HW 16

OK TO POST

1) a) $x(n) = x(n-1)\left(5/3 - x(n-2)\right)$

i) $x_1(n) = x_1(n-1)\left(5/3 - x_2(n-1)\right)$, $x_2(n) = x_1(n-1)$

ii) $z = z\left(5/3 - z\right)$

$z = 0 \quad z = 2/3$

iii) $(0,0)$, $(2/3, 2/3)$ eq pts

$(z_1, z_2) \to \left(z_1\left(5/3 - z_2\right), z_1\right)$

$f(z_1, z_2) = \left(z_1(5/3 - z_2)\right) \quad g(z_1, z_2) = z_1$

$$J = \begin{vmatrix} 5/3 - z_2 & -z_1 \\ 1 & 0 \end{vmatrix}$$

$$J(0,0) = \begin{vmatrix} 5/3 & 0 \\ 1 & 0 \end{vmatrix}$$

$z = 5/3, 0$ unstable

$$J(2/3, 2/3) = \begin{vmatrix} 5/3 - 2/3 & -2/3 \\ 1 & 0 \end{vmatrix}$$

$z = \frac{1}{2} + i\frac{\sqrt{15}}{6}, \quad \frac{1}{2} - i\frac{\sqrt{15}}{6}$

unstable

b) i) $x(n) = x(n-1)(2 - x(n-2))$

$x_1(n) = x_1(n-1)(2 - x_2(n-1))$, $x_2(n) = x_1(n-1)$

ii) $z = z(2-z)$

$z = 0, 1$

iii) $(0,0)$, $(1,1)$

$(z_1, z_2) \to (z_1(2 - z_2), z_1)$

$$J = \begin{vmatrix} 2 - z_2 & -z_1 \\ 1 & 0 \end{vmatrix}$$

$$J(0,0) = \begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix}$$ $z = 2, 0$

Unstable

$$J(1,1) = \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix}$$ $z = \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

Stable

2) $x(n) = x(n-1)(a - x(n-2))$

$z = z(a - z)$

$\boxed{z = 0, \quad z = a - 1}$ eq pts

$$J = \begin{vmatrix} a - z_2 & -z_1 \\ 1 & 0 \end{vmatrix}$$

$$J(0,0) = \begin{vmatrix} a & 0 \\ 1 & 0 \end{vmatrix} \qquad \boxed{z = a, 0}$$

$|a| < 1$ in order for $(0,0)$ to be stable

$$J(a-1) = \begin{vmatrix} a - (a-1) & a-1 \\ 1 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 1 & -(a-1) \\ 1 & 0 \end{vmatrix}$$

in order to be stable abs of Eigenvalues must be less than 1

4) a) $x'(t) = x(t)(3-x(t))(5-x(t))$

i) $x(3-x)(5-x)$

$x = 0, 3, 5$ eq pts

ii) $F'(x) = 3x^2 - 16x + 15$

$F'(0) = 0 - 0 + 15$ unstable

$F'(3) = 27 - 48 + 15 = -6$ stable

$F'(5) = 75 - 80 + 15 = 10$ unstable

iii) a) plot(DisI(z·(3-z)(5-z), z, 0.01, 0.01, 20)

b) plot(DisI(z·(3-z)(5-z), z, 0.01, 3.01, 20)

c) plot(DisI(z·(3-z)(5-z), z, 0.01, 5.01, 20)

a) $x = 0$ is unstable because we see a horizontal asymtote at 3

b) $x = 3$ is stable by the graph

c) $x = 5$ is unstable by the graph

b) $x'(t) = x(t)^2(3-x(t))(5-x(t))(7-x(t))$

i) $0 = x^2(3-x)(5-x)(7-x)$

$\boxed{x = 0, 3, 5, 7}$

ii) $F'(x) = -5x^4 + 60x^3 - 213x^2 + 210x$

$F'(0) = 0$ so unstable

$F'(3) = -405 + 1620 - 1927 + 630$

$= -82$ stable

$F'(5) = -3125 + 7500 - 5325 + 1050$

$= 100$ unstable

$F'(7) = -12005 + 20580 - 10437 + 1470$

$= -392$ stable

iii) plot(Dis1($x^2(3-x)(5-x)(7-x)$, x, 0.01, 0.01, 20))

plot(Dis1($x^2(3-x)(5-x)(7-x)$, x, 0.01, 3.01, 20))

plot(Dis1($x^2(3-x)(5-x)(7-x)$, x, 0.01, 5.01, 20))

plot(Dis1($x^2(3-x)(5-x)(7-x)$, x, 0.01, 7.01, 20))

x=0, horizontal asymtote at 3

so unstable

x=3, stable as graph goes neg to

x=5, unstable

x=7, stable