmatrix} 1 \\ 1 \end{bmatrix} + B \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = A \begin{bmatrix} 1 \\ 1 \end{bmatrix} + B \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$A + B = 2, \quad A + 2B = 3$$

$$A = 1, \quad B = 1$$

$$X(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^t$$

$$X(t) = \begin{bmatrix} e^{2t} + e^t \\ e^{2t} + 2e^t \end{bmatrix}$$

2) 18500 4050

$$x'(t) = x(t) + 8y(t)$$

$$y'(t) = 5x(t)$$

$$y''(t) = 5x'(t)$$

$$\frac{1}{5} y''(t) = x(t) + 8y(t)$$

$$\frac{1}{5} r^2 - 8 = 0$$

$$A e^{2\sqrt{10}t} + B e^{-2\sqrt{10}t}$$

$$x(0) = 0$$

$$y(0) = 5$$

$$x'(t) = \begin{bmatrix} 1 & 8 \\ 5 & 0 \end{bmatrix}$$

$$x(0) = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$\lambda = \frac{1 \pm \sqrt{161}}{2}$$

$$V_1 = \begin{bmatrix} 1 + \sqrt{161} \\ 10 \end{bmatrix}, \quad V_2 = \begin{bmatrix} 1 - \sqrt{161} \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 5 \end{bmatrix} = A \begin{bmatrix} 1 + \sqrt{161} \\ 10 \end{bmatrix} + B \begin{bmatrix} 1 - \sqrt{161} \\ 10 \end{bmatrix}$$

$$A = \frac{\sqrt{161} - 1}{4\sqrt{161}}, \quad B = \frac{161 + \sqrt{161}}{644}$$

$$x(t) = \frac{\sqrt{161} - 1}{4\sqrt{161}} \begin{bmatrix} 1 + \sqrt{161} \\ 10 \end{bmatrix} e^{\frac{1 + \sqrt{161}}{2}t} + \frac{161 + \sqrt{161}}{644} \begin{bmatrix} 1 - \sqrt{161} \\ 10 \end{bmatrix} e^{\frac{1 - \sqrt{161}}{2}t}$$

$$3) \quad x'(t) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$x(0) = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$\lambda_1 = .55, \begin{bmatrix} .55 \\ -1.25 \\ 1 \end{bmatrix}$$

$$\lambda_2 = -.8, \begin{bmatrix} -.8 \\ .44 \\ 1 \end{bmatrix}$$

$$\lambda_3 = 2.25, \begin{bmatrix} 2.25 \\ 1.8 \\ 1 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = A \begin{bmatrix} .55 \\ -1.25 \\ 1 \end{bmatrix} + B \begin{bmatrix} -.8 \\ .44 \\ 1 \end{bmatrix} + C \begin{bmatrix} 2.25 \\ 1.8 \\ 1 \end{bmatrix}$$