> *#OK to post homework*
*#Shreya Ghosh, 11-01-2021, Assignment 16*

> **read** "/Users/shreyaghosh/Documents/M15.txt"

> *Help15*( )

*HW3(u,v,w), HW2(u,v) , Dis1(F,y,y0,h,A), ToSys(k,z,f,INI)* **(1)**

> *#1a.*
$$Orbk\left(2, z, z[1]\cdot\left(\frac{5}{3} - z[2]\right), [0.1, 0.1], 1000, 1010\right)$$

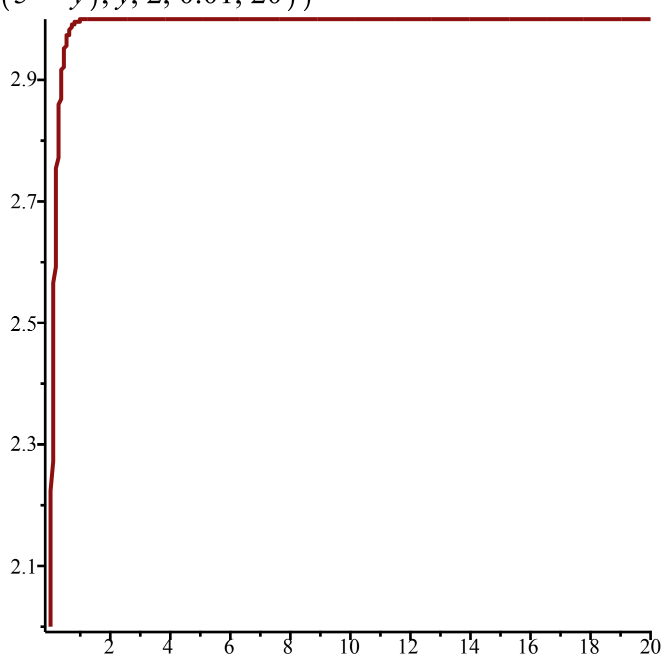
[0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667, 0.6666666667] **(2)**

> *#1b.*
$Orbk(2, z, z[1]\cdot(2 - z[2]), [0.1, 0.1], 1000, 1010)$

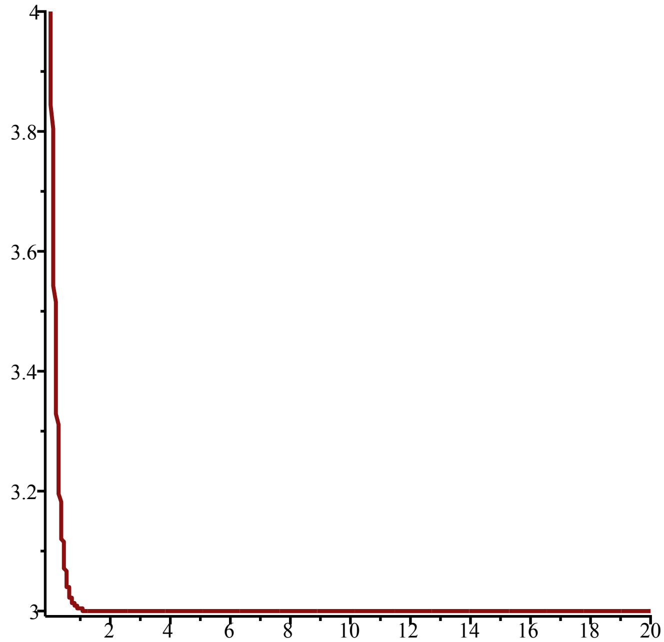
[1.041271875, 1.060759433, 1.016979902, 0.9551887798, 0.9389697679, 0.9810461487, 1.040919623, 1.060649058, 1.017247698, 0.9555525834, 0.9390715010] **(3)**

>

> *#4aiii.*
$plot(Dis1(y\cdot(3 - y)\cdot(5 - y), y, 2, 0.01, 20))$




> $plot(Dis1(y\cdot(3 - y)\cdot(5 - y), y, 4, 0.01, 20))$




> #4bii.

$diff\left(x^2 \cdot (3-x) \cdot (5-x) \cdot (7-x), x\right)$

$$2\,x\,(3-x)\,(5-x)\,(7-x) - x^2\,(5-x)\,(7-x) - x^2\,(3-x)\,(7-x) - x^2\,(3-x)\,(5-x) \quad \mathbf{(4)}$$

> $subs\left(x=0, 2\,x\,(3-x)\,(5-x)\,(7-x) - x^2\,(5-x)\,(7-x) - x^2\,(3-x)\,(7-x) - x^2\,(3-x)\,(5-x)\right)$

$$0 \quad \mathbf{(5)}$$

> $subs\left(x=3, 2\,x\,(3-x)\,(5-x)\,(7-x) - x^2\,(5-x)\,(7-x) - x^2\,(3-x)\,(7-x) - x^2\,(3-x)\,(5-x)\right)$

$$-72 \quad \mathbf{(6)}$$

> $subs\left(x=5, 2\,x\,(3-x)\,(5-x)\,(7-x) - x^2\,(5-x)\,(7-x) - x^2\,(3-x)\,(7-x) - x^2\,(3-x)\,(5-x)\right)$

$$100 \quad \mathbf{(7)}$$

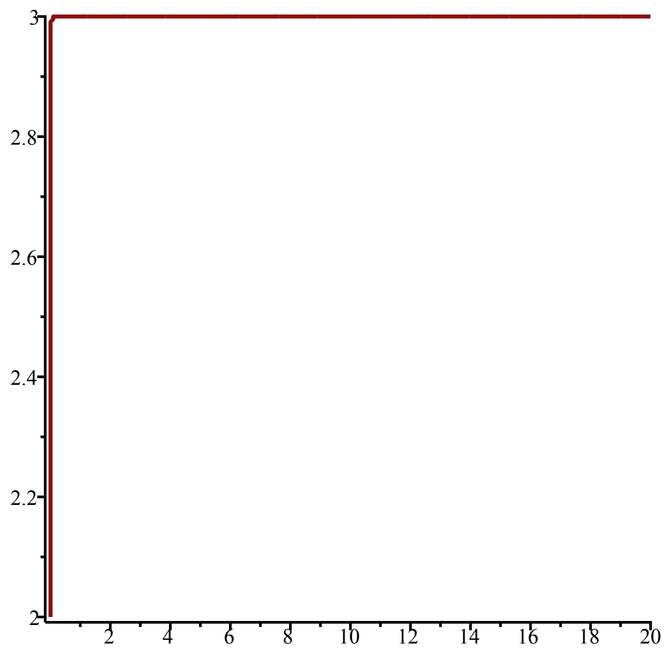
> $subs\left(x=7, 2\,x\,(3-x)\,(5-x)\,(7-x) - x^2\,(5-x)\,(7-x) - x^2\,(3-x)\,(7-x) - x^2\,(3-x)\,(5-x)\right)$

$$-392 \quad \mathbf{(8)}$$

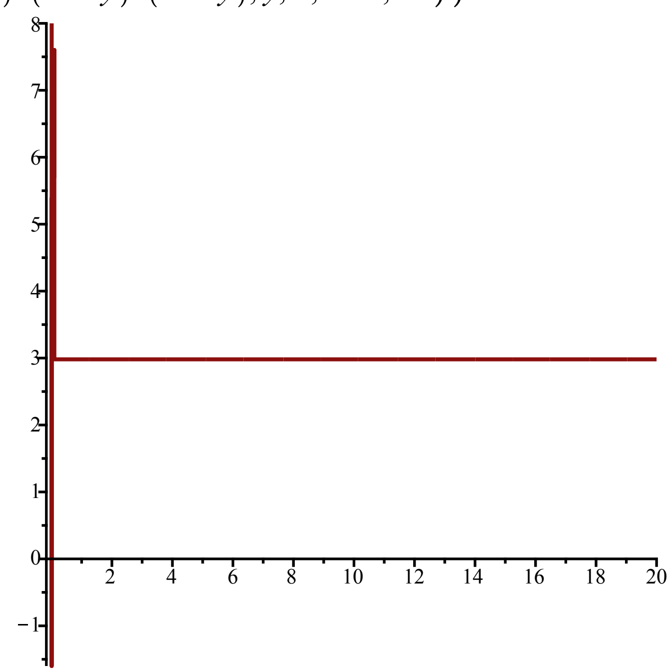
> #3, 7 are stable fixed points

> #4biii.

$plot\left(Dis1\left(y^2 \cdot (3-y) \cdot (5-y) \cdot (7-y), y, 2, 0.01, 20\right)\right)$




> $plot\left(Dis1\left(y^2\cdot(3-y)\cdot(5-y)\cdot(7-y), y, 8, 0.01, 20\right)\right)$




>

Shreya Ghosh

OK to post homework

HW 16

1.a) $x(n) = x(n-1)\left(\frac{5}{3} - x(n-2)\right)$

i) $x(n) \Rightarrow x_1(n) \quad x_2(n) = x_1(n-1)$

$x_1(n) = x_1(n-1)\left(\frac{5}{3} - x_2(n-1)\right), \; x_2(n) = x_1(n-1)$

ii) $z = z\left(\frac{5}{3} - z\right)$

$0 = z\left(\frac{5}{3} - z - 1\right)$

$0 = z\left(\frac{2}{3} - z\right)$

$z = 0, \frac{2}{3}$

iii) $(z_1, z_2) \longrightarrow \left(z_1\left(\frac{5}{3} - z_2\right), z_1\right)$

$$J = \begin{pmatrix} \frac{5}{3} - z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$$

$$J(0,0) = \begin{pmatrix} \frac{5}{3} & 0 \\ 1 & 0 \end{pmatrix}$$

$\left(\frac{5}{3} - \lambda\right)(-\lambda) - 0 = 0$

$\lambda^2 - \frac{5}{3}\lambda = 0$

$\lambda\left(\lambda - \frac{5}{3}\right) = 0 \Rightarrow \lambda = 0, \frac{5}{3} \Rightarrow$ not stable because both are not $< 1$ in absolute value

$$J\left(\tfrac{2}{3}, \tfrac{2}{3}\right) = \begin{pmatrix} 1 & -\frac{2}{3} \\ 1 & 0 \end{pmatrix}$$

$(1 - \lambda)(-\lambda) + \frac{2}{3} = 0$

$\lambda^2 - \lambda + \frac{2}{3} = 0$

$\lambda = \frac{1 \pm \sqrt{1 - 4\left(\frac{2}{3}\right)}}{2} = \frac{1 \pm \sqrt{1 - \frac{8}{3}}}{2} = \frac{1 \pm \sqrt{-\frac{5}{3}}}{2} = \frac{1}{2} \pm \frac{\sqrt{\frac{5}{3}}}{2} i \Rightarrow \sqrt{\frac{1}{4} + \frac{5}{12}} = \sqrt{\frac{7}{12}} \Rightarrow$ stable because $\sqrt{\frac{7}{12}} < 1$

1\. b) i) $x(n) = x(n-1)(2 - x(n-2))$

$x_1(n) = x(n), \quad x_2(n) = x_1(n-1)$

$x_1(n) = x_1(n-1)(2 - x_2(n-1))$

ii) $z = z(2-z)$

$0 = z(2-z-1)$

$0 = z(1-z)$

$z = 0, 1$

iii) $(z_1, z_2) \longrightarrow (z_1(2-z_2), z_1)$

$$J = \begin{pmatrix} 2-z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$$

$$J(0,0) = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix}$$

$(2-\lambda)(-\lambda) = 0$

$\lambda^2 - 2\lambda = 0$

$\lambda(\lambda - 2) = 0 \Rightarrow \lambda = 0, 2 \Rightarrow$ not stable because both are not $< 1$ in absolute value

$$J(1,1) = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$$

$(1-\lambda)(-\lambda) + 1 = 0$

$\lambda^2 - \lambda + 1 = 0$

$\lambda = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i \Rightarrow \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1 \Rightarrow$ not stable because not $< 1$ in absolute value

2. $x(n) = x(n-1)(a - x(n-2))$

$x_1(n) = x_1(n-1)(a - x_2(n-1))$

$z = z(a-z)$

$0 = z(a-z-1) = z(a-1-z)$

$z = a-1,\ 0$

$(z_1, z_2) \to (z_1(a-z_2), z_1)$

$$J = \begin{pmatrix} a-z_2 & -z_1 \\ 1 & 0 \end{pmatrix}$$

$$J(0,0) = \begin{pmatrix} a & 0 \\ 1 & 0 \end{pmatrix}$$

$(a-\lambda)(-\lambda) = 0$

$\lambda^2 - a\lambda = 0$

$\lambda(\lambda - a) = 0$

$-1 < \lambda - a < 1 \Rightarrow -1-\lambda < -a < 1-\lambda \Rightarrow \lambda - 1 < a < \lambda + 1$

$$J(a-1, a-1) = \begin{pmatrix} a-(a-1) & -(a-1) \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1-a \\ 1 & 0 \end{pmatrix}$$

$(1-\lambda)(-\lambda) - (1-a) = 0$

$\lambda^2 - \lambda - 1 + a = 0$

$\lambda = \frac{1 \pm \sqrt{1-4(-1+a)}}{2} = \frac{1 \pm \sqrt{5-4a}}{2}$

$-1 < \frac{1+\sqrt{5-4a}}{2} < 1$ $\qquad$ $-1 < \frac{1-\sqrt{5-4a}}{2} < 1$

$-2 < 1 + \sqrt{5-4a} < 2$ $\qquad$ $-2 < 1 - \sqrt{5-4a} < 2$

$-3 < \sqrt{5-4a} < 1$ $\qquad$ $-3 < -\sqrt{5-4a} < 1$

$1 < 5-4a < 9$ $\qquad$ $-1 < \sqrt{5-4a} < 3$

$-4 < -4a < 4$ $\qquad$ $1 < 5-4a < 9$

$-1 < a < 1$ $\qquad$ $-4 < -4a < 4$

$-1 < a < 1$

4. a) $x'(t) = x(t)(3 - x(t))(5 - x(t))$

$F(x) = x(3 - x)(5 - x)$

i) $0 = x(3-x)(5-x) \Rightarrow x = 0, 3, 5$

ii) $F(x) = (3x - x^2)(5-x)$

$= 15x - 3x^2 - 5x^2 + x^3$

$= x^3 - 8x^2 + 15x$

$F'(x) = 3x^2 - 16x + 15$

$F'(0) = 15 \Rightarrow > 0$ so unstable

$F'(3) = -6 \Rightarrow < 0$ so stable

$F'(5) = 10 \Rightarrow > 0$ so unstable

b) i) $x'(t) = x(t)^2(3 - x(t))(5 - x(t))(7 - x(t))$

$F(x) = x^2(3 - x)(5 - x)(7 - x)$

$0 = x^2(3-x)(5-x)(7-x) \Rightarrow x = 0, 3, 5, 7$

ii) $F'(x) = -5x^4 - 60x^3 - 213x^2 + 210x$

$F'(0) = 0 \Rightarrow$ unstable

$F'(3) = -72 \Rightarrow$ stable

$F'(5) = 100 \Rightarrow$ unstable

$F'(7) = -342 \Rightarrow$ stable